Review of Recent Literature on Static Analyses of Composite Shells: 2000-2010

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Received April 19th, 2012; revised May 15th, 2012; accepted May 31st, 2012

ABSTRACT

Laminated composite shells are frequently used in various engineering applications including aerospace, mechanical, marine, and automotive engineering. This article reviews the recent literature on the static analysis of composite shells. It follows up with the previous work published by the first author [1-4] and it is a continuation of another recent article that focused on the dynamics of composite shells [3]. This paper reviews most of the research done in recent years (2000-2010) on the static and buckling behavior (including postbuckling) of composite shells. This review is conducted with an emphasis on the analysis performed (static, buckling, postbuckling, and others), complicating effects in both material (e.g. piezoelectric) and structure (e.g. stiffened shells), and the various shell geometries (cylindrical, conical, spherical and others). Attention is also given to the theory being applied (thin, thick, 3D, nonlinear…). However, more details regarding the theories have been described in previous work [1,3].

Keywords: Review; Composite; Static Analysis

1. Introduction

The use of laminated composite shells in many engineering applications has been expanding rapidly in the past few decades due to their higher strength and stiffness to weight ratios when compared to most metallic materials. Composite shells now constitute a large percentage of recent aerospace or submarine structures. They are used increasingly in areas such as automotive engineering, biomedical engineering and other applications.

Literature on composite shell research can be found in many national and international conferences and journals. A recent article [3] focused on the recent research done on the dynamic behavior of composite shells wherein problems of free vibration, shock, wave propagation, dynamic stability, damping and viscoplastic behavior related to laminated shells are discussed. Several review articles on the subject, such as Qatu [2,4], Kapania [5], Noor and Burton [6,7], Noor et al. [8], and Soldatos [9] covered much of the research done in past decades. Computational aspects of the research were covered by Noor and Burton [6,7], Noor et al. [8,10] and Noor and Venneri [11]. Carrera [12] presented a historical review of zigzag theories for multilayered plates and shells. He also reviewed the theories and finite elements for multilayered, anisotropic, composite plates and shells [13]. Among the recent books on the subject are those by Reddy [14], Ye [15], Lee [16], and Shen [17].

Present article reviews only recent research (2000 through 2010) done on the static and buckling analyses of composite shells. It includes stress, deformation, buckling and post buckling analyses under mechanical, thermal, hygrothermal or electrical loading. Since there are extensive papers on experimental and optimization studies in literature, those topics have not been discussed in this review separately. However, papers in those topics based on their obtained results are classified in the topics of this review.

This article classifies research based upon the typically used shell theories. These include thin (or classical) and thick shell theories (including shear deformation and three dimensional theories), shallow and deep theories, linear and nonlinear theories, and others. Most theories are classified based on the thickness ratio of the shell being treated (defined as the ratio of the thickness of the shell to the shortest of the span lengths and/or radii of curvature), its shallowness ratio (defined as the ratio of the shortest span length to one of the radii of curvature) and the magnitude of deformation (compared mainly to its thickness). Fundamental equations are listed for the types of shells used by most researchers in other publications [1-4].

The literature is reviewed while focusing on various
aspects of research. Focus will first be placed on the various shell geometries that are receiving attention in recent years. Among classical shell geometries are the cylindrical, spherical, conical shells and other shells of revolution; other shells like shallow shells are also included in this review. Stress and deformation analyses, in which various boundary conditions and/or shell geometries are considered, buckling and post-buckling problems, and finally research dealing with thermal and/or hygrothermal environments will be reviewed. The third aspect of research will focus on material-related complexities, which include piezoelectric or other complex materials. Structural-related complexities will be the final category that will be addressed. This will include stiffened shells, shells with cut-outs, shells with imperfections or other complexities.

2. Shell Theories

Shells are three dimensional bodies bounded by two, relatively close, curved surfaces. The three dimensional equations of elasticity are complicated when written in curvilinear, or shell, coordinates. Researchers simplify such shell equations by making certain assumptions for particular applications. Almost all shell theories (thin and thick, deep and shallow …) reduce the three-dimensional (3D) elasticity problem into a two dimensional (2D) problem. The accuracy of thin and thick shell theories is established when their results are compared to those of 3D theory of elasticity.

2.1. Three Dimensional Elasticity Theory

A shell is a three dimensional body confined by two parallel (unless the thickness is varying) surfaces. In general, the distance between those surfaces is small compared with other shell parameters. In this section, the equations from the theory of 3D elasticity in curvilinear coordinates are presented. The literature regarding Mechanics of laminated shells using 3D elasticity theory will then be reviewed.

Consider a shell element of thickness $h$, radii of curvature $R_\alpha$ and $R_\beta$ (a radius of twist $R_{\alpha\beta}$ is not shown here) (Figure 1). Assume that the deformation of the shell is small compared to the shell dimensions. This assumption allows us to neglect nonlinear terms in the subsequent derivation. It will also allow us to refer the analysis to the original configuration of the shell. The strain displacement relations can be written as [1]

$$
\varepsilon_\alpha = \frac{1}{(1+z/R_\alpha)} \left( \frac{1}{A} \frac{\partial u}{\partial \alpha} + \frac{v}{AB} \frac{\partial A}{\partial \beta} + \frac{w}{R_{\alpha\beta}} \right)
$$

$$
\varepsilon_\beta = \frac{1}{(1+z/R_\beta)} \left( \frac{1}{A} \frac{\partial v}{\partial \beta} + \frac{u}{AB} \frac{\partial B}{\partial \alpha} + \frac{w}{R_{\alpha\beta}} \right)
$$

The laminated composite shells are assumed to be composed of plies of unidirectional long fibers embedded in a matrix material. On a macroscopic level, each layer may be regarded as being homogeneous and orthotropic. However, the fibers of a typical layer may not be parallel to the coordinates in which the shell equations are expressed. The stress-strain relationship for a typical nth lamina in a laminated composite shell made of N laminae as shown in Figure 2 is given by Equation (2) [1].

$$
\begin{bmatrix}
\sigma_{\alpha} \\
\sigma_{\beta} \\
\sigma_{z} \\
\sigma_{\alpha\beta} \\
\sigma_{\alpha z} \\
\sigma_{\alpha\alpha} \\
\sigma_{\beta\beta} \\
\sigma_{zz} \\
\sigma_{\alpha\beta} \\
\sigma_{\alpha z} \\
\sigma_{\alpha\alpha} \\
\sigma_{\beta\beta} \\
\sigma_{zz}
\end{bmatrix}
= 
\begin{bmatrix}
\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} & 0 & 0 & \bar{Q}_{1n} \\
\bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{23} & 0 & 0 & \bar{Q}_{2n} \\
\bar{Q}_{13} & \bar{Q}_{23} & \bar{Q}_{33} & 0 & 0 & \bar{Q}_{3n} \\
0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} & 0 \\
0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} & 0 \\
0 & 0 & 0 & \bar{Q}_{56} & \bar{Q}_{66} & 0 \\
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{\alpha} \\
\varepsilon_{\beta} \\
\varepsilon_{z} \\
\gamma_{\alpha\beta} \\
\gamma_{\alpha z} \\
\gamma_{\alpha\alpha} \\
\gamma_{\beta\beta} \\
\gamma_{zz}
\end{bmatrix}
$$

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The positive notations of the stresses are shown in Figure 1.

