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The Redistribution Effect[1] of Social Security System under Birth Control

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Abstract

This paper establishes an OLG model to analyze the long-term effect of social security on inequality under birth control. We study the effect of population policy and social security on fertility and education of families with different income. Then this effect determines the evolution of inequality. The analyses suggest that the effect of social security on inequality in the next generation is depended on population policy and the differentiation of family care.

Keywords

Birth Control, Social Security, Family Care, Inequality

1. Introduction

In order to control the population quantity, China has implemented the one-child policy to forcefully stipulate the upper limit of the birth of each family since 1979. However, with the continuous decline of the fertility rate and the deepening of the aging of the population, population problems have brought many kinds of pressures to the economic and social development in China in recent year. The government has constantly adjusted the birth policy to cope with the challenges brought by population problems. The third plenary session of the 18th CPC central committee has relaxed the one-child policy to allow couples to have the second baby if either parent is from single-child family. And the fifth plenary session allowed every couple to have the second baby. China has begun to establish a social security system since the 1990s. Now China has entered into the period of institutional integration and deepening reform [1]. However, it can be seen that although the social security system has attracted

[1]The redistribution effect is that social security system will change the income difference between different families by changing the fertility rate and the education input.

much attention, there is still a lack of understanding of the social security system [2]. The social security system still presents a high "fragmented" state. The government usually adopts a "patch" way to make social security policy. Due to the lack of reasonable values, the development of the rural social security system is seriously lagging behind, and the unbalance of the social security system between urban and rural areas is serious. Due to the change of fertility policy, family will also change in raising children and making pension decisions. What is the impact of social security system on the redistribution of social resources in such a case? This is a question to be explored in this article.

To better explore the redistribution effect of social security system under birth control, we point out that the government's population policy and social security policy have different effects on the decision-making of childbearing and education for different income families. This will affect the dynamic evolution of inequality. Based on the theoretical analysis in this paper, we can understand the impact of social security on the different families under family planning in detail, and provide a basis for better population policy and social security policy.

The remainder of the paper is organized as follows. In Section 2, we establish a dynamic equilibrium model. Section 3 analyzes the model. In Section 4, we analyze the social security under birth control, and Section 5 concludes.

2. Theoretical Model

In this section, we establish a OLG dynamic equilibrium model. We can analyze the long-term impact of social old-supporting policy on inequality evolution under the family planning policy.

2.1. Personal Sector

We develop the model from Kaiming Guo, Quansheng Zhang, Liutang Gong (2011) [3], t is used to represent time, assuming that time is discrete, from 0 to infinity. Each person has three periods of life: youth, adulthood and old age. The utility function of each person's life is

$$U_t = \log c_t + \gamma \log\left(n_t h_{t+1}\right) + \beta \log d_{t+1} \tag{1}$$

where c_t is personal consumption in adulthood, d_{t+1} is consumption when old. h_t shows that the different levels of human capital that each person has in adulthood. It's the only difference between the same generation of individuals in the economy. n_t is the quantity of children, and h_{t+1} is the quality of children (human capital). $\gamma > 0$ is a measure of the degree of concern to the parents of their children. $0 < \beta < 1$ is time preference factor.

w_t is the effective labor wage rate. So an adult with a human capital level of h_t can obtain a $w_t h_t$ income from a unit of labor in adulthood. The difference in personal income between the same generation depends on the difference in the level of human capital. We assume that h_t obeys the distribution function $F_t\left(h_t\right)$. So the average human capital level for adults is

$$\bar{h}_t = \int_0^\infty h_t \mathrm{d}F_t(h_t)$$

We define $z_t = h_t / \bar{h}_t$ as relative human capital level. N_t is the total number of adults in the period *t*. And H_t is the total human capital in the period *t*. So,

$$H_t = N_t \int_0^\infty h_t \mathrm{d}F_t(h_t) = N_t \bar{h}_t$$

So relative human capital z_t can be regarded as relative income. Then we assume that the alimony χ_t for parents from adults is a proportion of the labor income. The government levies social security tax on labor income, and the tax rate is τ_t. Here the subscript *t* shows that the government's social security policy can be changed freely in every period. So, the real income of each person in the adulthood is $w_t h_t (1-\chi_t-\tau_t)$.

