Labor Productivity vs. Minimum Wage Level

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Abstract

Recognition of the abstract nature of capital has liberated some new possibilities for alternative human capital research. Human capital, that is to say the human ability of doing work, is under the authority of all fundamental laws established in respect of the general notion of capital as spontaneous, and possessing random diffusion and limited growth. The phenomenon of human capital’s natural dispersion is a starting point for the theory of minimum wage, which ought to be sufficient to counterbalance the natural thinning out of the initial human capital of an employee. In practice, the legal minimum wage is fixed at different levels, and sometimes it is very low. Labor productivity is one fundamental factor that enables the establishment of a proper minimum wage level. Each human capital is vanquished by spontaneous and random diffusion, which averages 8% of the initial capital. Therefore the 8% rule is applicable to each employee no matter how educated and experienced he or she is. The results show that the level and fairness of the legal minimum wages is conditioned by labor productivity measured by ratio Q. This ratio should be at least 3.0 so the minimum wage could set off spontaneous random diffusion of employee’s human capital.

Keywords: Capital, Human Capital, Minimum Wage, Constant Pay, Labor Productivity

1. Introduction

The considerations introduced in this paper belong to an alternative research program of human capital. This research differs from T. Shultz’s and G. Becker’s well-known program coined under the popular title, “Investing in People”, as described by M. Blaug [1, pp. 303-321]. The new program is anchored in a deeper understanding of capital, which is discerned as the abstract “ability to do work.” The potential growth of capital is determined by the discovered constant p. The kernel of this research program [Lakatos’ hard core] is a model of capital that discloses factors causing changes of the initial capital. In addition to the hard core there is the triad, “capital-labor-money,” where labor is the transfer of capital to objects of work, and money is the pay receivables for work done. Therefore, a labor-driven economy is the central focus of the research program.

The model of capital at moment \( t \) [2] identifies factors that influence an initial value of capital. Among the factors are ratios \( s \) and \( p \), where \( s \) measures the spontaneous diffusion of initial capital, and \( p \) denotes the economic constant of potential growth. The most important relationship shows that \( p = E(s) = 0.08 \) [1/year].

Initial capital \( C_0 \) and time \( t \) are the essential factors of the compound interest formula \( C_t = C_0e^{pt} \). But a true challenge is the theory of the rate \( r \). As stated by M. Dobija [2] the rate of growth of the initial capital has a three-factor structure. Namely, \( r = p – s + m \), so the initial capital \( C_0 \) is influenced by the three subsequent factors and flow of time \( t \) as follows:

\[
C_t = C_0e^{pt}e^{-st}e^{mt} = C_0e^{(p-s+m)t} \quad \text{and} \quad p = E(s) = 0.08 \, [1/\text{year}]
\]

The variables are defined as follows:

- \( t \)—is the coordinating (calendar) time measured by chosen cyclical movements, particularly of the Earth;
- \( e^{pt} \)—is the factor of natural potential growth determined by the economic constant \( p = 0.08 \); the \( p \) also serves as the capitalization rate;
- \( e^{-st} \)—is discount factor, \( s \), is the rate of spontaneous and random diffusion of the initial capital; \( s \) is a central part of discount rates.
- \( e^{mt} \)—is an inflow of capital by human labor and management, which can offset the natural diffusion of capital and can save the potential for growth changing it to profit.

Let us note that the variables \( s \) and \( m \) represent active
work of the natural forces ($-s$) and the active outer work that can restrain the dispersion ($m$). Instead, the constant $p$ symbolizes a potential. The potential $p$ can yield fruit, provided the diffusion $s$ is counterbalanced by the work $m$.

The powerful forces determining our reality become visible in the model of the initial capital changes. Both thermodynamic principles are present here. The first thermodynamic principle is present since the model contains initial capital $C_0$. This means that the initial capital can only be changed or transferred but never created. The Second Law is also present since capital constantly diffuses; the initial value spontaneously and randomly declines.

