

Price-Setting Mixed Duopoly Models with Complementary Goods

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Abstract

This paper considers domestic (resp. international) Bertrand mixed duopoly competition in which a state-owned welfare-maximizing public firm and a domestic (resp. foreign) profit-maximizing private firm produce complementary goods. The main purpose of the paper is to present and to compare the equilibrium outcomes of the two mixed duopoly models.

Keywords: Complementary Goods, Price Competition, Domestic Mixed Duopoly, International Mixed Duopoly

1. Introduction

The analysis of mixed market models that incorporate state-owned welfare-maximizing public firms has received considerable attention in recent years and has been widely performed by many researchers. However, most studies consider quantity competition, such as [1-16].¹

Some studies consider mixed markets with price competition, such as [22-26]. These studies examine price-setting mixed market models with domestic firms and do not include foreign firms. Ohnishi [27] considers an international mixed market in which a state-owned public firm competes on price with a foreign private firm. However, this study examines mixed duopoly competition in which public and foreign private firms produce imperfectly substitutable goods. To the best of my knowledge, the analysis of international Bertrand mixed markets with public and foreign private firms producing complementary goods has been ignored.

Therefore, we analyze the behavior of a state-owned public firm and a foreign private firm in an international price-setting model with complementary goods. We consider both domestic and international mixed duopoly models with complementary goods. The main purpose of this paper is to present and to compare the equilibrium outcomes of the two mixed duopoly models.

The remainder of the paper proceeds as follows. In Section 2, we formulate a domestic Bertrand mixed duopoly model in which a state-owned public firm and a domestic private firm produce complementary goods. We

show the slope of each firm's reaction curve in the domestic Bertrand mixed duopoly model with complementary goods. We present the equilibrium outcome of the domestic Bertrand mixed duopoly model with complementary goods. Section 3 analyzes the equilibrium outcome of an international Bertrand model in which a state-owned public firm and a foreign private firm produce complementary goods. Section 4 compares the equilibrium outcomes of the two mixed duopoly models. Finally, Section 5 concludes the paper.

2. Domestic Mixed Duopoly with Complementary Goods

In this section, we consider a market in which one state-owned welfare-maximizing public firm and one domestic profit-maximizing private firm produce complementary goods. The basic structure is from Bárcena-Ruiz [25]. In the remainder of this paper, subscripts S and D denote the state-owned firm and the domestic private firm, respectively. There is no possibility of entry or exit. On the consumption side, there is a continuum of consumers of the same type whose utility function is linear. The representative consumer maximizes $U(q_S, q_D) - p_S q_S - p_D q_D$, where q_i is the amount of good i and p_i is its price ($i = S, D$). The function $U(q_S, q_D)$ is quadratic, strictly concave and symmetric in q_S and q_D : $U(q_S, q_D) = a(q_S + q_D) - (q_S^2 - q_S q_D + q_D^2)/2$, where $a > 0$. The de-

¹See, for example, [17-21] for excellent surveys.

mand function is given by

$$q_i = \frac{2(3a - 2p_i - p_j)}{3} \quad (i, j = S, D; i \neq j). \quad (1)$$

Each firm's profit function is given by

$$\pi_i = (p_i - c)q_i \quad (i = S, D), \quad (2)$$

where $c > 0$ is the total cost for each unit of output. We assume $c < a$ to assure that the production levels of firms are positive. Domestic social welfare, which is the sum of consumer surplus (CS) and profits, is given by

$$W = CS + \pi_S + \pi_D. \quad (3)$$

From (2) and (3), we derive the following reaction functions in prices:

$$R_S = \frac{3c - p_D}{2}, \quad (4)$$

$$R_D = \frac{3a + 2c - p_S}{4}. \quad (5)$$

From (4) and (5), we can state the following:

Lemma 1. In the domestic Bertrand mixed market with complementary goods, each firm's reaction function is downward sloping.

From (4) and (5), the equilibrium can be derived as follows:

$$p_S = \frac{10c - 3a}{7}, \quad (6)$$

$$p_D = \frac{6a + c}{7}, \quad (7)$$

$$q_S = 2(a - c), \quad (8)$$

$$q_D = \frac{8(a - c)}{7}. \quad (9)$$

Furthermore, the profits and consumer surplus can be expressed as follows:

$$\pi_S = \frac{6(a - c)(c - a)}{7}, \quad (10)$$

$$\pi_D = \frac{48(a - c)^2}{49}, \quad (11)$$

$$CS = \frac{37(a - c)^2}{49}. \quad (12)$$

From (10), we see that the state-owned firm's profit is negative. Substituting (10), (11) and (12) into (3), social welfare is obtained as

$$W = \frac{43(a - c)^2}{49}. \quad (13)$$

From (12) and (13), we see that though the state-owned

firm's profit is negative, social welfare is higher than consumer surplus.

3. International Mixed Duopoly with Complementary Goods

In this section, we consider a market in which one state-owned welfare-maximizing public firm and one foreign profit-maximizing private firm produce complementary goods. In the remainder of this paper, subscript F denotes the foreign private firm. The utility, demand, and profit functions are the same as those of the previous section. Domestic social welfare, which is the sum of consumer surplus and the state-owned firm's profit, is given by

$$W = CS + \pi_S. \quad (14)$$

From (2) and (14), we derive the following best response:

$$R_S = c, \quad (15)$$

$$R_F = \frac{3a + 2c - p_S}{4}. \quad (16)$$

From (15), we can state the following:

Lemma 2. In the international Bertrand mixed market with complementary goods, the slope of the state-owned firm's reaction curve is zero.