In order to develop a consistent set of equations, the boundary conditions and the equilibrium equations will be derived using the principle of virtual work, which yields the following equilibrium equations:

\[
\begin{align*}
\frac{\partial (B \sigma_{\alpha z})}{\partial \alpha} + \frac{\partial (A \sigma_{\alpha y})}{\partial \beta} + \frac{\partial (AB \sigma_{\alpha z})}{\partial z} \\
+ \sigma_{\alpha y} B \sigma_{\alpha z} + \sigma_{\alpha z} B \sigma_{\alpha z} - \sigma_{\beta y} B \sigma_{\alpha z} + ABq_{\alpha} = 0 \\
\frac{\partial (B \sigma_{\alpha y})}{\partial \alpha} + \frac{\partial (A \sigma_{\beta z})}{\partial \beta} + \frac{\partial (AB \sigma_{\beta z})}{\partial z} \\
+ \sigma_{\alpha y} A \sigma_{\beta z} + \sigma_{\alpha z} A \sigma_{\beta z} - \sigma_{\beta y} A \sigma_{\beta z} + ABq_{\beta} = 0 \\
\frac{\partial (B \sigma_{\alpha z})}{\partial \alpha} + \frac{\partial (A \sigma_{\beta y})}{\partial \beta} + \frac{\partial (AB \sigma_{\beta z})}{\partial z} \\
- \sigma_{\alpha y} A \sigma_{\beta z} - \sigma_{\alpha z} A \sigma_{\beta z} - ABq_{\beta} = 0
\end{align*}
\]

The principle of virtual work will also yield boundary terms that are consistent with the other equations. The boundary terms for \( z = \text{constant} \) are:

\[
\begin{align*}
\sigma_{\alpha z} - \sigma_{\alpha z} = 0 & \quad \text{or} \quad w^\alpha = 0 \\
\sigma_{\alpha z} - \sigma_{\alpha z} = 0 & \quad \text{or} \quad u^\beta = 0 \\
\sigma_{\beta z} - \sigma_{\alpha z} = 0 & \quad \text{or} \quad v^\beta = 0
\end{align*}
\]

where \( \sigma_{\alpha z} \), \( \sigma_{\alpha z} \), and \( \sigma_{\beta z} \) are surface tractions and \( u^\beta \), \( v^\beta \), and \( w^\alpha \) are displacement functions at \( z = \text{constant} \). Similar results are obtained for the boundaries \( \alpha = \text{constant} \) and \( \beta = \text{constant} \). A three dimensional shell element has six surfaces. With three equations at each surface, a total of 18 equations can be obtained for a single-layered shell.

The above equations are valid for single-layered shells. To use 3D elasticity theory for multi-layered shells, each layer must be treated as an individual shell. Both displacements and stresses must be continuous between each layer (layer \( k \) to layer \( k + 1 \)) in a n-ply laminate to insure that there are no free internal surfaces (i.e., delamination) between the layers.

\[
\begin{align*}
u (\alpha, \beta, z = h_k / 2)_{\alpha = \text{const}} = u (\alpha, \beta, z = -h_k / 2)_{\alpha = \text{const}} \\
v (\alpha, \beta, z = h_k / 2)_{\beta = \text{const}} = v (\alpha, \beta, z = -h_k / 2)_{\beta = \text{const}} \\
w (\alpha, \beta, z = h_k / 2)_{z = \text{const}} = w (\alpha, \beta, z = -h_k / 2)_{z = \text{const}} \\
\sigma_{\alpha z} (\alpha, \beta, z = h_k / 2)_{k = \text{const}} = \sigma_{\alpha z} (\alpha, \beta, z = -h_k / 2)_{k = \text{const}} \\
\sigma_{\beta z} (\alpha, \beta, z = h_k / 2)_{k = \text{const}} = \sigma_{\beta z} (\alpha, \beta, z = -h_k / 2)_{k = \text{const}} \\
\end{align*}
\]

For \( k = 1, \ldots, N - 1 \).


2.2. Thick Shell Theory

Thick shells are defined as shells with a thickness smaller by at least one order of magnitude when compared with other shell parameters such as wavelength and/or radii of curvature (thickness is at least 1/10 of the smaller length
of the shell). The main differentiation between thick shell and thin shell theories is the inclusion of shear deformation and rotary inertia effects. Theories that include shear deformation are referred to as thick shell theories or shear deformation theories.

Thick shell theories are typically based on either a displacement or stress approach. In the former, the midplane shell displacements are expanded in terms of shell thickness, which can be a first order expansion, referred to as first order shear deformation shell theory, and will lead to first-order shear deformation theories.

The 3D elasticity theory is reduced to a 2D theory using the assumption that the normal strains acting upon the plane parallel to the middle surface are negligible compared with other strain components. This assumption is generally valid except within the vicinity of a highly concentrated force (St. Venant’s principle). In other words, no stretching is assumed in the z-direction (i.e., . Assuming that normals to the midsurface strains remain straight during deformation but not normal, the displacements can be written as [1]

\[ u(\alpha, \beta, z) = u_0(\alpha, \beta) + z\psi_{a}(\alpha, \beta) \]
\[ v(\alpha, \beta, z) = v_0(\alpha, \beta) + z\psi_{\beta}(\alpha, \beta) \]
\[ w(\alpha, \beta, z) = w_0(\alpha, \beta) \]

where \( u_0, v_0 \) and \( w_0 \) are midsurface displacements of the shell and \( \psi_{a} \) and \( \psi_{\beta} \) are midsurface rotations. An alternative derivation can be made with the assumption \( \varepsilon_z = 0 \). The subscript (0) will refer to the middle surface in subsequent equations. The above equations describe a typical first-order shear deformation shell theory, and will constitute the only assumption made in this analysis when compared with the 3D theory of elasticity. As a result, strains are written as [1]

\[ \varepsilon_{\alpha} = \frac{1}{(1 + z/R_{a})} \left( \varepsilon_{0\alpha} + z\kappa_{a} \right) \]
\[ \varepsilon_{\beta} = \frac{1}{(1 + z/R_{a})} \left( \varepsilon_{0\beta} + z\kappa_{\beta} \right) \]
\[ \varepsilon_{\alpha\beta} = \frac{1}{(1 + z/R_{a})} \left( \varepsilon_{0\alpha\beta} + z\kappa_{\alpha\beta} \right) \]