The third force that has its part in the game is the natural potential for growth. It is the setup of the Earth and the Sun that guarantees essential potential for growth ($p = 8\%$). Thanks to this potential, human capital grows, originating labor resources. Subsequently, human labor can prevent diffusion by wise, productive labor, setting off dispersion forces and causing that potential growth $p$ to become real economic value. The model shows, among others, that economy is a non-zero sum game, and the added value can achieve an average rate of $8\%$. This value concentrates in different resources, both material (goods, soil, devices) and immaterial as intellectual and institutional resources (laws and procedures, among others).

Much research has been done to measure the value of the constant $p$. This constant manifests in many fields of economic investigation, and is known on the capital market as the risk premium. A significant research on the subject has been done by B. Kurek [3], who also described related issues in a recent book “Deterministic Risk Premium Hypothesis” [4, p. 124]. The author found that a good estimator of the constant $p$ is a properly defined ROA. Examining financial statements of companies belonging to the S & P 1500 over a 20-year period, the author showed that the average value of ROA = 8%, 28% so the $p$ as a priori value is 8%. Financial statements show values at the end of the year so the initial capital compounded at a rate of 0.08 should yield $e^{0.08} = 1.08328$, that is to say 8.33%. The author established the confidence interval (8.25; 8.89). Many authors still examine the constant $p$ in the field of employees’ human capital and their compensations. The human capital model and derived compensation models are suitable for testing the fundamental relationship: $p = E(s) = 0.08 \times [1/\text{year}]$. For testing, a simple econometric model is suitable. Namely $W_a = AW_h + B$, where $W_a$ is the real pay, and $W_h = s \times H(T, p)$ is the pay calculated in line with the human capital model. The variables $H(T, p)$ and $T$ denote respectively: $H(T, p)$—employee’s human capital, and $T$—years of professional work.

Human capital is discerned as an employee’s ability to do work. Models of human capital are derived from the general model, and then compensation models stem from the model of human capital. The most important conclusion stemming from the model of capital concerns an amount of fair constant pay. An employee as a living creature has to waste some capital since heat engines working in his body [5, pp. 157-158] have to disperse some energy. To keep balance, the constant pay has to counterbalance this loss. Therefore the constant pay should not be less than 8% of the employee’s human capital ($s \times H(T, p)$). Then the employee’s human capital is maintained, whereas less pay leads to the depreciation of the employee’s human capital. In human capital calculations, both the diffusion rate $s$ is assumed as 8% and the deterministic constant $p = .08$ as the capitalization rate.

Research has shown that countries have different percentages of consistency regarding the legal minimum pay for employees’ human capital. Western democratic countries have constant pay consistent with the 8% rule. This means the natural diffusion of the employee’s human capital (ratio $s$) is set off by basic constant pay. If the constant pay ($W$) is $s \times H(T, p)$, where $H(T, p)$ is the employee’s human capital, where $T = years$ of professional work then it is a fair pay that preserves the employee’s human capital. This is not the case in many other countries, where consistency is not 100% and sometimes is as low as 50%. The migration of the labor force (human capital) in searching for better work and living conditions is a natural phenomenon. Poland and Ukraine, whose degree of consistency does not exceed 80% and 50%, respectively, are good examples.

It seems an important reason, for a low minimum wage is weak labor productivity of an economy. Although low labor productivity is not the only reason for the inconsistency of the minimum wage, with the pay level equal to $p = E(s) = 8\%$ of an employee’s human capital (proper Gini’s coefficient can show other reason), it is a fundamental condition. The research presented here proposes that to attain the desired level of constant pay, the macroeconomic ratio of labor productivity $Q$ ought to be sufficiently high. The $Q$ is the ratio of labor productivity [6,7] roughly determined as the quotient of real GDP to total compensation. The searched level of labor productivity is computed from a regression curve showing the percentage of consistency between real pay $W_a$ and theoretical pay $W_h = s \times H(T, p)$ as a function of $Q$. 