It is shown in Ohnishi [27] that the slope of the state-owned firm's reaction curve is zero in international Bertrand mixed duopoly competition with imperfect substitutable goods. From (15), we see that the result of international Bertrand mixed duopoly competition with complementary goods is similar to that of international Bertrand mixed duopoly competition with imperfect substitutable goods.

From (15) and (16), the equilibrium can be derived as follows:

$$p_S = c, \quad (17)$$

$$p_F = \frac{3a + c}{4}, \quad (18)$$

$$q_S = \frac{3(a - c)}{2}, \quad (19)$$

$$q_F = a - c. \quad (20)$$

From (17), we know that the state-owned firm produces an output such that price equals marginal cost. From (19) and (20), we see that the state-owned firm's output is higher than the foreign private firm's output.

Furthermore, the profits, consumer surplus, and social welfare can be expressed as follows:

$$\pi_S = 0, \quad (21)$$

$$\pi_F = \frac{3(a - c)^2}{4}, \quad (22)$$

$$CS = \frac{7(a-c)^2}{8}, \quad (23)$$

$$W = \frac{7(a-c)^2}{8}. \quad (24)$$

We see that since the state-owned firm's profit is zero, social welfare is equal to consumer surplus.

4. Comparisons

In this section, we begin by presenting comparative statics results. In the remainder of this paper, superscripts H and I denote the domestic and international mixed duopolies, respectively. We have the following results.

$$\frac{\partial p_S^H}{\partial a} < 0, \quad \frac{\partial p_D^H}{\partial a} > 0, \quad \frac{\partial q_S^H}{\partial a} > 0, \quad \frac{\partial q_D^H}{\partial a} > 0,$$

$$\frac{\partial p_S^I}{\partial a} = 0, \quad \frac{\partial p_F^I}{\partial a} > 0, \quad \frac{\partial q_S^I}{\partial a} > 0, \quad \frac{\partial q_F^I}{\partial a} > 0.$$

In each model, a rise in a raises the private firm's price. However, in the domestic mixed duopoly model, a rise in a lowers the state-owned firm's price, and in the international mixed duopoly model, a rise in a has no influence on the state-owned firm's price. Furthermore, we have

$$\frac{\partial p_S^H}{\partial c} > 0, \quad \frac{\partial p_D^H}{\partial c} > 0, \quad \frac{\partial q_S^H}{\partial c} < 0, \quad \frac{\partial q_D^H}{\partial c} < 0,$$

$$\frac{\partial p_S^I}{\partial c} > 0, \quad \frac{\partial p_F^I}{\partial c} > 0, \quad \frac{\partial q_S^I}{\partial c} < 0, \quad \frac{\partial q_F^I}{\partial c} < 0.$$

A rise in c raises the firms' prices and decreases their outputs. We see that the comparative statics results are almost the same in both models.

We now compare the equilibrium outcomes of the two models. The main result of this study is described by the following proposition.

Proposition 1. In the equilibrium outcomes of the domestic and international mixed duopoly models, 1) $\pi_S^H < \pi_S^I$; 2) $\pi_D^H > \pi_F^I$; 3) $CS^H < CS^I$; and 4) $W^H > W^I$.

Proof. 1) From (21), we see that $\pi_S^I = 0$. Furthermore, from (10) and $0 < c < a$, π_S^H is negative.

2) From (11), (22) and $0 < c < a$, both π_D^H and π_F^I are positive, and thus

$$\begin{aligned} \pi_D^H &= \frac{48}{49}(a-c)^2 \\ &= \frac{192}{196}(a-c)^2 > \pi_F^I = \frac{3}{4}(a-c)^2 = \frac{147}{196}(a-c)^2 \end{aligned}$$

The proofs of 3) and 4) are omitted, since they are the

same as the proof of 2). Q.E.D.

Proposition 1 states that consumer surplus is higher in the international mixed duopoly equilibrium than in the domestic mixed duopoly equilibrium.

5. Conclusions

We have first considered a domestic Bertrand model in which a state-owned welfare-maximizing public firm and a domestic profit-maximizing private firm produce complementary goods. We have shown that each firm's reaction function is downward sloping in domestic Bertrand mixed duopoly competition with complementary goods. In addition, we have found that the state-owned firm's profit is negative in equilibrium.

Second, we have considered an international Bertrand model in which a state-owned public firm and a foreign private firm produce complementary goods. We have then shown that the slope of the state-owned firm's reaction curve is zero, and so social welfare is equal to consumer surplus.

Third, we have compared the equilibrium outcomes of the two mixed models. We have demonstrated that though the state-owned firm's profit, the private firm's profit, and social welfare are higher in the domestic mixed duopoly equilibrium than in the international mixed duopoly equilibrium, consumer surplus associated with the international mixed duopoly equilibrium exceeds consumer surplus associated with the domestic mixed duopoly equilibrium.

6. References

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