\[ \varepsilon_{\alpha} = \frac{1}{(1 + z/R_{a})} \left( \varepsilon_{0\alpha} + z\kappa_{a} \right) \]
\[ \varepsilon_{\beta} = \frac{1}{(1 + z/R_{a})} \left( \varepsilon_{0\beta} + z\kappa_{\beta} \right) \]
\[ \varepsilon_{\alpha\beta} = \frac{1}{(1 + z/R_{a})} \left( \varepsilon_{0\alpha\beta} + z\kappa_{\alpha\beta} \right) \]

where the midsurface strains are:

\[ \varepsilon_{\alpha} = \frac{1}{A} \partial_{\alpha} u_0 + \frac{v_0}{A} \partial_{\alpha} A \frac{w_0}{R_{a}} \]
\[ \varepsilon_{0\alpha} = \frac{1}{B} \partial_{\alpha} v_0 + \frac{u_0}{B} \partial_{\alpha} B \frac{w_0}{R_{a}} \]
\[ \varepsilon_{\alpha\beta} = \frac{1}{A} \partial_{\alpha} \partial_{\beta} u_0 + \frac{w_0}{A} \partial_{\alpha} \partial_{\beta} R_{a} \]
\[ \varepsilon_{0\alpha\beta} = \frac{1}{B} \partial_{\alpha} \partial_{\beta} v_0 + \frac{w_0}{B} \partial_{\alpha} \partial_{\beta} R_{a} \]

and the curvature and twist changes are:

\[ \kappa_{a} = \frac{1}{A} \partial_{\alpha} \psi_{a} - \frac{\psi_{\beta}}{A} \partial_{\alpha} A \]
\[ \kappa_{\beta} = \frac{1}{B} \partial_{\beta} \psi_{\beta} - \frac{\psi_{a}}{B} \partial_{\beta} B \]
\[ \kappa_{\alpha\beta} = \frac{1}{A} \partial_{\alpha} \partial_{\beta} \psi_{a} - \frac{\psi_{\beta}}{A} \partial_{\alpha} \partial_{\beta} R_{a} \]

The force and moment resultants (Figures 3 and 4) are obtained by integrating the stresses over the shell thickness considering the \((1 + z/R)\) term that appears in the denominator of the stress resultant equations [5]. The stress resultant equations are:

\[ \begin{bmatrix} N_{\alpha} \\ N_{\beta} \\ N_{agg} \\ N_{agg} \\ M_{\alpha} \\ M_{\beta} \\ M_{agg} \end{bmatrix} \begin{bmatrix} A_{1} & A_{2} & A_{16} & A_{16} & A_{6} & A_{6} & B_{11} & B_{16} & B_{16} \\ A_{16} & A_{22} & A_{22} & A_{22} & A_{26} & A_{26} & B_{22} & B_{26} & B_{26} \\ A_{6} & A_{6} & A_{6} & A_{6} & A_{6} & A_{6} & B_{6} & B_{6} & B_{6} \\ B_{11} & B_{11} & B_{11} & B_{11} & B_{11} & B_{11} & D_{11} & D_{11} & D_{11} \\ B_{16} & B_{16} & B_{16} & B_{16} & B_{16} & B_{16} & D_{16} & D_{16} & D_{16} \\ B_{22} & B_{22} & B_{22} & B_{22} & B_{22} & B_{22} & D_{22} & D_{22} & D_{22} \\ B_{26} & B_{26} & B_{26} & B_{26} & B_{26} & B_{26} & D_{26} & D_{26} & D_{26} \\ B_{6} & B_{6} & B_{6} & B_{6} & B_{6} & B_{6} & D_{6} & D_{6} & D_{6} \\ B_{16} & B_{16} & B_{16} & B_{16} & B_{16} & B_{16} & D_{16} & D_{16} & D_{16} \end{bmatrix} \begin{bmatrix} \varepsilon_{\alpha} \\ \varepsilon_{\beta} \\ \varepsilon_{agg} \\ \varepsilon_{agg} \\ \kappa_{a} \\ \kappa_{\beta} \\ \kappa_{agg} \end{bmatrix} \]

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\[
\begin{bmatrix}
Q_{\alpha} \\
Q_{\beta} \\
P_{\alpha} \\
P_{\beta}
\end{bmatrix}
= \begin{bmatrix}
\bar{A}_{33} & A_{33} & B_{33} & B_{33} \\
A_{33} & \bar{A}_{44} & B_{44} & B_{44} \\
B_{33} & B_{44} & \bar{D}_{33} & D_{33} \\
B_{33} & B_{44} & D_{33} & \bar{D}_{44}
\end{bmatrix}
\begin{bmatrix}
\gamma_{\alpha x} \\
\gamma_{\beta x} \\
\gamma_{\alpha y} \\
\gamma_{\beta y}
\end{bmatrix}
\]

(10)

where \( A_{ij}, B_{ij}, D_{ij}, \bar{A}_{ij}, \bar{B}_{ij}, \bar{D}_{ij}, \) and \( \bar{D}_{ij} \), are defined in [1].

It has been shown [1,5] that the above Equations (9) and (10) yield more accurate results when compared with those of plates and those traditionally used for shells [18].

The principle of virtual work can be used to derive the consistent equilibrium equations and boundary conditions. The equilibrium equations are [1-4]:

\[
\frac{\partial}{\partial \alpha} (BN_\alpha) + \frac{\partial}{\partial \beta} (AN_\beta) + \frac{\partial A}{\partial \alpha} N_{\alpha} + \frac{\partial B}{\partial \beta} N_{\beta} + \frac{AB}{R_\alpha} Q_\alpha + \frac{AB}{R_\beta} Q_\beta + AB q_\alpha = 0
\]

\[
\frac{\partial}{\partial \beta} (AN_\beta) + \frac{\partial}{\partial \alpha} (AN_\alpha) + \frac{\partial B}{\partial \alpha} N_{\alpha} + \frac{\partial A}{\partial \beta} N_{\beta} + \frac{AB}{R_\beta} Q_\beta + \frac{AB}{R_\alpha} Q_\alpha + AB q_\beta = 0
\]

\[
-AB \left( \frac{N_\alpha}{R_\alpha} + \frac{N_\beta}{R_\beta} + \frac{N_{\beta \alpha}}{R_{\beta \alpha}} \right) + \frac{\partial}{\partial \alpha} (BQ_\alpha) + \frac{\partial}{\partial \beta} (AQ_\beta) + AB q_\beta = 0
\]

\[
\frac{\partial}{\partial \alpha} (BM_\alpha) + \frac{\partial}{\partial \beta} (AM_\beta) + \frac{\partial A}{\partial \alpha} M_{\alpha} - \frac{\partial B}{\partial \beta} M_{\beta} - \frac{AB}{R_\alpha} Q_\alpha + \frac{AB}{R_\beta} P_\beta + AB m_\alpha = 0
\]

\[
\frac{\partial}{\partial \beta} (AM_\beta) + \frac{\partial}{\partial \alpha} (BM_\alpha) + \frac{\partial B}{\partial \alpha} M_{\alpha} - \frac{\partial A}{\partial \beta} M_{\beta} - \frac{AB}{R_\beta} Q_\beta + \frac{AB}{R_\alpha} P_\alpha + AB m_\beta = 0
\]