ν is the ratio of the cost of raising a child to the income of a family. And we assume that ν is a constant and is the same for people of different incomes. So the cost of raising a child for a family is $\nu w_t h_t$. e_t is the degree of education which parents choose for their children. So e_t can be seen as teachers' working time [4]. We assume the price of e_t is determined by the labor return for the average human capital. That is, every child's educational input is $e_t w_t \bar{h}_t$. Therefore, the budgetary constraint of the individual in the adulthood is

$$c_t + e_t w_t \bar{h}_t n_t + \nu w_t h_t n_t = w_t h_t (1-\chi_t-\tau_t-s_t) \tag{2}$$

where, s_t is savings rate.

Therefore, the income of the individual in the old age mainly comes from three aspects: The first is the return of savings $R_{t+1} w_t h_t s_t$, here R_{t+1} is the rate of return on savings. The second is χ_{t+1} from each their child's income $w_{t+1} h_{t+1}$ which is transferred to himself as a family pension. The third is the social pension provided by the government f_{t+1}. r_{t+1}^τ is the rate of return on social pension. That is the ratio of the discounted value of social pension and the social security tax paid in the adulthood:

$$r_{t+1}^\tau = \frac{f_{t+1}/R_{t+1}}{\tau_t w_t h_t}$$

Therefore, the budgetary constraint of the individual in the old age is

$$d_{t+1} = R_{t+1} w_{t+1} h_t s_t + \chi_{t+1} w_{t+1} h_{t+1} n_t + R_{t+1} r_{t+1}^\tau \tau_t w_t h_t \tag{3}$$

Each child's level of human capital h_{t+1} depends on the level of education e_t chosen by parents, human capital of parents h_t and the average level of human capital in society $\bar{h}_t$. So

$$h_{t+1} = B e_t^\theta h_t^\kappa \bar{h}_t^{1-\kappa} \tag{4}$$

Here, $B>0$ is a constant. $0<\theta<1$ measures the importance of education to the formation of human capital. κ measures the degree of intergenerational transfer of human capital. This part of the intergenerational transfer can not only include the parents' congenital inheritance of their children, but also their parents' influence on their children's human capital in postnatal life. $1-\kappa$ is

the externality in the accumulation of human capital. Further, we assume that $\theta+\kappa<1$. According to the following solution, this assumption will ensure that the level of human capital of children is marginal decreasing with regard to the level of human capital of parents [5].

2.2. Government

The government levies the social endowment insurance tax τ_t on adults. τ_t^P of it enters into social pooling funds, used as Pay-As-You-Go. The other part $\tau_t-\tau_t^P$ enters into individual account, for fund accumulation. We suppose that the rate of return on a individual pension account is the same as the rate of return on savings R_{t+1}. So the social pension of young people in the t period f_{t+1} comes from two parts: one part of which is the return of individual pension accounts $R_{t+1}\left(\tau_t-\tau_t^P\right)w_t h_t$. Another part of which is from social pooling funds $\tau_{t+1}^P w_{t+1}\bar{h}_{t+1}N_{t+1}/N_t$. Social pension income rate is

$$r_{t+1}^{\tau}=\frac{R_{t+1}\left(\tau_t-\tau_t^P\right)w_t h_t+\tau_{t+1}^P w_{t+1}\bar{h}_{t+1}N_{t+1}/N_t}{R_{t+1}\tau_t w_t h_t} \tag{5}$$