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1A farmer’s crop does not spoil and it yields economic value since the farmer harvests it in the proper time.
2. A Capital Theory of Minimum Wage

From the human capital point of view, the minimum wage is assigned to a sufficiently mature individual with the least ability to do work. It is usually an individual aged 17 or 18, who has just completed mandatory education but has not done professional work. On the one hand, the individual’s ability to work (human capital) can be computed in line with the general model. In the next step we calculate its human capital’s annual diffusion. On the other hand, in most states the legal minimum wage is established as mandatory law. The two abovementioned amounts are compared and assessed.

Thus, in the case of minimum pay, the receiver—an employee—is an individual without professional education and experience. The source of human capital is only the stream of outlays on expenses of an employee’s parents and society. In this case, the human capital model $H(T, p) = K(p)$, where $H(T, p)$—denotes the human capital of a person with $T = 0$ years of professional work; and $p = 0.08$ is the economic constant which serves as the capitalization rate. $K(p)$—denotes the capitalized cost of living (future value) through the period, ending at the moment mandatory education is completed.

To illustrate how the model works we apply human capital calculations for computing a fair minimal pay in the case of the USA. Let us assume a child is born in an American family ($2 + 2$ persons). This child would die soon if the parents and society did not care for it (The Second Law and dispersion rate $s$). Fortunately, they do this, and the inflow of capital (ratio $m$) offsets the $s$ at least. Therefore, the human capital of the child can grow at rate $p = 8\%$. In the course of life, human capital is funded by outlays for costs of living\(^2\) estimated at $\$500$ per person per month. Parents’ labor is not included.

We calculate the human capital at the end of the 17th year of life ($6 + 10 + 1$) and then compute an adequate fair pay resulting from human capital theory (HCT). The issues are presented in Table 1. Thus, the legal pay meets the theoretical one. The above calculations authorize the conclusion that the current minimum pay in the USA is (on average) fixed at a fair level, and the percentage of consistency between practice and theory is close to 100%.

It is a canon that Western democratic and capitalistic countries execute the 8% rule, and so employees’ human capital is preserved. One can say that it is an essence of adult democratic capitalism. In contrast to Western democratic countries, many other countries apply legal minimum pay beneath the 8% theoretical level. This causes a migration of the labor force to countries with a higher degree of consistency, because compensation less than 8% of employee’s human capital means its depreciation. However, the legal minimum payment of less than 8% of employee’s human capital is not only an agenda of a bad policy and the policymakers fault but also a sort of real economic boundaries. One of them is too low labor productivity.

In the body of Table 2 are data illustrating a situation affecting a significant part of the labor force in Ukraine, where the consistency of legal minimum pay with fair pay computed on the basis of human capital theory does not exceed 50% through the past five years. All numbers in the body of Table 2 are expressed in the Ukrainian national currency unit Hrn or Hrn per period.

Data presented in Table 2 show a difficult economic situation for numerous groups of young employees starting their first job and workers paid on a minimum wage level. Since the minimum wage is the pillar of all compensations one can conclude that most compensations do not preserve human capital. As a result, Ukraine loses a significant part of the labor force through migrations motivated by the desire to earn more. In addition, the Ukrainian population has significantly declined because human capital is not preserved, so people do not see a good future. E. Libanova [8] called this state of affairs a “demographic collapse.”

The above considerations show that the simplest model of employees’ human capital $H_i$ is $H_i = K(p)$, where $K(p)$ is the capitalized cost of living at the capitalization rate $p = 0.08$. The fair pay that preserves the capital $H_i$ is $W = s \times H_i$. Indeed, the present value of the stream of pays $W$ yields $H_i$ if the discount rate is equal to $p$. \(^{\text{Cost of living denotes the minimal outlays needed for a child to grow along with social standards developing its personal human capital.}}\)

| Table 1. Computation of human capital and the minimum wage for the USA. |
|--------------------------|------------------|
| Monthly cost of living (rough estimation) | $500 |
| Future value of stream of outlays: $6000 for 17 years capitalized at the rate of 8% | $202,501 |
| Fair annual pay is equal to annual diffusion of employee’s human capital ($s \times 0.08 \times 202,501 = 16,200 = 0.08$) | |
| Monthly pay | $16,200/12 = $1350 per month |
| Hourly pay | $1,350/176 hour = $7.670 per hour |
| Current legal minimum pay in the USA | $7.25 |
| Current social security rate paid by employer | 6.2% |
| Hourly cost of labor | $7.25 \times 1.062 = 7.70 |
| Percentage of consistency between legal and HCT pay | $7.70/7.674 = 1.003 (100%) |