(11)

The boundary terms for the boundaries with \( \alpha = \) constant are

\[
N_{\alpha a} - N_{\alpha} = 0 \quad \text{or} \quad u_{0} = 0
\]

\[
N_{\alpha \beta} - N_{\beta} = 0 \quad \text{or} \quad v_{0} = 0
\]

\[
Q_{\alpha a} - Q_{\alpha} = 0 \quad \text{or} \quad \psi_{0} = 0
\]

\[
M_{\alpha a} - M_{\alpha} = 0 \quad \text{or} \quad \sigma_{\alpha} = 0
\]

\[
M_{\alpha \beta} - M_{\beta} = 0 \quad \text{or} \quad \psi_{\beta} = 0
\]

(12)

Similar equations can be obtained for \( \beta = \) constant.

Equations (9) and (10) are significantly different from those that cover most of first order shear deformation theories (FSDTs) for shells which neglect the effect of \( z/R \) in the stress resultant equations. Asadi et al. [31] studied static and free vibration of composite shells using Equations (9) and (10) and compared their results with other FSDTs and 3D elasticity results. They showed that presented FSDT improves the prediction of displacements, force resultants and moment resultants significantly.

Shear deformation theories were used by many authors (e.g. Qatu [4]). Chaudhuri [32] presented a nonlinear zigzag theory for finite element analysis of shear-deformable laminated shells. Krejaa and Schmidt [33] studied large rotations in shear deformation finite element analysis of laminated shells. Non-linear buckling and postbuckling of a moderately thick anisotropic laminated cylindrical shell of finite length subjected to lateral pressure, hydrostatic pressure and external liquid pressure based on a higher order shear deformation shell theory with von Kármán-Donnell-type of kinematic non-linearity and including the extension/twist, extension/flexural and flexural/twist couplings were presented by Li and Lin [34] wherein the material property of each layer could be linearly elastic, anisotropic and fiber-reinforced. A mixed meshless computational method based on the Local Petrov-Galerkin approach for analysis of plate and shell structures was presented by Sorić and Jarak [35]. They overcame the undesired locking phe-
nominal and demonstrated that this meshless method is numerically more efficient than the available meshless fully displacement approaches. Shen [36,37] investigated postbuckling of shear deformable cross-ply laminated cylindrical shells under combined loading.

Piskunov et al. [38] were interested in a ratational higher order shear deformation theory of anisotropic laminated plates and shells. Iozzi and Gaudenzi [39] studied shear deformable shell elements for adaptive laminated structures. Han et al. [40] performed a geometrically nonlinear analysis of laminated composite thin shells using a modified first-order shear deformable element. Other studies that used a shear deformation shell theory include those of Li [41], Zenkour [42], Shen [43], Shen and Li [44], Balah and Al-Ghamedy [45], and Ferreira [46].

Zhen and Wanji [47] presented a higher order theory for multilayered shells and performed analysis on laminated cylindrical shell panels. Khare et al. [48] discussed closed-form thermo-mechanical solutions of higher-order theories of cross-ply laminated shallow shells. Khare and Rode [49] showed similar solutions for thick laminated sandwich shells. Ferreira et al. [50] modeled cross-ply laminated elastic shells by a higher-order theory. Aljani and Aghdam [51] presented a semi-analytical solution for stress analysis of moderately thick laminated cylindrical panels with various boundary conditions. Pinto Correia et al. [52] analyzed laminated conical shell structures for buckling using higher order models. Matsunaga [53] studied thermal buckling of cross-ply laminated composite shallow shells according to a higher order deformation theory. Oh and Cho [54] investigated a higher order zigzag theory for smart composite shells under mechanical thermo-electric loading. Yaghoubshahi et al. [55] and Asadi and Faribrz [56] employed general higher-order shear deformation theory and formulated it to analyze deep composite shells and plates with mixed boundary conditions. Benson et al. [57] presented a Reissner-Mindlin shell formulation based on a degenerated solid is implemented for NURBS-based isogeometric analysis. They constructed a user-defined element in LS-Dyna for industrial purposes to analyze elasto-plastic behavior of shells.

In general, layer-wise laminate theories are used to properly represent local effects, such as interlaminar stress distribution, delaminations, etc. These theories are typically employed for cases involving anisotropic materials in which transverse shear effects cannot be ignored. Recent studies include Yuan et al. [58] in which a stress projection, layer-wise-equivalent formulation was used for accurate predictions of transverse stresses in laminated plates and shells. Kim and Chaudhuri [59,60] and Chaudhuri and Kim [61] described a layer-wise linear displacement distribution theory and based their analysis on it to investigate the buckling and shear behavior of a long cross-ply cylindrical shell (ring). Leigh and Tafreshi [62] used layerwise shell finite element based on first order shear deformation theory to investigate delamination buckling of composite cylindrical shells. A static analysis of thick composite circular arches using a layerwise differential quadrature technique was performed by Malekzadeh [63]. Roh et al. [64,65] investigated the thermo-mechanical behavior of shape memory alloys using a finite element method based on layerwise theory. The theory of layerwise displacement field was used to perform a finite element analysis of aero-thermally buckled composite shells by Shin et al. [66]. The displacement field of a layerwise theory was also used to develop laminated beam theories by Tahani [67].