Supposing that the government implements the policy of birth control, the fertility rate in each family cannot be more than $\bar{n}$. Therefore, every family is restrained to the policy of population in the best choice of birth and education

$$n_t\le\bar{n} \tag{6}$$

2.3. Family's Pension Security

A part of the child's income is used to support their parents. It is assumed that the support ratio χ_t is different for families with different incomes. And with the increase of income, the support ratio has gradually decreased. That is to say, the elderly are more dependent on their children's support in low income families. To simulate this relationship, we assume

$$\chi_t=\bar{\chi}_t z_t^{-\eta} \tag{7}$$

where, $\bar{\chi}_t$ is the family support ratio of the average human capital level in the society. Parameter $\eta>0$ measures the differentiation of family pension in families with different income. In fact, it can also be seen as the elasticity of family support for income. That is, how much percentage of the dependence on family's pension with an increase of one percent of household income.

This paper assumes that the recessive contract of the children's support to their parents comes from the social habits of the moral norms of the family [6]. This social habit enables the elderly in the period $t+1$ to transfer payments from the younger generation (calculated according to the average family income in the current period), which is equal to the part μ of their labor income when they are adults:

$$\bar{\chi}_{t+1}w_{t+1}\bar{h}_{t+1}N_{t+1}+\tau_{t+1}^P w_{t+1}\bar{h}_{t+1}N_{t+1}=\mu w_t\bar{h}_t N_t \tag{8}$$

According to the analysis of Becker and Murphy (1988) [7], urbanization and

regional population flow will reduce the income transfer of intergeneration, that is, parameter μ will gradually decline. In this paper, there is no distinction between urban and rural and regional differences, so μ will be regarded as a constant.

2.4. The Dynamic Evolution and Equilibrium

For families with different human capital h_t, the fertility rate n_t and the human capital of the next generation h_{t+1} are different. Because h_t obeys the distribution function $F_t(h_t)$, the number of adults in the next period is satisfied

$$N_{t+1} = N_t \int_0^{\infty} n_t \mathrm{d}F_t(h_t) \tag{9}$$

The distribution of human capital in the next period is satisfied

$$F_{t+1}(h) = \frac{N_t}{N_{t+1}} \int_0^{\infty} n_t q(h_{t+1} \le h) \mathrm{d}F_t(h_t) \tag{10}$$

Here, $q(\cdot)$ is a logical judgment function. When $h_{t+1} \le h$, the value is 1, otherwise 0. It is important to note that n_t and h_{t+1} in Equation (9) and (10) are the functions of h_t.

Based on the model framework established above, we define the dynamic equilibrium of the economy: Given the initial distribution of human capital $F_0(h_0)$, the population quantity N_0, the sequence of wage $\{w_t\}$, the sequence of return on savings $\{R_{t+1}\}$, the government's social security policy in each period $\{\tau_t, \tau_t^P\}$, the population policy n_t, the population sequence required by the dynamic equilibrium of the economy N_{t+1}, the distribution of human capital $F_{t+1}(h_{t+1})$, the sequence of the family support ratio $\{\chi_{t+1}\}$, and the sequence of family's selection variables $\{c_t, s_t, d_{t+1}, n_t, e_t, h_{t+1}\}$ should satisfy:

1) Given the wage rate w_t, w_{t+1}, the rate of return on savings R_{t+1}, the family support ratio χ_{t+1}, the average level of human capital in society $\bar{h}_t$, $\bar{h}_{t+1}$, the rate of the return on social pension r_{t+1}^{τ}, the population policy $\bar{n}_t$, Under the constraint Equation (2), (3), (4), (6), by choosing the consumption in the adulthood c_t, the consumption in the old age d_{t+1}, the savings rate s_t, the fertility rate n_t, the education cost e_t, the family Maximizes own utility Equation (1).

2) The rate of return on social pension for each period satisfies Equation (5), the support ratio of family pension satisfies Equation (7) and (8).

3) The population quantity and the distribution of human capital are dynamically evolving in terms of Equation (9) and (10).