\(^{\text{HCT—human capital theory.}}\)

\(^{2}\text{Cost of living denotes the minimal outlays needed for a child to grow along with social standards developing its personal human capital.}}\)
Table 2. Ukrainian minimum wages in comparison to pay computed in line with HCT\(^\dagger\) (data are publicly available).

<table>
<thead>
<tr>
<th>Years</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost of living (4-person family)</td>
<td>400</td>
<td>440</td>
<td>500</td>
<td>780</td>
<td>870</td>
<td>997</td>
</tr>
<tr>
<td>Value of human capital (H)</td>
<td>162001</td>
<td>178201</td>
<td>224700</td>
<td>350532</td>
<td>390980</td>
<td>448055</td>
</tr>
<tr>
<td>Yearly cost of labor (0.08 × (H))</td>
<td>12960</td>
<td>14256</td>
<td>17976</td>
<td>28042</td>
<td>31278</td>
<td>35844</td>
</tr>
<tr>
<td>Monthly cost of labor (MCL)</td>
<td>1080</td>
<td>1188</td>
<td>1498</td>
<td>2339</td>
<td>2606</td>
<td>2987</td>
</tr>
<tr>
<td>Legal minimum wages (LMW)</td>
<td>400.0</td>
<td>440.0</td>
<td>545.0</td>
<td>780.0</td>
<td>888.0</td>
<td>960.0</td>
</tr>
<tr>
<td>Monthly (LMW) increased by pension charge 36.6% (LMCL)</td>
<td>400 × 1.366 = 546.440</td>
<td>1.366 × 601.545 = 728.744</td>
<td>1.366 × 1016.888 = 1213.960</td>
<td>1.366 × 1311 = 1311</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ratio of LMCL to MCL</td>
<td>51%</td>
<td>50%</td>
<td>49%</td>
<td>44%</td>
<td>46%</td>
<td>44%</td>
</tr>
</tbody>
</table>

\(\dagger\)HCT—human capital theory.

rate \(s\), which represents random and natural diffusion of capital, that is to say a decline of the initial capital.

\[
P V = W / s = s \times K(p) \times (1 + \varepsilon) = H \times (1 + \varepsilon)
\]

— close to zero random factor.

The above proof may be illustrated by simple but significant calculations. In the case of the USA a couple with two children who earn the minimum wage have a monthly revenue of \(2 \times $ 7.70 \times 176\) hours = $ 2,710.4. Let as assume that health care for a four-person family requires 9% of the total. In addition, they pay 20% for pension funds capitalized at a modest rate of 3%. Thus, the couple have a residual income of $ 1,924.4, which, divided by 4, yields $ 481 per person. Since the pays will grow together with professional experience, an amount of $ 500 is accessible as their cost of living. Moreover, at the age of 65 years, a pension fund for the parents will amount to \(0.2 \times 2710.4 \times 12 \times 100.4 = $ 653,075\). The amount for one person is then $ 326,538. Thus, if the pension payments are preserved by a right policy, fair money for older age is also guaranteed.

3. The Theory of Constant Pay Preserving Employee’s Human Capital

The 8% rule is applicable to each employee no matter how educated and experienced he or she is. Each capital is vanquished by spontaneous and random diffusion, which averages 8% of the initial capital. Therefore, a constant part of compensation should exist, and its amount ought to be able to counterbalance the effect of dispersion. Such a level of constant pay is subordinated to the 8% rule, and is called fair. The 8% rule has been tested many times in numerous empirical researches, which show that pays lower than 8% of employees’ capital trigger workers’ protests. In addition, their requests for a pay rise halts at a level close to 8%. Research shows that irrespective of the country or place, employees expect compensation equal to at least 8% of their human capital. The constant \(p = 0.08\) determines a border, since this value allows employees’ to keep their human capital intact. If the percentage is higher, the employees feel safer and they have a greater possibility of development.