### 2.3. Thin Shell Theory

If the shell thickness is less than 1/20 of the other shell dimensions (e.g., length) and/or radii of curvature, a thin shell theory, where shear deformation and rotary inertia are negligible, is generally acceptable. Depending on various assumptions made during the derivation of the strain-displacement relations, stress-strain relations, and the equilibrium equations, various thin shell theories can be derived [5]. All these theories were initially derived for isotropic shells and expanded later for laminated composite shells by applying the appropriate integration through laminas, and stress-strain relations. For very thin shells, the shell is thin such that the ratio of the thickness compared to any of the shell’s radii or any other shell parameter, i.e., width or length, is negligible when compared to unity. Also, for thin shells, the normals to the middle surface remain straight and normal when the shell undergoes deformation. This assumption assures that certain parameters in the shell equations (including the $z/R$ term mentioned earlier in the thick shell theory) can be neglected. The shear deformation can be neglected in the kinematic equations allowing the in-plane displacement to vary linearly through the shell’s thickness as given by

$$
\varepsilon_{\alpha} = \varepsilon_{0\alpha} + 2K_{\alpha}, \quad \varepsilon_{\beta} = \varepsilon_{0\beta} + 2K_{\beta},
$$

$$
\gamma_{\alpha\beta} = \gamma_{0\alpha\beta} + 2T_{\alpha\beta},
$$

where the mid-surface strains, curvature and twist changes are

$$
\varepsilon_{0\alpha} = \frac{1}{A} \frac{\partial u_{0\alpha}}{\partial \alpha} + \frac{v_{0\beta}}{A} \frac{\partial A}{\partial \beta} + \frac{w_{0\alpha}}{R_{\alpha}},
$$

$$
\varepsilon_{0\beta} = \frac{1}{B} \frac{\partial v_{0\beta}}{\partial \beta} + \frac{u_{0\alpha}}{B} \frac{\partial B}{\partial \alpha} + \frac{w_{0\beta}}{R_{\beta}},
$$

$$
\gamma_{0\alpha\beta} = \frac{1}{A} \frac{\partial u_{0\beta}}{\partial \alpha} + \frac{1}{B} \frac{\partial v_{0\alpha}}{\partial \beta} - \frac{v_{0\beta}}{B} \frac{\partial B}{\partial \alpha} + \frac{2}{A} \frac{w_{0\alpha}}{R_{\alpha\beta}} - \frac{2}{B} \frac{w_{0\beta}}{R_{\alpha\beta}},
$$

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\[ \kappa_\alpha = \frac{1}{A} \frac{\partial \psi_\alpha}{\partial \alpha} + \frac{\psi_\beta}{AB} \frac{\partial A}{\partial \beta} + \frac{1}{B} \frac{\partial \psi_\alpha}{\partial \beta} + \frac{1}{AB} \frac{\partial B}{\partial \alpha} \]
\[ \kappa_\beta = \frac{1}{A} \frac{\partial \psi_\beta}{\partial \alpha} + \frac{\psi_\alpha}{AB} \frac{\partial A}{\partial \beta} + \frac{1}{B} \frac{\partial \psi_\beta}{\partial \beta} + \frac{1}{AB} \frac{\partial B}{\partial \alpha} \]
\[ \tau = \frac{1}{A} \frac{\partial \psi_\beta}{\partial \alpha} + \frac{\psi_\alpha}{AB} \frac{\partial A}{\partial \beta} + \frac{1}{B} \frac{\partial \psi_\alpha}{\partial \beta} + \frac{1}{AB} \frac{\partial B}{\partial \alpha} \]

and where

\[ \psi_\alpha = \frac{u}{R_\alpha} + \frac{v}{R_\alpha} - \frac{1}{A} \frac{\partial \psi}{\partial \alpha}, \]
\[ \psi_\beta = \frac{u}{R_\beta} + \frac{v}{R_\beta} - \frac{1}{A} \frac{\partial \psi}{\partial \beta}. \]

Applying Kirchhoff hypothesis of neglecting shear deformation and the assumption that \( \varepsilon_z \) is negligible, the stress-strain equations for an element of material in the kth lamina may be written as [1]

\[ \begin{bmatrix} \sigma_\alpha \\ \sigma_\beta \\ \sigma_{\alpha\beta} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & \gamma_{\alpha\beta} \end{bmatrix} \begin{bmatrix} \varepsilon_\alpha \\ \varepsilon_\beta \\ \gamma_{\alpha\beta} \end{bmatrix} \]

(15)

where \( \sigma_\alpha \) and \( \sigma_\beta \) are normal stress components, \( \sigma_{\alpha\beta} \) is the in-plane shear stress component [1], \( \varepsilon_\alpha \) and \( \varepsilon_\beta \) are normal strains, and \( \gamma_{\alpha\beta} \) is the in-plane engineering shear strain. The terms \( Q_{ij} \) are the elastic stiffness coefficients for the material. If the shell coordinates (\( \alpha, \beta \)) are parallel or perpendicular to the fibers, then the terms \( Q_{16} \) and \( Q_{26} \) are both zero. Stresses over the shell thickness (h) are integrated to get the force and moment resultants as given by

\[ \begin{bmatrix} N_{\alpha} \\ N_{\beta} \\ N_{\alpha\beta} \\ M_{\alpha} \\ M_{\beta} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \end{bmatrix} \begin{bmatrix} \varepsilon_\alpha \\ \varepsilon_\beta \\ \gamma_{\alpha\beta} \\ k_\alpha \end{bmatrix} \]

(16)

where \( A_{ij}, B_{ij}, \) and \( D_{ij} \) are the stiffness coefficients arising from the piecewise integration over the shell thickness (Equation (14b)). For shells which are laminated symmetrically with respect to their midsurfaces, all the \( B_{ij} \) terms become zero. Note that the above equations are the same as those for laminated plates, which are also valid for thin laminated shells. Using principle of virtual work yields the following equilibrium equations.

\[ \frac{\partial}{\partial \alpha} \left( BN_\alpha \right) + \frac{\partial}{\partial \beta} \left( AN_{\beta} \right) + \frac{\partial A}{\partial \alpha} N_{\alpha\beta} + \frac{\partial B}{\partial \alpha} N_{\beta} + \frac{AB}{R_\alpha} Q_\alpha + \frac{AB}{R_\alpha} Q_\beta + AB q_\alpha = 0 \]

\[ \frac{\partial}{\partial \beta} \left( AN_{\beta} \right) + \frac{\partial}{\partial \alpha} \left( AN_{\alpha} \right) + \frac{\partial A}{\partial \beta} N_{\alpha\beta} - \frac{\partial A}{\partial \beta} N_{\alpha} + \frac{AB}{R_\beta} Q_\beta + \frac{AB}{R_\beta} Q_\alpha + AB q_\beta = 0 \]

\[ -AB \left( N_{\alpha} \frac{R_\beta}{R_\alpha} + N_{\beta} \frac{R_\alpha}{R_\beta} + N_{\alpha\beta} \right) + \frac{\partial}{\partial \alpha} \left( Q_{\alpha} \right) + \frac{\partial}{\partial \beta} \left( Q_{\beta} \right) + AB q_\alpha = 0 \]

(17)

The following boundary conditions can be obtained for thin shells for \( \alpha = \) constant (similar equations can be obtained for \( \beta = \) constant).

\[ N_{\alpha} \beta \| = 0 \text{ or } u_\alpha \| = 0 \]

\[ N_{\alpha\beta} \| - \frac{N_{\alpha\beta}}{R_\beta} = 0 \text{ or } v_\alpha \| = 0 \]

\[ Q_{\alpha\beta} \| - \frac{Q_{\alpha\beta}}{B} = 0 \text{ or } w_\alpha \| = 0 \]

\[ M_{\alpha\beta} \| = 0 \text{ or } \psi_\alpha \| = 0 \]

\[ M_{\alpha\beta} \| \frac{R_\alpha}{\beta} = 0 \]

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Buckling and post-buckling analysis of a laminated composite spherical shell panel embedded with shape memory alloy fibers using nonlinear finite element methods. Sze and Zheng [94] studied a hybrid-stress solid element for geometrically nonlinear laminated shell analyses. Andrade et al. [95] and Kima et al. [96] performed geometrically nonlinear analysis of laminated composite plates and shells using various shell elements. Huang [97] performed nonlinear buckling of composite shells of revolution. Ferreira et al. [98] conducted a nonlinear finite element analysis of rubber composite shells. Material nonlinearity was discussed by Khoroshun et al. [99,100].

