Gravitation

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Abstract

In this paper, it is proved that the small deformation strain tensor can be used instead the fundamental metric tensor of the *General Theory of Relativity*, in order to formulate a *Dynamic Theory of Gravitation*. Also, a solution of the velocity of the gravitational interactions is given in terms of the escape velocity due to the apparent size of the heavenly bodies. This last paragraph is the motivation and the importance of the study here presented. Thus, when it has a couple of celestial bodies separated by a distance in space, its apparent sizes as seemed at a distance plays a special role in the gravitational interactions. This is so because of some effect over the size due to the very big distances in space. In that situation, the values of their escape velocities are dependent on their mass, and critically on their apparent radius. It is proved that they are the medium used by the gravity to transmit its effects like propagating force of nature. Then, when the escape velocities meets in some point of the space between the bodies, they pull each other; because they are the carriers of the respective attractive gravitational fields. In other words, the escape velocity due to the apparent size is the exchanging coin in the gravitational interactions. Also it is proposed that such a dynamic process is the responsible for the strong link which is established between any couple of interacting heavenly objects in the *Universe*.

Keywords

Small Deformation Strain Tensor, Velocity of the Gravitational Interactions, Escape Velocity

Subject Areas: **Classical Mechanics**

1. Introduction

Within the theoretical frame of classical fluid dynamics [1], it is demonstrated that under the action of applied forces, the region occupied by some continuous medium exhibit deformations; which can be described mathematically by the small deformation strain tensor.

Consider an ordered set of N real variables x_i, $i = 1, 2, \ldots N$. These variables are called the coordinates of a point, in such a way that with all the points corresponding to an all values of the coordinates it has an *N-dimentional space*. In it let's consider a region, and also two points close together. If over the boundary surface of the

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region is applied an external dynamic agent, the region is deformed. In order to mathematical describe that deformation, the procedure is as follows. The distance between the point before and after deformation is

$$\mathrm{d}s^2 = \mathrm{d}x_i^2$$
$$\mathrm{d}s'^2 = \mathrm{d}x_i'^2 \tag{1}$$

where $\mathrm{d}x_i' = \mathrm{d}x_i + \mathrm{d}u_i$ and $u_i = x_i' - x_i$ are the components of the displacement vector while $\mathrm{d}x_i$ the i-component of the radius vector joining the points before the deformation, and $\mathrm{d}u_i$, the radius vector joining them after deformation [2]. For small deformations, and from the second of the Equations (1), it can be obtained that

$$\mathrm{d}s'^2 = \mathrm{d}s^2 + 2u_{ik}\mathrm{d}x_i\mathrm{d}x_k \tag{2}$$

where

$$u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) \tag{3}$$

are the components of the small deformation strain tensor. On the other hand, after deformation the interval ds'^2 between the nearby points can be written as follows:

$$\mathrm{d}s'^2 = \mathrm{d}x_m'\mathrm{d}x_m' = \frac{\partial x_m'}{\partial x_i}\frac{\partial x_m'}{\partial x_k}\mathrm{d}x_i\mathrm{d}x_k = g_{ik}\mathrm{d}x_i\mathrm{d}x_k \tag{4}$$

where

$$g_{ik} = \frac{\partial x_m'}{\partial x_i}\frac{\partial x_m'}{\partial x_k} \tag{5}$$

are the components of the fundamental metric tensor. Then, if instead considering that the points are separate; we make them coincide in the undeformed initial situation; it is obtained that

$$\mathrm{d}s^2 = \mathrm{d}x_i^2 = 0 \tag{6}$$

So that in (2) it has that

$$\mathrm{d}s'^2 = 2u_{ik}\mathrm{d}x_i\mathrm{d}x_k \tag{7}$$

If the Equations (4) and (7) are compared it is obtained that [2].

$$u_{\mu k} = \frac{1}{2}g_{\mu k}; \tag{8}$$

The Equation (8) must be considered as a ***relation of congruence*** between ***Physics and Geometry***, more than and equality [2]. This is so, because the ***General Theory of Relativity***, constructed in terms of the fundamental metric tensor, is a geometrical description of the ***Universe***, where forces and momenta are not taken into account; in such a way that is very difficult to explain the ***space-time*** deformation, and the movement of the bodies, without the action of some dynamical agent; whereas the ***Dynamic Theory of Gravitation*** consider dynamical agents by means of the small deformation strain tensor. Those theories can be equally applied to object that travel at velocities lesser than the speed of light in vacuum; in such a way that the relation (8) makes congruent the geometrical description of the ***Universe***, and the correspondent dynamical descriptions. From the mathematical viewpoint, both tensors are equivalents. That means that both tensors have the same properties [2]. However, from the physical viewpoint, those tensors have different significance. The small deformation strain tensor describes the geometrical properties of any region of the ***continuous space-time*** when is deformed because of the effect of some dynamic agent. On the other hand, the fundamental metric tensor describes the metrical properties of the same region of the ***continuous space-time***, but once it was deformed. Thus, in one member of the relation (8), it has a dynamical object, and in the other a geometrical entity. Then, that fact gives the possibility to formulate a ***Dynamical Theory of Gravitation*** **[2]**. It is possible to demonstrate that from the small deformation strain tensor, all the mathematical structures needed to formulate that theory can be obtained.