3. Analysis of the Model

In this part, we first find out the optimal choice of families restricted by population policy and families unrestricted in consumption, savings, childbirth and education. On this basis, through comparative static analysis, giving the difference in fertility rate and children's education input in families with different income, this determines the dynamic evolution of inequality.

3.1. The Solution of the Model

Under the constraint condition Equations (2), (3), (4) and (6), families with human capital h_t can maximize their utility (1) by choosing the consumption in the adulthood c_t, the consumption in the old age d_{t+1}, the savings rate s_t, the fertility rate n_t and the education cost e_t. When the fertility rate is not restricted, that is, Equation (6) does not take the equal sign, the first order conditions of s_t, n_t and e_t are given:

$$\frac{1}{c_t} = \frac{\beta R_{t+1}}{d_{t+1}} \tag{11}$$

$$\frac{v w_t h_t + e_t w_t \bar{h}_t}{c_t} = \frac{\beta \chi_{t+1} w_{t+1} h_{t+1}}{d_{t+1}} + \frac{\gamma}{n_t} \tag{12}$$

$$\frac{w_t \bar{h}_t n_t}{c_t} = \theta \left(\frac{\beta \chi_{t+1} w_{t+1} h_{t+1} n_t}{d_{t+1} e_t} + \frac{\gamma}{e_t} \right) \tag{13}$$

Equation (11) shows that the utility loss caused by savings from the unit of the income in the adulthood is equal to the increase in the utility of a unit of savings return in the old age. Equation (12) shows that the utility loss of raising one more child (raising cost and education cost) is equal to the increase in the utility of it. Equation (13) shows that the utility loss caused by the child's education cost is equal to the utility improvement from this. It can be seen that the return of raising and educating children consists of two parts: a part is from direct utility brought by children, another part is from the alimony in the old age.

Making $g_t^w = w_{t+1}/w_t$, $g_t^h = h_{t+1}/h_t$ and $g_t^H = H_{t+1}/H_t$ to represent the growth rate of w, h and H in the period t. Taking the family budget constraint Equations (2), (3), (4), the function (5) for determining the rate of return on social pension and Equation (7), (8) for determining the rate of family support into Equation (11), (12) and (13), so we can get

$$n_t^0 = \frac{\gamma}{1+\beta+\gamma} \cdot \frac{m_t}{p_t} \tag{14}$$

$$e_t^0 = \frac{\theta v}{1-\theta} \cdot z_t \tag{15}$$

Here, Superscript 0 means that the fertility rate does not reach the upper limit of population policy.

$$m_t = 1 - \chi_t + \left(r_{t+1}^{\tau} - 1 \right) \tau_t = 1 - \tau_t^P - \frac{\bar{\chi}_t}{z_t^{\eta}} + \frac{g_t^w g_t^H}{R_{t+1} z_t} \cdot \tau_{t+1}^P$$

$$p_t = \frac{v}{1-\theta} - \frac{\Omega \left(\mu / g_t^H - \tau_{t+1}^P g_t^w \right)}{R_{t+1} z_t^{1+\eta-\theta-\kappa}}, \quad \Omega = B \left(\frac{\theta v}{1-\theta} \right)^{\theta}$$

When the fertility rate is limited by population policy, the first order (12) about n_t is no longer established, and is replaced by $n_t = \bar{n}_t$. Making the solution in accordance with the above process, Get the best choice of the family at this time

$$n_t^1 = \bar{n}_t \tag{16}$$

$$e_t^1 = \left[\frac{\theta v}{1-\theta} + \frac{\theta\gamma}{(1+\beta+\theta\gamma)\bar{n}_t}\cdot m_t - \frac{(1+\beta+\gamma)\theta}{1+\beta+\theta\gamma}\cdot p_t\right] z_t \tag{17}$$

Here, Superscript 1 means that the fertility rate does reach the upper limit of population policy.