Other nonlinear analyses include Chaudhuri [32], Khosravi et al. [72], Han et al. [40], Hsia [101], Wang et al. [102], Moitaa et al. [103], Jakomina et al. [80], Li and Lin [34], and Razzaq and El-Zafrany [104].

2.5. Shell Geometries

Shells may have different geometries based mainly on their curvature characteristics. In most shell geometries, the fundamental equations have to be treated at a very basic level. The equations are affected by the choice of the coordinate system, the characteristics of the Lame parameters and curvature [1-4]. Equations for cylindrical, spherical, conical and barrel shells can be derived from the equations of the more general case of shells of revolution. Equations for cylindrical, barrel, twisted and shallow shells can also be derived from the general equations of doubly curved shells. Cylindrical shells, doubly curved shallow shells, spherical and conical shells are the most treated geometries in research.


Shin et al. [115] investigated thermal post-buckled behaviors of cylindrical composite shells with viscoelastic damping treatments. Bhaskar and Balasubramanyam [116]
showed accurate analysis of end-loaded laminated orthotropic cylindrical shells. Merglyakov and Gatishin [117] performed analysis of the thermoelastoplastic non-axisymmetric laminated circular cylindrical shells. Weaver et al. [118] investigated anisotropic effects in the compression buckling of laminated cylindrical shells. Huang and Lu [119], Shen and Xiang [120] studied buckling and postbuckling of cylindrical shells under combined compression and torsion. Diaconu et al. [121] studied buckling characteristics and layup optimization of long laminated composite cylindrical shells subjected to combined loads. Fu and Yang [122] and Yang and Fu [123] described delamination growth for composite laminated cylindrical shells under external pressure. Shen [124] conducted a study on the hygrothermal effects on the postbuckling of laminated cylindrical shells. Wang and Dong [125] were interested in local buckling for triangular delaminations near the surface of laminated cylindrical shells under hygrothermal effects. Goldfeld and Ejgenberg [126] were interested in linear bifurcation analysis of laminated cylindrical shells. Shen [127,128] and Shen and Li [129] analyzed postbuckling of axially-loaded laminated cylindrical shells with piezoelectric actuators. Panda and Ramachandra [130] studied postbuckling analysis of cross-ply laminated cylindrical shell panels under parabolic mechanical edge loading. Rahman and Jansen [131] presented a finite element formulation of Koiter’s initial post-buckling theory using a multi-mode approach for coupled mode initial post-buckling analysis of a composite cylindrical shell.

Studies on buckling of cylindrical shells include Wang and Xiao [132], Shen [133-135], Wang et al. [136], Geier et al. [137], Weaver et al. [138], Wang and Dai [139], Zhu et al. [140], Patel et al. [141], Yang and Fu [142], Hilburger and Starnes [143], Semenyuk et al. [144], Tafreshi [145], Solaimurugan and Velmurugan [146], Semenyuk and Zhkova [147], Tafreshi [148], Weaver and Dickenson [149], Kere and Lyly [150], Vaziri [151], Semenyuk et al. [152], Tafreshi [153,154], Babich and Semenyuk [155], Biagi and Medico [156], Sheinman and Jabareen [157], Prabu et al. [77], Li and Lin [34], and De Faria [158].


Other analyses include those of Shen and Ye [18], Li and Shen [21,25,26], Shen [36,37], Li [41], Zenkour [42], Shen and Li [44], Zen and Wanji [47], Chaudhuri et al. [71], Sofiyev et al. [73], Patel et al. [90], Khoroshun et al. [99,100], Wang et al. [102], Zhu et al. [140], Seif et al. [170], Burgueño and Bhide [171], Belozerov and Kireev [172], Alibeigloo and Nouri [27], Semenyuk and Trach [173], Paris and Costello [174] and Movsumov and Shamiev [175]. As can be seen from the above review, cylindrical shells received the most attention (as compared with other shell geometries).

Khare et al. [48] presented closed-form thermo-mechanical solutions of cross-ply laminated shallow shells. Soldatos and Shu [70] discussed modeling of perfectly and weakly bonded laminated plates and shallow shells. Zang et al. [176] were interested in nonlinear dynamic buckling of laminated shallow spherical shells. Kioua and Mirza [177] investigated piezoelectric induced bending and twisting of laminated shallow shells. Niemi [178] developed a four-node bilinear shell element of arbitrary quadrilateral shape and applied that to find the solution of static and vibration problems of shallow shells. Zarinvay [179] researched the probability of the critical state of glue joints of a shallow laminated shell. Other studies on shallow shells include those of Grigorenko et al. [180] Matsunaga [53], Wang et al. [86], Ye and Zhou [112], Jakomina et al. [80], Xu et al. [92], Gupta [181], and Zhu et al. [140].

Conical shells are other special cases of shells of revolution. For these shells, a straight line revolves about an axis to generate the surface. Wu et al. [182] discussed a refined asymptotic theory of laminated circular conical shells. Das and Chakravorty [183] suggested selection guidelines of point-supported composite conoidal shell roofs based on a finite element analysis. Mahdi et al. [184] investigated the effect of material and geometry on crushing behavior of laminated conical shells. Goldfeld [185] studied the imperfection sensitivity of laminated conical shells. Goldfeld et al. [186] performed a multi-fidelity optimization of laminated conical shells for buckling. Mahdi et al. [187] were interested in the effect of residual stresses in a filament wound laminated conical shell. Singh and Babu [188] studied thermal buckling of
laminated piezoelectric conical shells. Wu and Chiu [189] picked up the problem of thermoelastic buckling of laminated conical shells. Rezadoust et al. [190] investigated the crush behavior of conical composite shells. Goldfeld et al. [191] presented design and optimization of laminated conical shells for buckling. Kosonen [192] described specification for mechanical analysis of conical composite shells. Other studies on conical include Patel et al. [90,193,194], and Pinto Correia [52].