Mathematical Structures

It is possible to prove that the expressions obtained are equal of those obtained using the fundamental metric tensor; that is to say

The Christoffel Symbols

The Differential Equation of a Geodesic

The Covariant Derivative of a Vector

The Covariant Derivative of a Tensor

The Riemann-Christoffel Tensor

Christoffel's symbols are two quantities which are not tensors, called some times as the *three-index symbols* of the first and second kind [2]. They has the following form

$$[mn, p] = \frac{1}{2}\left(\frac{\partial u_{np}}{\partial x^m} + \frac{\partial u_{pm}}{\partial x^n} - \frac{\partial u_{mn}}{\partial x^p}\right)$$

and

$$\{mn, r\} = u^{rp}[mn, p] \tag{9}$$

Those symbols defined in terms of the small deformation strain tensor, can be written as follows $[mn, p]$ and $\{mn, r\}$. They have the important properties that there are symmetrical in m, n [2].

The ***differential equation of a geodesic***, which is the path between two points in the ***Riemannian space***, is obtained using the calculus of variation, and the following condition

$$\int \mathrm{d}s \text{ is stationary.} \tag{10}$$

This absolute track is of fundamental importance in dynamics. Keeping the beginning and the end of the path fixed, we give every intermediate point an arbitrary infinitesimal displacement δx_σ so as to deform the path. The stationary condition is

$$\int \delta(\mathrm{d}s) = 0 \tag{11}$$

According to definition (9) and making some algebra, it can be obtained the following result, which is the looked for differential equation

$$\frac{\mathrm{d}^2 x_\alpha}{\mathrm{d}s^2} + \{\mu\upsilon, \alpha\}\frac{\mathrm{d}x_\mu}{\mathrm{d}s}\frac{\mathrm{d}x_\upsilon}{\mathrm{d}s} = 0 \tag{12}$$

Now to obtain the ***covariant derivative of a vector***, the procedure is as follows. Since $\mathrm{d}x_\mu$ is a contravariant vector, and $\mathrm{d}s$ an invariant, $\mathrm{d}x_\mu/\mathrm{d}s$ a kind of velocity, is a contravariant vector. Hence if A_μ is any covariant vector, the inner product

$$A_\mu \frac{\mathrm{d}x_\mu}{\mathrm{d}s} \text{ is invariant.} \tag{13}$$

The rate of change of this expression per unit interval $\mathrm{d}s$ along any assigned curve must also be independent of the coordinate system; that is to say

$$\frac{\mathrm{d}}{\mathrm{d}s}\left(A_\mu \frac{\mathrm{d}x_\mu}{\mathrm{d}s}\right) \text{ is invariant.} \tag{14}$$

Again, and making some algebra, it can be obtained that [2]

$$A_\upsilon^\mu = \frac{\partial A^\mu}{\partial x_\upsilon} + \{\varepsilon v, u\} A^\varepsilon \tag{15}$$

This is called the ***covariant derivative*** of A^μ. The tensors A_μ^υ and $A^{\mu\upsilon}$ are called the contravariant derivative of A_μ and A^μ.

The covariant derivatives of tensors of the second rank are formed as follows [2]

$$A^{\mu\upsilon}_{\sigma} = \frac{\partial A^{\mu\upsilon}}{\partial x_{\sigma}} + \{\alpha\sigma,\mu\} A^{\alpha\upsilon} + \{\alpha\sigma,\upsilon\} A^{\mu\alpha} \tag{16}$$

$$A^{\upsilon}_{\mu\sigma} = \frac{\partial A^{\upsilon}_{\mu}}{\partial x_{\sigma}} - \{\mu\sigma,\alpha\} A^{\upsilon}_{\alpha} + \{\alpha\sigma,\upsilon\} A^{\alpha}_{\mu} \tag{17}$$

$$A_{\mu\upsilon\sigma} = \frac{\partial A_{\mu\upsilon}}{\partial x_{\sigma}} - \{\mu\sigma,\alpha\} A_{\alpha\upsilon} - \{\upsilon\sigma,\alpha\} A_{\mu\alpha} \tag{18}$$

Finally, it is possible to construct the following expression [2]

$$B^{\varepsilon}_{\mu\upsilon\sigma} = \{\mu\sigma,\alpha\}\{\alpha\upsilon,\varepsilon\} - \{\mu\upsilon,\alpha\}\{\alpha\sigma,\varepsilon\} + \frac{\partial}{\partial x_{\upsilon}}\{\mu\sigma,\varepsilon\} - \frac{\partial}{\partial x_{\sigma}}\{\mu\upsilon,\varepsilon\}. \tag{19}$$

This is called the ***Riemann-Christoffel Tensor*** [2][1].