3.2. Family Different Decision under the Free Choice of Fertility Rate

When the fertility rate is freely chosen, the fertility rate and education input are determined by Equation (14) and (15). It can be seen from Equation (15) that families with high income have higher education input to their children. This is because for high income families, the unit price of education $w_t\bar{h}_t$ is lower than that of its income $w_t h_t$. As a result, higher education input e_t^0 and the level of human capital of parents h_t in high income families make their children's human capital level higher than those of low income families. Taking Equation (15) into Equation (4), we can get the growth rate of human capital $g_t^h = \Omega z_t^{-(1-\theta-\kappa)}$. We can see that as the formation of human capital is marginal diminishing ($\theta+\kappa<1$), the growth rate of human capital in high income families is lower, and the low income families enjoy the externality in the process of the accumulation of human capital, the growth rate will be higher than the high income families. This means that the gap between human capital is shrinking. If there is no difference in fertility rate, it also means that inequality will continue to decline.

By budget constraint Equation (2) and (3), the cost of raising and educating a child in the adulthood is $(v+e_t)w_t h_t$, in the old age alimony $\chi_{t+1}w_{t+1}h_{t+1}$ will be obtained by this child. So, the ratio of the actual cost of raising and educating a child (Discounted at interest rate of R_{t+1}) and the labor income is $v+e_t-(\chi_{t+1}w_{t+1}h_{t+1})/(R_{t+1}w_t h_t)$. Taking Equation (7), (8) and (15) into, it is equal to p_t. Based on this, we make p_t as residual relative price. It can be seen that family support reduces the relative price of childbearing [8]. Further, if the alimony given in the old age is understood as changing the remaining price rather than the income, then the disposable income of a person in a lifetime is $\left[1-\chi_t+\left(r_{t+1}^{\tau}-1\right)\tau_t\right]w_t h_t$. So, m_t is the ratio of the disposable income to the labor income of a person's life.

So, it can be seen from (14) that the family spends $\gamma/(1+\beta+\gamma)$ of the disposable income m_t at the price of p_t for raising and educating children when the fertility rate is not restricted by the population policy. The greater the parameter γ for determining the degree of concern to the children, the more input in children, and the higher the fertility rate is.

The relative price of raising children and the ratio of the disposable income are different in families with different income, so the fertility rate is different. According to the relative price of raising children, low income families' ratio of family support χ_{t+1} and the rate of human capital of their children g_t^h are

higher, making the ratio of alimony from their children and their income $\left(\chi_{t+1}w_{t+1}h_{t+1}\right)/\left(R_{t+1}w_t h_t\right)$ is higher. This will reduce the relative price of raising children p_t more, so low income families face a lower birth cost.

It can be seen from m_t that the Pay-As-You-Go part of social pension is same in families of all income, thus the redistribution effect is realized, making the ratio of the pension from Pay-As-You-Go and the income $\left(r_{t+1}^{\tau}-1\right)\tau_t$ is higher in families with low income. However, because of the high family support in pension χ_t/z_t^{η}, the relationship between the ratio of the disposable income and family income is uncertain.

From the point of view of the economic mechanism of family pension, on the one hand, it is equivalent to reducing the cost of childbirth because of the children's alimony in the future. This is the price effect of family pension, reflecting the support from the next generation. The lower the income of the family, the lower the price of childbirth, so the price effect is a negative relationship between fertility and income. On the other hand, because of the need to support the parents, the disposable income will reduce. This is the income effect of family pension, reflecting the support for the last generation. The lower the income of families, the greater the ratio of support to the parents, the less the disposable income, so the income effect is a positive relationship between fertility and income. Considering that it is a negative relationship between fertility and income in actual date [9], in the remainder of this analysis, we further assume that the price effect of family pension is always greater than the income effect. In theoretical calculations, this hypothesis makes it possible for us to ignore the difference in χ_t in the short term analysis. Thus the impact of government policies on the next phase of the economy can be more clearly analyzed.

