Open Journal of Applied Sciences, 2014, 4, 497-500
Published Online September 2014 in SciRes. http://www.scirp.org/journal/ojapps
http://dx.doi.org/10.4236/ojapps.2014.410048

Analytical Solution of Kolmogorov Equations for Four-Condition Homogenous, Symmetric and Ergodic System

Victor V. Kravets[1], Konstantin M. Bass[1], Vladimir V. Kravets[1], Larisa A. Tokar[2]

[1]Department of Automobiles and Automobile Sector, National Mining University, Dnepropetrovsk, Ukraine
[2]Foreign Languages Department National, Mining University, Dnepropetrovsk, Ukraine
Email: prof.w.kravets@gmail.com

Received 5 July 2014; revised 22 August 2014; accepted 4 September 2014

Abstract

Technical system consisting of two independent subsystems (e.g. hybrid car) is considered. Graduated state graph being homogenous ergodic system of symmetric structure is constructed for the system. Differential Kolmogorov equations, describing homogenous Markovian processes with discrete states and continuous time, are listed in symmetric matrix form. Properties of symmetry of matrix of subsystem failure and recovery flow intensity are analyzed. Dependences of characteristic equation coefficients on intensity of failure and recovery flows are obtained. It is demonstrated that the coefficients of characteristic equation meet the demands of functional dependence matching proposed visible analytical solution of complete algebraic equation of fourth order. Depending upon intensity of failure and recovery flows, four roots of characteristic equation are analytically found out. Analytical formulae for state probability of interactive technical system depending upon the roots of characteristic equation are determined using structurally ordered symmetric determinants, involving proper column of set initial data as well as subsystem failure and recovery flow intensity are proposed.

Keywords

State Graph, Markovian Process, Kolmogorov Equations, Intensity Matrix, Characteristic Equations, State Probabilities

1. Introduction

Studies of homogenous Markovian processes with discrete states and continuous time in ergodic systems result in generating and solving Kolmogorov equations [1]. Kolmogorov equations are homogenous system of ordi-

How to cite this paper: Kravets, V.V., Bass, K.M., Kravets, Vl.V. and Tokar, L.A. (2014) Analytical Solution of Kolmogorov Equations for Four-Condition Homogenous, Symmetric and Ergodic System. *Open Journal of Applied Sciences*, **4**, 497-500. http://dx.doi.org/10.4236/ojapps.2014.410048

nary linear differential equations with constant coefficients; their analytical solution amounts to determining roots of characteristic degree equation equal to system order of differential equations [2]. Analytical solutions of characteristic equations (complete algebraic equations) are limited by fourth order (Cardano formulae and Ferrari formulae); however, they are slightly used due to their awkwardness. It is of practical interest applying specific analytical solution for complete algebraic equations reflecting qualitatively harmonic root distribution within complex plane [3]. The paper illustrates the use of such a specific analytical solution [4] for biquadratic ergodic system which state graph is of symmetrical structure. A new compact record of analytical solution for Kolmogorov equation involved simply ordered determinant contrary to classic solving systems of differential equations resulting in immense, awkward structureless record of desirable formulae. Determined formulae of random state of interactive technical system make opportunity of deep analysis of random processes with the help of analytical methods, and numerical ones having applied computer.

2. Setting up a Problem

Markovian processes with discrete states and continuous time within technical system are considered; its state graph is of following symmetrical type (**Figure 1**):

Where $S_i (i = 1, 2, 3, 4)$ are states of technical system (TS) involving two independent subsystems. The subsystems are in two conditions: $\oplus$ —operational condition, and Θ —inoperable one.

Intensity of failure flows (λ_1, λ_2) and recovery flows (μ_1, μ_2) of the two subsystems is set as given and constant. It is required to determine state probability of interactive technical system $P_i(t)$ ($I = 1, 2, 3, 4$) under certain condition of the system in start time $P_i(0)$.

3. Kolmogorov Equations

Kolmogorov equations are set up in accordance with set state graph taking following symmetrical matrix form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{Vmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{Vmatrix} = \begin{Vmatrix} -(\lambda_1+\lambda_2) & \mu_2 & \mu_1 & 0 \\ \lambda_2 & -(\lambda_1+\mu_2) & 0 & \mu_1 \\ \lambda_1 & 0 & -(\mu_1+\lambda_2) & \mu_2 \\ 0 & \lambda_1 & \lambda_2 & -(\mu_1+\mu_2) \end{Vmatrix} \begin{Vmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{Vmatrix}$$

where the matrix of failures and recoveries is ordered, *i.e.* when it is asymmetrical to the main diagonal and symmetrical to secondary diagonal, then it has central asymmetry. Subsequently, coefficients of characteristic equations are:

$$a_3 = 2\left[(\lambda_1+\mu_1)+(\lambda_2+\mu_2)\right];$$

$$a_2 = \left[(\lambda_1+\mu_1)+(\lambda_2+\mu_2)\right]^2 + (\lambda_1+\mu_1)(\lambda_2+\mu_2);$$

$$a_1 = \left[(\lambda_1+\mu_1)+(\lambda_2+\mu_2)\right](\lambda_1+\mu_1)(\lambda_2+\mu_2);$$