Thus, a stage has been reached at which the results obtained can be applied to the theory of gravitation. From the contracted ***Riemann-Christoffel Tensor*** it is obtained that [2].

$$D_{\mu\nu} = -\frac{\partial}{\partial x_{\alpha}}\{\mu\nu,\alpha\} + \{\mu\alpha,\beta\}\{\nu\beta,\alpha\} + \frac{\partial^2}{\partial x_{\mu}\partial x_{\nu}}\log\sqrt{-u} - \{\mu\nu,\alpha\}\frac{\partial}{\partial x_{\alpha}}\log\sqrt{-u}; \tag{20}$$

where u is the determinant of $u_{\mu\upsilon}$. The expression

$$D_{\mu\nu} = 0 \tag{21}$$

in empty space, is chosen as the ***law of gravitation*** in a ***Dynamic Theory of Gravitation*** [2].

At is easy to see, all those mathematical formulae have the same structure that those of the ***General Theory of Relativity***, but in terms of ***the small deformation strain tensor***. So that, it is possible to formulate a dynamical description of the ***Universe***, instead of the ***General Relativity*** geometrical description [2].

However, both theories give the same results when the speed of the objects is lesser than the speed of light in vacuum. But in the case of the gravitational interactions, the situation is different, because the speed of gravity is always greater than the light velocity. Apparently, gravity in contrast to light, has no detectable aberration or propagation delay for its action.

Newton's Law of Gravitation

The most amazing thing about the celestial mechanics is that all gravitational interactions between bodies in all dynamical systems had to be taken as instantaneous. However, experiment shows that instantaneous interactions do not exist in nature. No one has expressed the objection to a such situation than ***I. Newton***. He says that it is absurd to consider that one body may act upon another at a distance through a vacuum, without the mediation of anything else. But mediation requires propagation, and finite bodies should be incapable to propagate at infinite speeds since that would require infinite energy [3]. ***Einstein's Special Theory of Relativity***, an experimentally well established theory, proved that nothing with ***proper mass non-zero***, could propagate at a speed greater than light in vacuum [3].

Indeed, it is widely accepted that the velocity of gravity in ***Newton's Universal Law*** is unconditionally infinite. This fact is not mentioned in the statement that ***General Relativity*** reduces to ***Newtonian Gravity*** in low-velocity, weak-field limit, because of the question it begs about how that can be true if the propagation speed in one model is the speed of light, and in the other model is infinite [3]. There are many objections related to that topic; in such a way that in discussions of gravity, the most frequently asked question is, what is the speed of gravity? and also, what is the exchanging object in the gravitational interactions?

The problem arises since ***Newton's Discovery*** of the ***Law of Universal Gravitation***,

$$F = G\frac{m_1 m_2}{r^2} \tag{22}$$

That law expressed the fact that ***the force in magnitude between two particles having mases m_1 and m_2 se-***

[1]The derivation process of the Equation (19) is not the main objective of the study. However, in reference [2], that derivation process is given in my paper title ***The small deformation strain tensor as a fundamental metric tensor*** published by the ***Journal of High Energy Physics, Gravitation and Cosmology***in 28 July 2015.

parated by a distance r is an attraction acting along the line joining the particles. In that relationship *G* ***is a universal constant having the same value for all pairs of particles*** [4].

It is important to say at once some features of this law to understand it clearly. The gravity forces between two particles are an ***action-reaction pair***. On the other hand, the question of what means the distance between heavenly bodies arises immediately. In the case of the gravitational interaction between the ***Moon*** and the ***Earth***, ***Newton*** regarded every particle of the ***Earth*** as contributing to the gravitational attraction it had on the ***Moon***; but from different distances and directions. Then, ***Newton*** made the daring assumption that the ***Earth*** could be treated for such a proposal as if all the mass were concentered at its center. In other words, the ***Earth*** and also the ***Moon*** could be treated as interacting particles. That assumption was proved by ***Newton*** when he invent the ***Calculus***.

However, that assumption is very important for any couple of interacting celestial objects. Perhaps, ***Newton*** considered that the apparent size of the bodies as seemed at a distance, as a useful concept.

The Speed of Gravity

The gravitational interactions between bodies is described in ordinary mechanics by means of a potential energy which appears as a function of the coordinates of the interacting bodies; and contains the assumption of instantaneous propagation, which is unaccepted because it seems to be a form of ***action at a distance***. However, it can be shown that the gravity is a very fast propagating force greater than the light velocity in vacuum.

Let us consider now, the fundamental quadratic form, but in terms of the components of the small deformation strain tensor.

The interval between two neighboring events with coordinates (x_1, x_2, x_3, x_4) and ($x_1 + \mathrm{d}x_1$, $x_2 + \mathrm{d}x_2$, $x_3 + \mathrm{d}x_3$, $x_4 + \mathrm{d}x_4$)

$$\begin{aligned}\mathrm{d}s^2 &= u_{11}\mathrm{d}x_1^2 + u_{22}\mathrm{d}x_2^2 + u_{33}\mathrm{d}x_3^2 + u_{44}\mathrm{d}x_4^2 + 2u_{12}\mathrm{d}x_1\mathrm{d}x_2 + 2u_{13}\mathrm{d}x_1\mathrm{d}x_3 \\ &\quad + 2u_{14}\mathrm{d}x_1\mathrm{d}x_4 + 2u_{23}\mathrm{d}x_2\mathrm{d}x_3 + 2u_{24}\mathrm{d}x_2\mathrm{d}x_4 + 2u_{34}\mathrm{d}x_3\mathrm{d}x_4;\end{aligned} \tag{23}$$

where the coefficients u_{11}, etc. are functions of x_1, x_2, x_3, x_4. That is to say, $\mathrm{d}s^2$ is some quadratic function of the differences of coordinates [5].