Research in Poland and the Ukraine has confirmed this opinion. Author M. Dobija [9], among others, examined compensation in a chosen institution and other companies as well as the salaries of some professionals, such as school and academic teachers, nurses, and physicians. W. Koziol [10] examined compensations in companies and universities. The author has shown that the constant pay was close to 8% of human capital, whereas the average percentage of compensations paid in the Polish companies examined was 10.1% in respect of employees’ human capital. Similarly, J. Renkas [11] found that this percentage was 9.1% in the Ukrainian company examined, so the 8% rule also applied. J. Barburski [12] examined a set of companies and showed that the average percentage of compensation over 8% (premium pay) was 1.74; in other words 21.75% of the constant pay. I. Cięślak [13] in Poland and J. Renkas in Ukraine examined the expectations of unemployed people searching for work through employment offices. Both researches have shown that expectations of constant pay have averaged 8% with respect to employee’s human capital.

Generally, an employee also has capital from professional education and experience. Considering not only the cost of living \(K(t, p)\) capitalized through \(t\) years, but also capital originating from a professional education \(E(t, p)\), the employee’s human capital can be expressed as \(H_t = K(t, p) + E(t, p)\). This is human capital on the threshold of a professional career. When years of employment \(T\) are taken into account, then employee capital increases through experience. Quantifying experience by a semi learning curve \(Q(T)\), the typical model of employee capital is as follows:
\[ H(T) = \left[ K(t,p) + E(t,p)(1+Q(T)) \right] \]

where: \( T \) — years of employment, \( w \) — learning parameters of the employee, \( Q(T) \) — coefficient of experience idealizing an idea of a learning curve.

The above model can be reshaped to an additive form as follows:

\[ H(T) = K(t,p) + E(t,p) + D(T) \]

where: \( D(T) \) — denotes capital from experience.

\[ D(T) = H(T) = \left( K(t,p) + E(t,p) \right) = H(0)Q(T) \]

Perceiving a human being as a triad: “body-mind-spirit,” the above model gains one more factor, namely “creativity capital.” The last is measured as the present value of a stream of earnings exceeding fair pay stemming from employee’s human capital. Thus, the entire model of human capital is as follows:

\[ H(T) = K + E + D(T) + R \]

where: \( R \) — denotes creativity capital. The above model illustrates that the intellectual capital \( I(T) \) of an employee is as follows:

\[ I(T) = H(T) - K = E + D(T) + R \]

Having determined models of an employee’s capital, one can prove that the minimum basic pay, which preserves initial capital, is determined by the 8% rule. By applying the IRR concept to employee’s human capital measurement, we get the equation:

\[ H(T)(1+r) = W + H(T+1) \]

where: \( r \) — the expected rate of return on capital, \( W \) — market pay received by the employee during one year in the form of wages and fringe benefits. By finding variable \( W \), we can derive an adequate earning model:

\[ W = H(T)r - H\left[ Q(T+1) - Q(T) \right] = \cdots = H(T)\times r - \Delta D(T) \]

Here \( \Delta D(T) \) measures the annual increase of employee’s experience. The last factor \( \Delta D(T) \) is significant at the start of a professional career, and it tends to zero if \( T \) grows.

Thus, the basic wage is determined by the employee’s capital and rate of return. This amount is decreased by the experience the employee got in the last year. The above model confirms Sunder’s [14, p. 36] opinion that experience is a “by-product of doing a job”; and thus an employer is justified in modifying an employee’s earnings in the short run. It is an interesting phenomenon that earnings can be lower in some cases, because of non-monetary benefits in terms of experience the employee gains during the course of a year. An employer may be aware of the resources, opportunities, and benefits enjoyed by an employee as well as on-the-job-training opportunities. According to the above model, the experience gained is capitalized, increasing the earnings potential in the subsequent period. Factor \( \Delta D(T) \) diminishes quickly in time; it affects the first years of employment. Since it quickly disappears, the general basic wage model stemming from human capital measurement can be limited to formula \( W = r \times H(T, p) \).