As a result, low income families face lower fertility prices, making their fertility rates higher. The relationship between fertility and income means that the next generation of low and middle income people is more. This will deteriorate the degree of inequality. **Figure 1** firstly gives the case that when the fertility rate is not restricted to the upper limit of the population policy, the relationship between fertility rate n_t^0 and education input e_t^0 and relative income z_t. The part of the dotted line indicates that the fertility rate is beyond the upper limit of population policy. n_t^1 is the fertility rate of the low income families, n_t^0 is the fertility rate of the high income families. e_t^1 is the education input of the low income families, e_t^0 is the education input of high income families.

3.3. Family Differentiation Decision under Birth Control Population Policy

According to the analysis of the previous section, the fertility rate n_t rises with the decrease of relative income z_t, so when z_t is less than a certain critical value (set as z^*), $n_t > \overline{n}_t$, the fertility rate is beyond the upper limit of population policy. So the fertility rate and education input of these families are determined by Equation (16) and (17).

It can be seen that under the restriction of birth control, the fertility rate of

families with different income is the same, and the difference is reflected in the education for future generations. When the income effect of family pension is small, $\mathrm{d}e_t^1/\mathrm{d}z_t > 0$, $\mathrm{d}g_t^h/\mathrm{d}z_t < 0$, low income families have lower education input, but the growth rate of human capital for their children is high. Consistent with the free choice of fertility, this will help reduce the income gap.

Compared with the case under the unrestricted fertility rate, we can get $e_t^1 > e_t^0$. Birth control has prompted the family's investment in future generations from childbearing to education. Compared with the case under the unrestricted fertility rate, each child enjoys a higher education cost, and the level of human capital will grow faster. Further, $\mathrm{d}\left(e_t^1 - e_t^0\right)/\mathrm{d}z_t < 0$, birth control has encouraged lower income families to increase their education input in relation to high income families, so the degree of differential education is lower than it under the free choice of the fertility rate, this will reduce the inequality of the next generation. This is because in the free birth, low income families have a higher fertility rate, the more the number of births is reduced when the birth control is limited. These inputs, originally used for childbirth, have turned to consumption and education, and their children have enjoyed more cost in education.

So, under birth control, every families' education input will increase, it is good for the increase of human capital, and the gap between human capital is smaller and the inequality is lower between different income families. Figure 1 shows the relationship between the fertility rate n_t^1 and the education input e_t^1 and the relative income z_t in the case of population policy constraints, the part of the dotted line indicates that the fertility rate is lower than the upper limit of the population policy.

4. Social Security System under Birth Control

In the upper part, by solving the problem of family optimization, we get the choice of the fertility rate and education input of different income families under birth control (decided by Equation (14)-(17)). In this part, we will carry out a

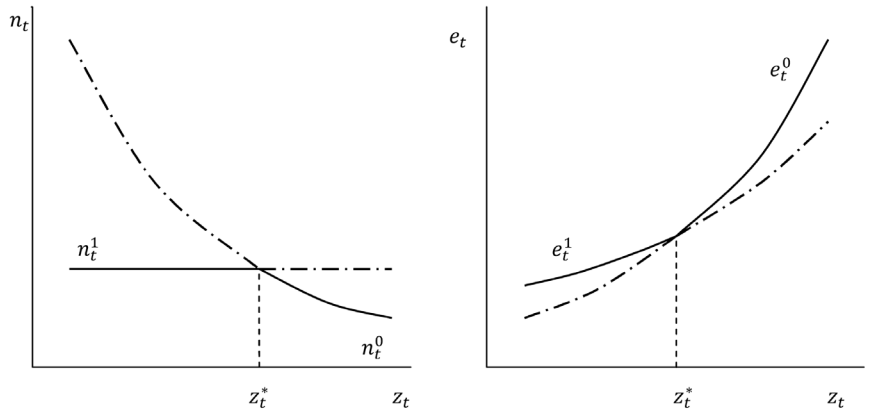

Figure 1. Family fertility and education input under different income.

policy analysis on this basis, Considering that the impact of increasing social security on income distribution. This paper mainly studies the influence of social pension on the inequality of the next generation, that is, the evolution of inequality.