Let us assume that there is a small region of the space throughout which the u's can be treated as constants. In that case, and making a change of variables [5], the right-hand side of (23) can be broken up into the sum of squares; in such a way that it is obtained [5]

$$\mathrm{d}s^2 = \mathrm{d}y_1^2 + \mathrm{d}y_2^2 + \mathrm{d}y_3^2 + \mathrm{d}y_4^2 \tag{24}$$

However, that reduction is not in general possible for a large region, where the u's have to be considered as functions, not as constants. Consider all the events for which y_4 has some specified value. These will form a three-dimensional space. Since $\mathrm{d}y_4$ is zero for every pair of events, their mutual intervals are given by

$$\mathrm{d}s^2 = \mathrm{d}y_1^2 + \mathrm{d}y_2^2 + \mathrm{d}y_3^2 \tag{25}$$

But this is exactly like familiar space in which the interval is given by [3]

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 \tag{26}$$

where x, y, z are rectangular coordinates.

Hence a section of the world by $y_4 = constant$ would appear as space, and y_1, y_2, y_3 will appear as rectangular coordinates. The coordinate-frames y_1, y_2, y_3 and x, y, z are examples of two system*s s* and *s*' for which the intervals between corresponding pairs of mesh-corners are equal [5]. The two systems are therefore exactly alike observationally; and if one appears to be a rectangular frame in space, so also must the other [5]. Clearly, the coordinates for real events must be real. Taken into account the last observation, the reduced general expression is

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 + \mathrm{d}y_4^2 \tag{27}$$

Clearly y_4 must involve the time, otherwise the location of events by the four coordinates would be incomplete. Consider that the following definition of equal time-intervals is accepted, if it has some mechanism capable of cyclic motion, in which its cycles will measure equal durations of time ***anywhere*** and ***anywhen***, provided the mechanism, its laws of behavior, and all outside influences remain similar [5]. To this, the theory of relativity would add the condition that the mechanism must be at rest in the space-time frame considered, because it is well known that a clock in motion goes slower in comparison with a fixed clock [5]. Since then, it is agreed that the mechanism as a whole is to be at rest, and the moving parts return to the same positions after a complete cycle, it has for any pair of events marking the beginning and the end of the cycle that

$$\mathrm{d}x, \mathrm{d}y, \mathrm{d}z = 0$$

in such a way that according to (27) it has that

$$\mathrm{d}s^2 = \mathrm{d}y_4^2.$$

Then, the cycles in all cases correspond to equal intervals ds. Hence, they correspond to equal values of dy_4. But by the above definition of time, they also correspond to equal lapses of time dt. Then, it must have dy_4 proportional to dt; in such a way that it is proposed the following proportionality relationship

$$\mathrm{d}y_4 = i\,nc\mathrm{d}t; \tag{28}$$

this is so, because there are some observational results which points out toward the fact that the velocity of the gravitational interactions is always greater than the speed of light in vacuum [3]. In the relation (28), $i = \sqrt{-1}$, n a number which for the case of gravity is $n > 1$ always; and $n \le 1$ for the case of the material finite bodies, and also for de photons; and c the light speed, considered as the ***fundamental velocity*** in the ***Special Theory of Relativity***. Then (27) becomes

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 - n^2c^2\mathrm{d}t^2 \tag{29}$$

The velocity of gravity

It is important to say that in relativity all the last it was said is subjected to the condition that it has dealing with a region of the space in which the g's, and also the u's for the gravity, are constants, or approximately constants. A region having that property is called a ***flat space***; and the appropriate theory, is called the ***Einstein´s Special Theory of Relativity***; in which is established the velocity of light as the greatest speed in the ***Universe***. That theory proved that no material finite body could propagate beyond the light barrier. But in the gravitational interactions, this is not the case, because the gravity is a propagating force of Nature without mediation of anything, apparently [5]. In other words, is not a pure geometrical effect of curved space-time, which travel at light speed, as it is suggested by the ***General Theory of Relativity***. In reality, the velocity of the gravitational interactions is the greatest speed of the ***Universe***.

Now from (29) it is follows that when

$$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)^2 = n^2c^2, \tag{30}$$

ds = 0. This is important because in the ***Special Theory of Relativity***, the track of a light-wave is a geodesic with ds = 0; and it is included there the identification of the ***light*** with the ***fundamental velocity*** [3]. Hence, in a ***Dynamic Theory of Gravitation***, the track of the ***escape velocity***, due to the apparent size of the celestial objects as a ***fundamental velocity***; and which is the medium used by the gravity to transmit its effects like propagating force of nature, must also be a kind of geodesic with ds = 0 [5]. Then it is proposed that

$$v_g^2 = \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)^2; \tag{31}$$

where v_g is the velocity of the gravitational interactions. Clearly.

$$v_g = nc. \tag{32}$$

Moreover, it can be proposed that v_g is equal to the ***escape velocity*** of any massive source of gravitational force, which has an apparent size due to some kind of effect produced by the very big distances between the bodies in space.