Now, the essential question is about the size of rate \( r \). The answer is that the rate of return should be equal to constant \( p = E(s) = 0.08 \). Then the employee’s human capital does not deteriorate. To prove this, let us compute the present value (PV) of a stream of wages. PV = \( ph(T, p)/s \approx ph(T, p)/p = H(T, p) \) since \( p = E(s) \). Thus the formula \( W = p \times H(T, p) \) estimates the fair basic pay. In other words, the basic pay is sufficient to cover all natural depreciation of the employee’s capital. Consequently, under average conditions, this pay allows a couple to cover the costs of living and the education of their two children. This means a couple have the resources needed to cover the cost of living and the cost of education of a four-person family, and their two children can attain the parent’s level of professional education.

Fair constant pay is a stabilizing factor of socio-economic life. The correct amount of this pay enables responsible family planning and is an essential variable of home economics equilibrium. This pay is paid independently from company performance, whereas other parts of compensation (premium pay) depend strongly on profitability and other ratios measuring company performance.

4. Labor Productivity Ratio \( Q \) as a Factor of Production Function

A deeper economic sense of the ratio \( Q \) can be explained by the production function arising in the analytical approach rather than an econometric one as discussed in earlier papers [15,16]. Reservations about the econometric production function result from observations of the features pertaining to a money-goods economy, in which production factors are measured in money units; therefore, the value of production outlays (labor costs, use of materials, etc.) is defined as an amount in a uniform unit of measurement. These production factors are summed up in the product according to the principles of cost calculation and common sense; therefore it is the grand total of product components as a result of combining production factors that could become the starting point for defining production function.

Taking into account the above fact, and applying a natural approach based on the calculus of costs, we ar-
derive at a production function with seven specified arguments. As a result, the structure of the arguments specifies all the significant variables, and the basic analytical formula of the function does not require an estimation of parameters. Production function expressed analytically may be a tool of economic analysis using differential calculus; or it may provide numerous non-linear models describing the behavior of a selected variable. The value of production in the historical prices of outlays may be expressed as follows:

\[
P = K \left(1 + \frac{Z}{K}\right) = K(1 + r)
\]

where: \(P\)—denotes yearly production in market prices, \(K\)—all costs of manufacturing and managing, and \(Z\)—annual income. Thus \(Z/K\) denotes cost profitability. This ratio can be introduced as a function of the ROA. The quotient \(w = K/A\), where \(A\) is the book value of assets is known as the turnover ratio. Thus \(K = wA\), and \(Z = K = Z/wA = ROA/w\).

Production cost \(K\) can be divided into two parts: cost of labor \(W\) and other costs \(M\). Therefore \(K = W + M = W + zA\), where \(z\) is the turnover ratio calculated in relation to other costs \(M\). Hence, we obtain product \(P\), expressed in market prices, as follows:

\[
P = [W + zA](1 + r)
\]

where: \(P\) denotes value of products in real market prices, \(z\)—index of annual assets’ turnover.

After reshaping, the value of production becomes:

\[
P = W \left[1 + \frac{zA}{uH}\right](1 + r)
\]

Because the variable \(W\) is related to human capital, we apply \(W = u \times H\), where: \(u\) is the rate of remuneration of human capital and \(H\) is the total value of human capital of all employees.

In the next step we attain the formula:

\[
P = W \left[1 + \frac{zA}{uH}\right](1 + r)
\]

Then, using approximation \((1 + x \approx e^x)\), the production function is described by the following formula:

\[
P = We^{\left[1 + \frac{zA}{uH}\right]} = W_0 \text{ROA/w} \left[1 + \frac{zA}{uH}\right] = W \times Q
\]

Here \(Q\) is the labor productivity. Thus the labor productivity \(Q\) is a dimensionless variable (multiplier) and as a function of several variables, it synthesizes all influences of material, labor factors, and skilled management. \(Q\) therefore depends upon the capacity to generate market value (ROA), technical equipment for the work \((A/H)\), assets rotation \((z\) and \(w)\), and the degree of remuneration for labor \((u)\).