The family fertility rate in this period determines the number of people in the next generation per income class, the family education input in this period determines the human capital gap between the next generation. They together determine the income distribution of the next generation. Therefore, we can predict the income distribution of the next generation by analyzing the impact of increasing social security on family fertility and education input.

4.1. Comparative Static Results

Considering changing the social pension policy for young people in the period *t*. There are two options for the government: One is to change the savings rate of a person in his pension fund account when he is in the adulthood, that is, to change τ_t in the case that τ_t^P is constant. The two is to change the standard of pension that individuals get from the social fund in the old age, that is, to change τ_{t+1}^P. It can be seen from Equation (14) to (17) that τ_t does not change the family planning and education decisions. In other words, as long as the social pension fund does not affect private savings, it will not affect the inequality distribution of the next generation. Therefore, we mainly analyze the changes in the family's fertility rate and educational choice after increasing the proportion τ_{t+1}^P of social coordinate.

It can be seen from Equation (14) and (15) that τ_{t+1}^P will only affect the fertility rate n_t^0, but it will not change the choice of education for families that are not restricted by fertility policy ($z_t \geq z_t^*$). This is because, by Equation (19), part of the motivation for education input comes from the pension of their children in the future. Although increasing social pension τ_{t+1}^P can reduce the enthusiasm of families to increase educational investment by reducing family pension χ_{t+1}, it will change the fertility rate at the same time. The fertility rate has not only changed the cost of education e_t, but also changed the return of the family pension. Through Equation (12) and (13), the proportion of education expenditure for the total expenditure of children is always θ. This means that the two effects will cancel each other out, and the change in fertility will not change the educational input for each child. And for families restricted by the birth control policy, the fertility rate is always $\overline{n}_t$.

A comparative static analysis of n_t^0 and e_t^1 to τ_{t+1}^P can be obtained. So,

$\mathrm{d}n_t^0/\mathrm{d}\tau_{t+1}^P \lessgtr 0$ is equivalent to $\dfrac{z_t^{\eta-\theta-\kappa}}{1-\chi_t-\tau_t^P+\mu/\left(R_{t+1}z_t\right)} \lessgtr \dfrac{(1-\theta)\Omega}{\upsilon g_t^H}$

$\mathrm{d}e_t^1/\mathrm{d}\tau_{t+1}^P \lessgtr 0$ is equivalent to $z_t^{\eta-\theta-\kappa} \lessgtr \dfrac{(1+\beta+\gamma)\Omega}{\gamma g_t^H}\cdot\overline{n}_t$

4.2. The Case of $\eta > \theta + \kappa$

When $\eta > \theta + \kappa$, critical families with income level z_t^n and z_t^e will be existed,

making $\mathrm{d}n_t^0/\mathrm{d}\tau_{t+1}^P \lesseqgtr 0$ is equivalent to $z_t \gtreqless z_t^n$, $\mathrm{d}e_t^1/\mathrm{d}\tau_{t+1}^P \lesseqgtr 0$ is equivalent to $z_t \lesseqgtr z_t^e$. According to this result, Figure 2 gives how to change the fertility rate and education input by different income families in the case that the government increases social pension τ_{t+1}^P.

It can be seen that families who are not restricted by birth control (relative income $z_t \geq z_t^*$) will only change the fertility rate when social security is raised (as the left in Figure 2, n_t^0 becomes $n_t^{0\prime}$), and the high income families will raise the fertility rate, while low income families will reduce the fertility rate, and the birth rate gap decreases. Families that are restricted by birth control ($z_t < z_t^*$) will only change the education input (as the right in Figure 2, e_t^1 becomes $e_t^{1\prime}$), and the high income families will increase the education input, while low income families will reduce the education input, the inequality of education will expand.