The Lorentz transformation

Let us consider the following transformation of coordinates

$$\begin{aligned} x &= \beta\left(x' - wt'\right);\ y = y';\ z = z' \\ t &= \beta\left(t' - wx'/c^2\right); \end{aligned} \tag{33}$$

where

$$\beta = \left(1 - \frac{w^2}{c^2}\right)^{-1/2}, \tag{34}$$

and, w is any real constant not greater than c. It can be demonstrate that

$$\mathrm{d}x^2 - c^2\mathrm{d}t^2 = \mathrm{d}x'^2 - c^2\mathrm{d}t'^2. \tag{35}$$

Hence, from the condition about equal intervals, $\mathrm{d}s^2 = \mathrm{d}s'^2$, and taking into account that

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 - c^2\mathrm{d}t^2, \tag{36}$$

it has that

$$\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 - c^2\mathrm{d}t^2 = \mathrm{d}x'^2 + \mathrm{d}y'^2 + \mathrm{d}z'^2 - c^2\mathrm{d}t'^2 \tag{37}$$

This is the Lorentz transformation. The essential property of the foregoing transformation is that it leaves de formula for ds^2 unaltered. It is valid when the velocity of any object with ***proper massnon-zero*** is lesser than the speed of light in vacuum. That is to say, $w < c$ for all finite objects, and $w = c$, only for the photon [5].

However, let us consider that in the case of the speed of gravity $w= nc$, with n a number which can be very big. Then, the new transformation of coordinates is

$$\begin{aligned} &x = \beta\left(x' - nct'\right); y = y'; z = z' \\ &t = \beta\left(t' - \frac{n}{c}x'\right); \end{aligned} \tag{38}$$

in such a way that

$$\beta = \left(1 - n^2\right)^{-\frac{1}{2}} \tag{39}$$

It can be proved that

$$\mathrm{d}x^2 - \frac{w^2}{n^2}\mathrm{d}t^2 = \mathrm{d}x'^2 - \frac{w^2}{n^2}\mathrm{d}t'^2 \tag{40}$$

But, $w^2/n^2 = c^2$, which is again the Equation (35)

So, in that case it is always true that the intervals ds^2 and $\mathrm{d}s'^2$ are equal; and clearly the Lorentz transformation are valid even when the speed of gravity is taking into account; because that law is referred to objects which travel at a velocities lesser than the speed of light in vacuum.

The velocity of the gravitational interactions

When the traditional experimental and scientific research methods are used to determine the velocity of gravity, they always yield propagation speeds of gravity faster than light speed. This is so, because the gravitational force, in contrast to light, apparently has no any propagation delay for its action. The finite value of light speed causes not only ***aberration***, but also radiation pressure forces to have a non-radial component proportional to ***v/c***. The last ratio is the deviation of light in transforming to a new reference system, and is a phenomenon known as the ***aberration of light***. In that ratio, v is the orbital velocity of the ***Earth***, and c the velocity of light in vacuum [6]. The gravity has no any force proportional to ***v/c***. That means that the gravitational interactions apparently have no detectable aberration.

The gravitational interactions between bodies in all dynamic systems had always to be taken as instantaneous. This is unacceptable, due that, it was said before, it seemed to be a form of ***action at a distance*** through vacuum, without the mediation of anything. But mediation means propagation of something, and finite bodies should be incapable of travel faster than light speed, since that would require infinite energy.

The ***Einstein's Special Theory of Relativity*** proved that no material body with a ***proper mass*** non-zero could propagate at speed greater than the velocity of light in vacuum. Then, when planetary orbits are determined, the astronomers use instantaneous interactions; they extract the position of some planet along its orbit at a time of interest, and calculate where that position would appear as seen from ***Earth*** by allowing for the finite propagation speed of light from there to here. It seems incongruous to allow for the finite propagation speed of light from the planet to ***Earth***, but to take the effect of ***Earth's*** gravity on the same planet as propagating from here to there, instantaneously [3].

This kind of problems is not new. They have been discussed many times. Even today, the most frequently debated topic is centered in the question about the velocity of propagation of gravity.

2. Gravitational Interactions

Einstein's Theory of Relativity suggests that gravitation, unlike electromagnetic forces, is a pure geometric effect of curved space-time, not a force of nature that propagates. In that case, the gravitational radiation, which surely does travel at light speed, but like a possible effect of the fifth order in ***v/c***, is too small to play a role in explaining the difference in behavior between gravitational interactions and ordinary forces of nature. In this connection, in ***General Relativity***, also exist problems with the ***causality principle***, such as explaining how the external gravity fields between binary black holes [6] manage update without communication with the material bodies hidden behind event horizons [3].