In macroeconomic interpretation \(P\) can be replaced by real GDP, so that it forms the relationship: \(\text{GDP} = W \times Q\).

Here the factors that influence GDP are divided into two groups. The first, represented by \(W\), involves labor and demand created by labor costs. The second is \(Q\) showing how efficiently one dollar of pay is changed into GDP.

Of course the least value of \(Q\) is 1. The more the \(Q\) is greater than 1, the better. An increase of the productivity ratio \(Q\) means an increase in the society’s wealth. \(Q = 1\) means that the prehistoric individual gathers food necessary for survival, and this alone constitutes the wage. Then products equal earnings, and \(Q = 1\). These days, that index is usually higher than one; for example, in the USA it approximates 3.5.

There is one defect hidden in the \(Q\). This number cannot be computed simply by dividing real GDP by an empirical number of the total compensations \(W\). As a matter of fact, total compensations are the sum of the private sector pays and the public sector pays. But compensation in the public sector has its source in taxes. This is not right since labor is self financing as explained in [17]. Therefore, empirical data need some corrections done while calculating the \(Q\) for a group of states (Table 3). As labor share like factor the \(Q\) is pretty stable.

It is important to discern the wide applications of the ratio \(Q\) both macroeconomically as discussed in [17] and microeconomically as presented by J. Barburski [18], W. Koziol [19], M. Dobija [20], and others. Ratio \(Q\) is, after, an important ratio for evaluating company performance. It serves, among others, to determine the right fund for premium pay consistent with company economic performance.

## 5. The Empirical Relationship between Labor Productivity (the Ratio \(Q\)) and Minimum Wage Level

There is no question about the positive relationship between the ratio \(Q\) and employees’ earnings. Instead, an

<table>
<thead>
<tr>
<th>Country</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>3.458</td>
<td>3.470</td>
<td>3.560</td>
<td>3.500</td>
<td>3.452</td>
</tr>
<tr>
<td>Japan</td>
<td>3.069</td>
<td>3.093</td>
<td>3.186</td>
<td>3.433</td>
<td>3.279</td>
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<tr>
<td>UK</td>
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<td>3.517</td>
<td>3.444</td>
<td>3.082</td>
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</tr>
<tr>
<td>Switzerland</td>
<td>3.534</td>
<td>3.645</td>
<td>3.748</td>
<td>3.650</td>
<td>3.509</td>
</tr>
<tr>
<td>Germany</td>
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<td>3.389</td>
<td>3.276</td>
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</tr>
<tr>
<td>Czech Republic</td>
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<td>2.355</td>
<td>2.210</td>
<td>2.134</td>
</tr>
<tr>
<td>Poland</td>
<td>1.881</td>
<td>1.992</td>
<td>1.854</td>
<td>1.869</td>
<td>1.903</td>
</tr>
<tr>
<td>China</td>
<td>1.415</td>
<td>1.512</td>
<td>1.685</td>
<td>1.762</td>
<td>1.768</td>
</tr>
</tbody>
</table>

interesting question is, what minimum level of the Q still guarantees the preservation of employees’ human capital? An answer to this question requires an examination of pays and of the Q using a representative sample for every country of the world. Here the agenda in narrowed to minimum wages only and to the European group of states together with the USA. Therefore, the issues can give an orientation rather than an exact answer for every country of the world. Table 4 includes sample data, which enables the application of a regression relationship (conditional mean) between variables Con and Q.