Spherical shells are other special cases of shells of revolution. For these shells, a circular arc, rather than a straight line, revolves about an axis to generate the surface. If the circular arc is half a circle and the axis of rotation is the circle’s own diameter, a closed sphere will result. Smithmaitree and Tzou [195] discussed actions of actuator patches laminated on hemispherical shells. Marchuk and Khomyak [196] presented refined mixed finite element solutions of laminated spherical shells. He and Hwang [197] investigated identifying damage in spherical laminated shells. Kadoli and Ganesan [198] analyzed thermoelastic buckling of composite hemispherical shells with a cut-out at the apex. Saleh et al. [199] described crushing behavior of composite hemispherical shells subjected to axial compressive load. Other studies on spherical shells include those of Zang et al. [176], Wu and Lo [19], Xu et al. [92], Panda and Singh [93], and others.


Sai et al. [202,203] investigated shells with and without cut-outs. Other study includes Latifa and Sinha [204].

3. Types of Analyses

Analyses can be dynamic in nature. These include free and transient vibrations, wave propagation, dynamic stability, shock and impact loadings and others. These were covered in another review article [3]. The types of analyses that this work focuses on are static, buckling, post buckling, thermal and hygrothermal, and failure and damage.

3.1. Static Analysis


Other static analyses include Albieglloo and Nouri [27], Yuan et al. [58], Maksymyuk and Chernyshenko [84], Razzaq and El-Zafrany [104], Vasilenko et al. [109], Ye and Zhou [112], Tafreshi [145], Wang et al. [159], Albieglloo [164], Zenkour and Fares [166], Seif et al. [170], Semenyuk and Trach [173], Paris and Costello [174], Kioua and Mirza [177], Grigorenko et al. [180], Mahdi et al. [187], Marchuk and Khomyak [196], Saleh et al. [199], and Sai Ram and Sreedhar Batu [202,203].

3.2. Buckling Analysis


Studies on buckling of cylindrical shells include Wang and Xiao [132], Shen [133-135], Wang et al. [136], Geier et al. [137], Weaver et al. [138], Wang and Dai [139], Zhu et al. [140], Patel et al. [141], Yang and Fu [142], Hilburger and Starnes [143], Semenyuk [144], Tafreshi [145], Solaimurugan and Velmurugan [146], Semenyuk and, Zhukova [147], Tafreshi [148], Weaver and Dickenson [149], Kere and Lyly [150], Vaziri [151], Semenyuk et al. [152], Tafreshi [153,154], Babich and Semenyuk [155], Biagi and Medico [156], Sheinman and Jabareen [157], Prabu et al. [77], Li and Lin [34], and De Faria [158].

Other buckling analyses include Matsunaga [53], Shen [69], Sofiyev et al. [73], Wang et al. [86], Huang [97], Wang et al. [102], Weaver et al. [118], Huang and Lu [119], Shen and Xiang [120], Diaconu et al. [121], Wang and Dong [125], Hilburger and Starnes [143], Semenyuk.
et al. [144], Tafreshi [145], Zang et al. [176], Goldfeld et al. [186,191], Singh and Babu [188], Wu and Chiu [189], Kadoli and Ganesan [198], and Pinto Correiaa [205].

3.3. Postbuckling Analysis


Other studies on postbuckling analysis include Shen [36,37,43,69,124,127,128,133-135], Li and Shen [21,25,26], Li [41], Shen and Xiang [120], Shen and Li [44,129], Tafreshi [145,148], Semenyuk and Zhukova [147], Kere and Lyly [150], Sheinman and Jabareen [157], Patel et al. [193,194], Lee and Lee [217], Rahman and Jansen [131], Li and Lin [34], Panda and Ramachandra [130], and Sai Ram and Sreedhar [218].

3.4. Thermal and Hygrothermal Loading


Studies that treated thermal and/or hygrothermal effects include those of Li and Shen [21,25,26], Ruhi et al. [30], Li [41], Shen [43,44,69,124], Khare et al. [48], Matsunaga [53], Oh and Cho [54], Galishin and Shevchenko [85], Kundu et al. [87], Naidu and Sinha [88], Wang et al. [96,136], Merzlyakov and Galishin [111], Shin [115], Wang and Dong [125], Patel et al. [141,193,194,222], Shevchenko and Babeshko [213,214], Zenkour and Fares [166], Wang and Dai [139], Zhu et al. [140], Singh and Babu [188], Wu and Chiu [189], Kadoli and Ganesan [198], Panda and Singh [93], and in addition to articles that can be found on the dynamic problems in the review by Qatu [3].

3.5. Failure, Delamination and Damage Analyses


Other studies on failure of composite shells include those of Galishin [232], Xie and Biggers [233], He and Hwang [197], Khoroshun et al. [99,100], Khoroshun and Babich [108,114,163,245], Mahdi et al. [184], Rezadoust [190], Saleh et al. [199], Solaimurugan and Velmurugan [169], and Ghosh [238].

3.6. Other Analyses


4. Material Complexity

Material complexity in composites occurs in various ways. Composite shells can have active or piezoelectric layers. They can also be braided or made of wood or natural fibers or a combination of materials.

4.1. Piezoelectric Shells


Other studies on piezoelectric shells include Santos et al. [22], Nosier and Ruhi [29], Kioua and Mirza [177], Shen and Xiang [120], Shen [124,127], Alibeigloo [164], Alibeiglo and Nouri [27], Kulikov and Plotnikova [242], Singh and Babu [188], as well as others that dealt with dynamic response [3].

4.2. Other Materials


5. Structural Complexity

Structural complexity occurs when the geometry or boundary conditions of the shells deviate from the classical shells described earlier. These include stiffened shells, shells with internal boundaries from cracks, imperfect shells as well as other types of complexities.