Many teachers and some books head off the question, assuring their students that the gravitational waves propagate at the speed of light, leaving the impression that the topic of gravitational propagation speed has already been solved [3].

Anyone can verify the consequences of introducing a delay into gravitational interactions. The effect on the planetary orbits, is usually disastrous due to the fact that the conservation of angular momentum is destroyed [3].

Indeed, it is widely accepted that the speed of gravity in ***Newton's Law of Gravity*** is unconditionally infinite. This is not mentioned in the statement that ***General Relativity*** reduces to ***Newtonian Gravity*** in the low-velocity, weak-field limit, because of the obvious question it begs about how that can be true the propagation speed in one model is the velocity of light in vacuum, and in the other model it is infinite [3]. The same kind of question comes up in many other situations; that is to say, how can black holes have gravity when nothing can get out because escape velocity is greater than the speed of light?, and also, why do total eclipses of the ***Sun*** by the ***Moon*** reach the maximum about ***40 seconds*** before the ***Sun*** and the ***Moon's*** gravitational forces align? [3]. If gravity is a propagating force, the system ***Sun-Moon-Earth*** test implies that gravity propagates faster than light.

3. Escape Velocity

According to ***Einstein's Special Theory of Relativity***, no material body could possible travel with a velocity greater than the velocity of light in vacuum. Thus, it becomes obvious that none of them could be accelerated beyond the light barrier. This argument still stands. In its original paper, ***Einstein*** said that there is an upper limit of the velocity for the material bodies [7]. The only particle that always travels at that velocity is the photon. Thus, for a given distance, it can be written that

$$x = ct_\ell \tag{41}$$

where x is any distance, c the velocity of light in vacuum, and t_ℓ the time that takes the photon to travel the given distance.

Let's consider now that for the same distance x, it can be written the following relationship between the speed of gravity v_g, and the time transit t_t

$$x = v_g t_t \tag{42}$$

Then, for a given distance the speed of gravity grows with the distance between the interacting bodies, whereas the transit time diminish, in such a way, that their product is always constant. In that case it is easy to see that

$$v_g = \frac{ct_\ell}{t_t} \tag{43}$$

Now, it is necessary another independent equation in order to calculate v_g. That relationship is the escape velocity; which is the medium used by the gravity to transmit its effects like a force of nature that propagates. By its definition, it has that

$$v_o = \left(\frac{2GM}{R}\right)^{1/2} \tag{44}$$

where G is the ***Gravitational Constant***, M the mass of the object which is the source of gravitational interaction, and R its radius. Hence, the value of v_o depends on the mass of the gravity source, and critically on its radius.

Let's consider the following situation. A distant galaxy, per example, as appear seen from ***Earth***, looks like a light point. In that apparent very small region of space, it is concentered the total mass of that galaxy. That fact is very important because the apparent size of heavenly bodies plays a special role in the gravitational interactions. This is so because of a some kind of effect over the size due to the very big distances in space. So that, when a couple of celestial bodies interact each other, they can be considered as sources of gravitational force, because there are certain degrees of reciprocity in that situation. So that, their escape velocities takes values that depends critically on its apparent size. Hence, in Equation (44) is has that

$$v'_o = \left(\frac{2GM}{R'}\right)^{1/2} \tag{45}$$

where v'_o means the real escape velocity and R' the radius as they are seen at a galactic distances.

In gravitational interactions, the question of what means by ***distance between the heavenly bodies*** arises immediately; because every of their particles can be considered as a contributing to the gravity attraction. But, and according to ***I. Newton***, the distance and direction over each body, are different from each particle [4]. Then, it is possible make the assumption that the interacting bodies could be treated as if all their masses where concentrated at its apparent sizes. However, and by definition, a geometrical point is an object that has no dimension; so that it is not a good idea to take the size of a very far object as zero; in such a way that it is necessary to take another approach. Suppose that as the limit in size of the massive source of the gravitational force is taken as the size of the atom, or the atomic nucleus.

Thus, the value of v'_o in such a condition, grows with the mass of the source concentered in a minimum region of space which has the radius R'.

4. The Total Solar Eclipse

This is an example of the difference in behavior between the velocity of the gravitational interactions and the velocity of light in vacuum.

The ***Moon*** and ***Earth*** are relatively nearby and sharing the ***Earth*** orbital motion around the ***Sun***. It is well known that it has an aberration of just **20 *arc seconds***. It takes to the ***Moon*** about **38 *seconds of time*** to move **20 *arc seconds*** on the sky relative to the ***Sun*** [3]. Since the total solar eclipse agree with predicted times, to within a couple of seconds, it can be used the orbits of the ***Sun*** and the ***Moon*** near times of maximum total solar eclipse to compare the time of predicted gravitational maximum with the time of visible maximum eclipse. Thus, the maximum gravitational perturbation by the ***Sun*** on the orbit of the ***Moon*** near eclipse may be taken as the time when the **Moon** and **Sun** sizes are equal. In those conditions, the maximum eclipse occurs roughly **38.19 *seconds of time*** before the time of gravity maximum [3].