The data from columns 5 and 6 allow for the determination of a conditional relationship between the variables Con and Q according to regression curve: \( F(q) = E(\text{Con} \mid Q = q) \). In order to avoid a mistake by applying linear regression or a personal choice sort of curve, we use the nonparametric approach. We use an estimator of the conditional mean as introduced in papers [21,22]. The estimator is the function as follows:

\[
F(q) = \frac{\sum_{i=1}^{\text{n}} \text{Con}_i \cdot \phi(q_i, \sigma_{qn})}{\sum_{i=1}^{\text{n}} \phi(q_i, \sigma_{qn})}
\]

where

\[
\sigma_{qn}^2 = \frac{1}{n(n-1)} \sum_{i=1}^{\text{n}} (q_i - \overline{q})^2.
\]

The functions of weight \( \phi(q, q_i, \sigma_{qn}) \) are the density function of the normal distribution. Having determined a suitable estimator of the conditional mean we find that 90% of consistency between the legal minimum pay and the constant pay determined by HCT is achieved at the \( Q \geq 2.7 \), while the \( Q \geq 3.2 \) guarantees a consistency close to 100%.

**6. Conclusions**

The presented empirical study concerned mainly minimum wages. Assuming that the legal minimum wage does influence the level of all compensation, and it is an indicator of a country’s economy, the presented research authorizes some conclusions. The most general reflection suggests that the level of preservation of employees’ human capital depends among others on labor productivity. Therefore, an increase in rising labor productivity is a constant task of management and the state authorities. It is not an easy task since each real growth of GDP is a reason to request more pay, as happened in Poland in 2007, or increasing the budget sector. The effects are apparent in Table 3. Therefore, Poland still has a remarkable gap compared to countries with a \( Q \) of not less than 3.0. The earning migration trend still continues, although it has declined slightly compared to ten years ago. Poland needs to increase the \( Q \) to about 1 in order to become a member of the welfare states. On the other hand, the Polish situation is much better than that of Ukraine, which is still in a state of stagnation.

Productivity in the private sector, which operates in a free market environment, is driven by market forces and market competitions. A public sector that is too large and does not have proper control of productivity is undoubtedly a reason for general low productivity as measured by the ratio \( Q \). Therefore, comprehensive control of the public sector and deep reform, in particular, as discussed in [23] are ways of attaining better preservation of employees’ human capital. This reform requires an under-

<table>
<thead>
<tr>
<th>Country</th>
<th>Legal Minimum Wage</th>
<th>Cost of Labor*</th>
<th>Pay in line with HCT#</th>
<th>Con§</th>
<th>Labor Productivity Ratio Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. USA</td>
<td>$ 1,276</td>
<td>$ 1,355</td>
<td>$ 1,350</td>
<td>100%</td>
<td>3,452</td>
</tr>
<tr>
<td>2. France</td>
<td>€ 1,365</td>
<td>€ 1,837</td>
<td>€ 1,947</td>
<td>101%</td>
<td>3,070</td>
</tr>
<tr>
<td>3. Germany</td>
<td>€ 1,467</td>
<td>€ 1,687</td>
<td>€ 1,700</td>
<td>100%</td>
<td>3,169</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>27 Czech Rep.</td>
<td>8000 Krn</td>
<td>9440 Krn</td>
<td>11,000 Krn</td>
<td>86%</td>
<td>2134</td>
</tr>
<tr>
<td>28 Poland</td>
<td>1386 zł</td>
<td>1636 zł</td>
<td>2097 zł</td>
<td>78%</td>
<td>1903</td>
</tr>
<tr>
<td>29 Ukraine</td>
<td>915 Hrn</td>
<td>1234 Hrn</td>
<td>2741 Hrn</td>
<td>46%</td>
<td>1455</td>
</tr>
</tbody>
</table>

*Legal Minimum Wage increased by pension payment charging employer; #HCT—Human Capital Theory; §The quotient of column 3 to column 4 expressed as a percentage.
standing and application of labor self-financing as well as a limitation of total pays in the public sector according to the size of the \( Q \). We can conclude that labor productivity measured by ratio \( Q \) is significant factor influencing level of the minimum wage. Nevertheless it is not exclusive factor. The mentioned Gini’s coefficient of earnings distribution as well the too large public sector are also essential factors.

7. References