5.1. Stiffened Shells

Ambur and Janunky [271] demonstrated a design optimization process while investigating the local buckling behavior of stiffened structures with variable curvature. Optimum design of stiffened cylindrical shells with added T-rings subjected to external pressure was also performed by Bushnell [272]. The reliability of a postbuckled composite isogrid stiffened shell structure subjected to a compression load was studied by Kim [273]. Zeng and Wu [274] performed a post-buckling analysis of stiffened braided cylindrical shells subjected to combined external pressure and axial compression loads. For the same combined loading, Poorveis and Kabir [275] analyzed the static buckling of orthotropic stringer stiffened composite cylindrical shells. The postbuckling behavior of stringer stiffened panels by using strip elements was determined by Mock and Reimerdes [276]. Bisagni and Cordisco [277,278] tested stiffened carbon composite stringer-stiffened shells in the postbuckling range until failure. Rao [279] and Rickards et al. [224] used finite elements for buckling and vibration analysis of laminated composite stiffened shells. Prusty [266] used the finite element method to perform a linear static analysis of composite hat-stiffened laminated shells. Bai et al. [280] performed a numerical analysis using a finite element method to investigate the buckling behavior of an advanced grid stiffened structure. Kidane et al. [281] developed an analytical model to study the global buckling load of grid stiffened composite cylinders. De Vries [282] used a hierarchical method to analyze localized buckling of thin-walled stiffened or unstiffened metallic and composite shells. Accardo et al. [283] discuss the design of a combined loads test machine and test fixture to perform experimental investigations on curved reinforced metallic and composite stiffened panels. Linde et al. [284] discussed the development of a virtual test platform used for parametric modeling and simulation of stiffened test shells to study the static behavior in the buckling and postbuckling range. Park et al. [207] and Patel et al. [285] used shell elements to perform both linear and dynamic analysis of laminated stiffened composite shells. An optimization design procedure based on surrogate modeling of stiffened composite shells was presented by Rikards et al. [286]. Using the finite element method, Wong and Teng [287] investigated the buckling behavior of axisymmetric stiffened composite shell structures and Apicella et al. [288] studied the behavior of a stiffened bulkhead subjected to ultimate pressure load. Chen and Guedes Soares [289] modeled ship hulls as stiffened composite panels to perform a strength analysis under sagging moments. Rais-Rohani and Lokits [290] conducted an optimization study to study reinforcement layout and sizing parameters of composite submarine sail structures. Wu et al. [291] conducted an experimental investigation to study the behavior of grid stiffened steel-concrete composite panels under a buckling load. Chen et al. [292] used a nonlinear finite ele-
ment method to study the thermal mechanical behavior of advanced composite grid stiffened shells with multi-delaminations. The finite element method was used by Chen and Xu [293] and by Prusty [294] to study the buckling and postbuckling response of doubly curved stiffened composite panels under general loading. Sahoo and Chakravorty [295] used finite elements to solve a bending problem of a composite stiffened hypar shell subjected to a concentrated load. Zhang et al. [296] and Lu et al. [297] performed a stability analysis of advanced composite grid stiffened shells. A buckling load analysis of composite grid stiffened structures was investigated by the finite element method by Tafreshi [153]. Xie and Balzani [250], and others on dynamic analysis [3].

5.2. Shells with Cutouts

Several recent studies have focused on various composite shell structures with cutouts. Hillburger and Starnes [143] and Hillburger [300] performed numerical and experimental studies to determine the effects of unreinforced and reinforced cutouts in composite cylindrical shells subjected to compression loading. Li et al. [301] performed a three-dimensional finite element analysis to study the buckling response of sandwich composite shells with cutouts under axial compression. The principle of minimum potential energy was used by Madenci and Barut [302] to investigate the effects of an elliptical cut-out in a composite cylindrical shell subjected to compression. Nanda and Bandyopadhyay [303] looked at the nonlinear transient responses from static and dynamic analyses of composite cylindrical and spherical shell laminates with cutouts. The finite element method was used to study the bending behavior of laminated composite shells without a cutout [202] and with a central circular cutout [203]. Buckling and post-buckling due to internal pressure and compression loading of composite shells with various size cutouts was investigated through the finite element method by Tafreshi [153]. Xie and Biggers [230] performed analysis on tailored laminated plates and shells with a central cutout subjected to compressive buckling loads. Other studies include Kadoli and Ganesan [198] and Hillburger and Starnes [143]. Asadi et al. [304] considered a layer containing several cavities and cracks and solved the problem under static point forces on the layer.

5.3. Imperfect Shells

Starnes and Hillburger [305] conducted an experimental and analytical study to investigate the effects of initial imperfections on the buckling response of graphite-epoxy cylindrical shells. Arboz and Hillburger [306] used a probability-based analysis to investigate section properties such as geometric imperfections to determine more accurate buckling-load “knockdown factors”. Biagi and Perugini [307] investigated the buckling behavior of the front composite skirt using linear and nonlinear finite element analysis to study the relationship between various shapes of geometrical imperfections and amplitudes and failure modes. Bisagni [308] studied the buckling and post-buckling characteristics of carbon composite cylindrical shells with geometric imperfections under axial compression using eigenvalue analysis. Carvellet al. [309, 310] performed a non-linear buckling analysis to study the geometric imperfections of composite shells in an underwater environment. Hillburger and Starnes [311, 312] investigated the effects of imperfections such as shell-wall thickness variations, imperfections due to composite fabrication, shell-end geometric imperfections, and nonuniformly applied end-loads, on the buckling and post-buckling response of un-stiffened thin-walled graphite-epoxy cylindrical shells. Jayachandran et al. [313] also investigated the postbuckling behavior of imperfect thin shells by using secant matrices with the finite element method to study postbuckling behavior of thin composite shells with initial imperfections. Kere and Lyly [150] considered geometric shape imperfections and demonstrated that the best numerical-experimental correlation was achieved with diamond shape imperfections. Rahman and Jansen [131] investigated imperfection sensitivity of composite cylindrical shells under axial compression using a finite element method. Tafreshi and Bailey [314] investigated the effects of combined loading on imperfect composite shell structures. Wardle and La-gace [315] compared experimental and numerical computations of the buckling response from transversely loaded composite shell structures. Other studies on imperfect shells include Goldfeld [165, 185], Vasilenko et al. [107], Cheng and Batra [236], Shen and Li [44], Wang and Zhong [20], and Hillburger and Starnes [223].

Vasilenko et al. [109] studied contact interaction between a laminated shell of revolution and a rigid or elastic foundation.

6. Concluding Remarks

It is interesting to see that despite advances made in computational power, researchers avoided in general usage of 3D theory of elasticity. Experience shows that extensive usage of 3D elements in practical problems is not feasible even with advanced computers. Researchers looked for, developed and used thick shell theories to solve engineering problems. Finite element is the most
used method in the analysis. Its ability to treat general boundary conditions, loading and geometry have certainly attributed to its popularity.

Cylindrical shells are still the subject of research of most recent articles. Doubly curved shallow shells have also received considerable interest. These shells can be spherical, barrel, cylindrical, or other shape.

Complicating effects of various kinds have received considerable interest. The use of piezoelectric shells necessitated by various applications and certain advanced materials resulted in considerable literature in the field. Other complicating effect of stiffened shells received some attention.

Looking at recent innovations in the area of composite plates, the authors think that it is a matter of time before these composites start making strong presence in research on shells. Areas of innovation include the use of natural fiber, single-walled and multi-walled carbon nanotubes, varying fiber orientation (both short and long fibers) as we as others. Such innovation are becoming nanotubes, varying fiber orientation (both short and long fibers) as we as others. Such innovation are becoming more necessary as composite materials are required to deliver simultaneously structural functions (strength, stiffness, damping, toughness…) and non-structural ones (thermal and electrical conductivity). Both modeling and testing of such composites can be a corner-stone of future research on composite shells.

7. Acknowledgements

The authors thank Mr. Imran Aslam for his help gathering the papers.

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