Hence, let $t_t = 38.19$ seconds be, the transit time that takes the gravitational interaction to travel the distance between the ***Sun*** and the system ***Moon-Earth***. Given that the light from the ***Sun*** requires 4.98×10^2 sec. to reach the system before mentioned, from Equation (43) it is easy to obtain that

$$v'_o = 13c \tag{46}$$

that is to say,

$$v'_o = 3.9 \times 10^6 \text{ km} \cdot \text{sec}^{-1} \tag{47}$$

Now, from the relationship (45), and using the following data

$$\begin{aligned} G &= 6.66 \times 10^{-8} \text{ ergs} \cdot \text{cm} \cdot \text{gr}^{-2} \\ M_{\odot} &= 1.985 \times 10^{33} \text{ gr} \\ v'_o &= 3.9 \times 10^{11} \text{ cm} \cdot \text{sec}^{-1}; \end{aligned} \tag{48}$$

where the symbol $\odot$ indicates the ***Sun***, it is obtained that the apparent Sun's radius, as seen since the system ***Moon-Earth***, has the following value

$$R'_{\odot} = 1.738 \times 10^{-2} \text{ km} \tag{49}$$

It is proposed that this picture can be used to explain the behavior of any couple of interacting celestial objects; where one of them is the source of gravitational force that propagate with an escape velocity which depends on the apparent size of that body.

In conclusion, the ***Sun's field*** cannot be treated as ***static and unchanging***, even in the planetary system, because it propagates as a force of nature, with an escape velocity whose value depends on the ***Sun's size***, as seen from any planet of the ***Solar System***. Thus, its gravity field is continuously updated, but with very little gravitational aberration, difficult to determine.

5. Conclusions

The small deformation strain tensor can be used as a fundamental metric tensor instead the usual fundamental metric tensor, in order to formulate a ***Dynamic Theory of Gravitation***, in which the ***escape velocity*** is the greatest velocity of ***Nature*** [1] [2].

For the ***Sun*** and ***Earth***, or for any other planet of the ***Solar System*** and also for any other celestial body, the picture before given has certain degree of reciprocity. In other words, seen from the ***Sun*** any one planet or another body has an apparent size that generated its own appropriate apparent escape velocity. However, in the case of the planets, all of them have a very small size compared with ***Sun's size***, and also the mass of any of them is very small as compared with Sun's mass.

Then, its apparent escape velocity is orders of magnitude smaller than the apparent escape velocity of the ***Sun***. Hence, when the escape velocity of the ***Sun***, in some point of the space, meets with the escape velocity of any planet, they pull each other; because they are the carriers of the respective gravitational fields.

Given that those gravitational forces are attractive, that dynamic process is the responsible for the strong link which is established between the ***Sun*** and every of the planets and other bodies of the ***Solar System***. It is proposed that this argument is valid for any couple of objects in the ***Universe***. Then, it can be said that the gravitational interactions exchanging coin is the ***escape velocity*** due to the apparent size of the celestial bodies.

Due to the finite value of the velocity of the gravitational interactions, the gravity force have a propagation delay for its action too. That means that it has a little aberration which, in despite of it smallness, have a minute non-radial component capable to accelerate the planets along its orbital motions. That effect is accumulative, in such a way, that along of thousand million years will tend to increase the angular momentum of the system, and will soon cause an appreciable change of period.

Maybe the minute, but accumulative effect of the non-radial component of the gravitational force, due to the ***gravity aberration***, is the dynamic agent which produces the slow rotational motion of the galaxies.

Finally, given that the velocity of the gravitational interactions is in general terms, greater than the velocity of light in vacuum; and taken into account the fact that according to ***Einstein's Special Theory of Relativity***, no material particles or bodies could possible travel with a velocity greater than the light speed; the gravity is strong enough to slow down the ***Universal Expansion*** [5]. This argument points out toward a closed ***Universe Model***.

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Appendix

The condition for flat space-time: A formal proof

It was shown that when the components of the small deformation strain tensor $u_{\mu\upsilon}$ are constants, ds^2 can be reduced to the sum of four squares, and a ***Galilean coordinates*** can be constructed. Thus, an equivalent definition of ***flat space-time*** is that it is such that coordinates can be found for which the $u_{\mu\upsilon}$ are constants, and a region of the space is called ***flat*** if it is possible to construct in it a ***Galilean frame of reference***.

If the $\mu_{\mu\upsilon}$ are constants the ***3-index Christoffel simbols***, all vanishes; but since they do not form a tensor, will not continue to vanish when other coordinates are substituted in the same flat region. Then, in such a condition, the ***Reimann-Christoffel Tensor***, being composed of products and derivatives of the ***3-index simbols***, will vanish. Since it is a tensor, it will continue vanish when other coordinate system is used in the same region. Hence, the vanishing of that tensor is a ***necessary condition*** for ***flat space-time***. But, that condition is also ***sufficient***; in such a way that if that tensor is equal to zero, the ***space-time*** must be ***flat*** [2] [5]. Then, when

$$B^{\varepsilon}_{\mu\upsilon\sigma} = 0 \tag{A-1}$$

it is possible to construct a uniform vector-field by means of parallel displacements of a vector over the region.