$$a_0 = 0$$

meeting the requirement:

$$a_1 + \frac{a_3}{2}\left[\left(\frac{a_3}{2}\right)^2 - a_2\right] = 0,$$

Analytically, roots of characteristic equation are found out by [3] [4] formula:

$$\nu_{1,2,3,4} = -\frac{a_3}{4} \pm \frac{1}{\sqrt{2}}\sqrt{\left[\frac{3}{2}\left(\frac{a_3}{2}\right)^2 - a_2\right] \pm \sqrt{\left[\left(\frac{a_3}{2}\right)^2 - a_2\right]^2 - 4a_0}},$$

that is

$$\nu_1 = 0, \quad \nu_2 = -\frac{a_3}{2},$$

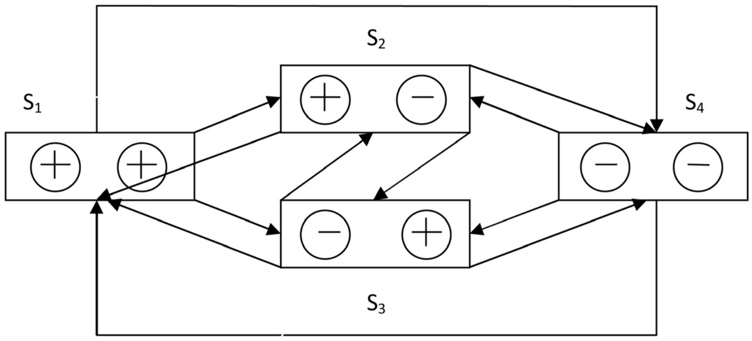

Figure 1. State graph of a technical system.

$$\nu_3 = -\frac{a_3}{4} - \sqrt{\frac{5}{4}\left(\frac{a_3}{2}\right)^2 - a_2}\,,\quad \nu_4 = -\frac{a_3}{4} + \sqrt{\frac{5}{4}\left(\frac{a_3}{2}\right)^2 - a_2}$$

Respectively, expressing roots of characteristic equation in terms of intensity of failures and recoveries of the two subsystems, we obtain:

$$\nu_1 = 0,\ \nu_2 = -(\lambda_1 + \mu_1) - (\lambda_2 + \mu_2),\ \nu_3 = -(\lambda_1 + \mu_1),\ \nu_4 = -(\lambda_2 + \mu_2)$$

4. Analytical Solution

Symbolize analytical solution of initial matrix fourth-order Kolmogorov equation in a normalized form based on structurally simple determinants:

$$P_i(t) = \sum_{k=1}^{4} \frac{\Delta_i(\nu_k)}{\prod\limits_{\substack{s=1\\(k\neq s)}}^{4} (\nu_k - \nu_s)} \mathrm{e}^{\nu_k t} \qquad (i = 1,2,3,4)$$

where

$$\Delta_1(\nu_k) = \begin{vmatrix} -P_1(0) & \mu_2 & \mu_1 & 0 \\ -P_2(0) & -(\lambda_1+\mu_2)-\nu_k & 0 & \mu_1 \\ -P_3(0) & 0 & -(\mu_1+\lambda_2)-\nu_k & \mu_2 \\ -P_4(0) & \lambda_1 & \lambda_2 & -(\mu_1+\mu_2)-\nu_k \end{vmatrix} \text{ etc.}$$

For example, it is supposed that: $P_1(0) = 1,\ P_2(0) = 0,\ P_3(0) = 0,\ P_4(0) = 0$.

5. Conclusion

Fourth-order Kolmogorov equations for state probabilities of TS consisting of two subsystems are represented in a symmetrical matrix form. The form is reflected in ordered symmetrical record of characteristic determinant. The characteristic determinant ordering is expressed in structurally simple formulae for coefficients of dependence-bonded characteristic equation. The dependence demonstrates the nature of symmetry under consideration. Functional dependence of characteristic equation coefficients motivates qualitatively harmonic distribution of roots within complex plane in the form of central symmetry. Specified property of the symmetry makes it possible to apply visible formula to calculate roots of characteristic equation of fourth order. Analytical solution for Kolmogorov equation is represented in the form of structurally simple and ordered determinants to analyze interactive state probability of TS.

References

[1] Venttsel, E.S. and Ovcharov L.A. (1991).Theory of Random Processes and Its Engineering Applications. Moscow, 384

p.

[2] Smirnov, V.I. (1974) A Course on Higher Mathematics. Moscow, Vol. 2, 656p.

[3] Kravets, V. and Chibushov, Y (1994) Method of Finding the Analytical Solution of the Algebraic Particular Aspect Equation, Poland, Rzeszow, Folia Sci. Univ. Tech. Resoviensis, Math. 16, 104-117.

[4] Kravets, V.V. (1972) On the Solving Certain Engineering Problems, Amounting to Find Roots of Particular-Type Algebraic Equation, Selected works "Best Practices of Electrification of Mining-and-Processing Integrated Works in Dnepropetrovsk Region", Dnepropetrovsk, 107-109 pp.