Let $A^{\mu}_{(\alpha)}$ be four uniform vector-fields given by $\alpha = 1, 2, 3, 4$; where α is not a tensor-suffix, but only distinguishes the four independent vectors. So that

$$\frac{\partial A^{\mu}_{(\alpha)}}{\partial x_{\sigma}} = 0 \tag{A.2}$$

or by the following equation [2]

$$\frac{\partial A^{\mu}_{(\alpha)}}{\partial x_{\sigma}} = -\{\varepsilon\sigma, \mu\} A^{\varepsilon}_{(\alpha)} \tag{A.3}$$

These four uniform vector-fields can be used in order to define a new coordinate system distinguishes by accents. The unit mesh is a geometrical figure contained by the four vector at any point, so that the complete system will be formed by successive parallel displacements of the unit mesh until the whole region is filled [5]. One edge of it, given in the old coordinates by

$$\mathrm{d}x_{\mu} = A^{\mu}_{(1)} \tag{A.4}$$

has to become in the new coordinate system by

$$\mathrm{d}x'_{\alpha} = (1, 0, 0, 0).$$

Similarly, the second edge

$$\mathrm{d}x_{\mu} = A^{\mu}_{(2)}$$

must become

$$\mathrm{d}x'_{\alpha} = (0, 1, 0, 0);\ etc.$$

Then, this requires the following law of transformation

$$\mathrm{d}x_{\mu} = A^{\mu}_{(\alpha)} \mathrm{d}x'_{\alpha} \tag{A.5}$$

Clearly, the new coordinate system depends on the possibility of constructing uniform vector-fields, and this depends on the relationship (A.1).

Since ds^2 is an invariant, it is easy to see that

$$u'_{\alpha\beta}\,\mathrm{d}x'_{\alpha}\,\mathrm{d}x'_{\beta} = u_{\mu\upsilon}\,\mathrm{d}x_{\mu}\,\mathrm{d}x_{\upsilon} = u_{\mu\upsilon}\,A^{\mu}_{(\alpha)}\,A^{\upsilon}_{(\beta)}\,\mathrm{d}x'_{\alpha}\,\mathrm{d}x'_{\beta};$$

by the Equation (A.5). Hence

$$u'_{\alpha\beta} = u_{\mu\upsilon}\,A^{\mu}_{(\alpha)}\,A^{\upsilon}_{(\beta)} \tag{A.6}$$

So that, by differentiation of (A.6) it has that

$$\begin{aligned}\frac{\partial u'_{\alpha\beta}}{\partial x_\sigma} &= u_{\mu\upsilon} A^{\mu}_{(\alpha)} \frac{\partial A^{\upsilon}_{(\beta)}}{\partial x_\sigma} + u_{\mu\upsilon} A^{\upsilon}_{(\beta)} \frac{\partial A^{\mu}_{(\alpha)}}{\partial x_\sigma} + A^{\mu}_{(\alpha)} A^{\upsilon}_{(\beta)} \frac{\partial u_{\mu\upsilon}}{\partial x_\sigma} \\ &= -u_{\mu\upsilon} A^{\mu}_{(\alpha)} A^{\varepsilon}_{(\beta)} \{\varepsilon\sigma,\upsilon\} - u_{\mu\upsilon} A^{\upsilon}_{(\beta)} A^{\varepsilon}_{(\alpha)} \{\varepsilon\sigma,\mu\} \\ &\quad + A^{\upsilon}_{(\alpha)} A^{\upsilon}_{(\beta)} \frac{\partial u_{\mu\upsilon}}{\partial x_\sigma};\end{aligned} \tag{A.7}$$

by the Equation (A.2). Now, by changing dummy suffixes, this become

$$\begin{aligned}\frac{\partial u'_{\alpha\beta}}{\partial x_\sigma} &= A^{\mu}_{(\alpha)} A^{\upsilon}_{(\beta)} \left[-u_{\mu\varepsilon} \{\upsilon\sigma,\varepsilon\} - u_{\varepsilon\upsilon} \{\mu\sigma,\varepsilon\} + \frac{\partial u_{\mu\upsilon}}{\partial x_\sigma} \right] \\ &= A^{\mu}_{(\alpha)} A^{\upsilon}_{(\beta)} \left[-[\upsilon\sigma,\mu] - [\mu\sigma,\upsilon] + \frac{\partial u_{\mu\upsilon}}{\partial x_\sigma} \right] = 0;\end{aligned} \tag{A.8}$$

by the following relationship [2] [5]

$$[\mu\upsilon,\sigma] + [\sigma\upsilon,\mu] = \frac{\partial u_{\mu\upsilon}}{\partial x_\sigma} \tag{A.9}$$

Hence, the components of the small deformation strain tensor $u'_{\alpha\beta}$ in the new coordinate system are constants throughout the region considered. We have thus constructed a coordinate-system fulfilling the condition that the u's are constant, and it follows that the ***space-time*** is ***flat***. [2] [5].