Therefore, in such a situation, social pension will increase the education inequality of middle and low income persons (limited by birth control), and increase the difference of the fertility rate of the high income people (not restricted by birth control). When the number of families restricted by birth control is in the majority, the social pension will expand the income inequality of the next generation by increasing educational inequality.

4.3. The Case of $\eta < \theta + \kappa$

When $\eta < \theta + \kappa$, critical families with income level z_{1t}^n, z_{2t}^n and z_t^e will be existed, making $\mathrm{d}n_t^0/\mathrm{d}\tau_{t+1}^P \lesseqgtr 0$ is equivalent to $\left(z_t - z_{1t}^n\right)\left(z_t - z_{2t}^n\right) \gtreqless 0$, $\mathrm{d}e_t^1/\mathrm{d}\tau_{t+1}^P \lesseqgtr 0$ is equivalent to $z_t \gtreqless z_t^*$. According to this result, Figure 3 gives how to change the fertility rate and education input by different income families in the case that the government increases social pension τ_{t+1}^P.

It can be seen that families that are restricted by birth control ($z_t < z_t^*$) will change the education input (as the right in Figure 3, e_t^1 becomes $e_t^{1\prime}$), contrary to the previous situation, the high income families will reduce the education input, while low income families will increase the education input, the

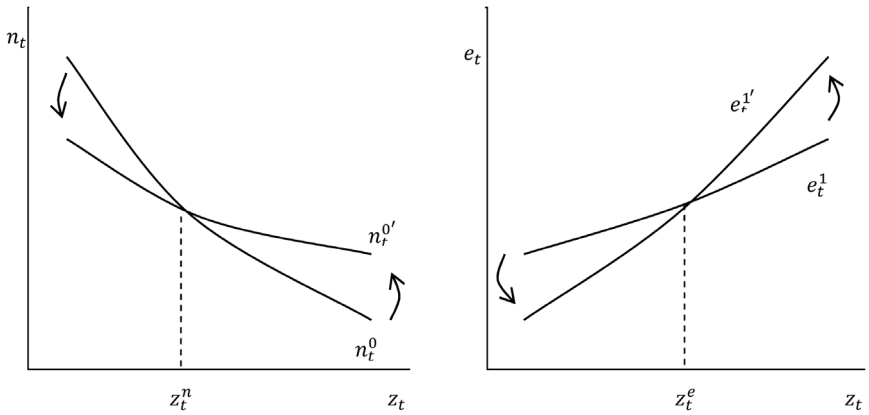

Figure 2. The influence of increasing social pension on the difference of fertility rate and educational inequality (when $\eta > \theta + \kappa$).

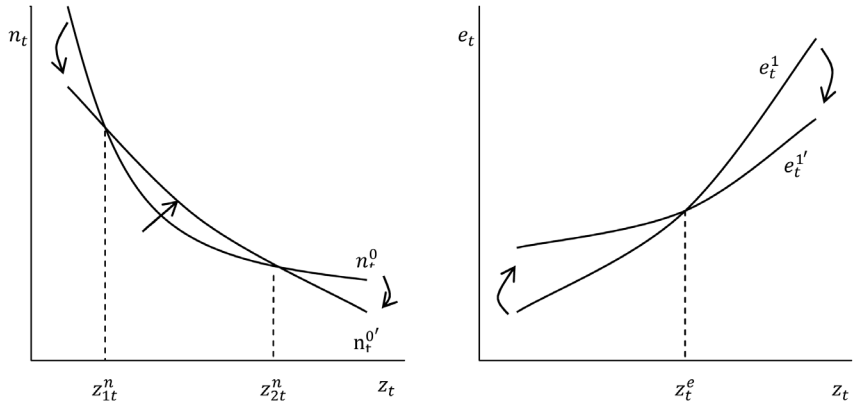

Figure 3. The influence of increasing social pension on the difference of fertility rate and educational inequality (when $\eta < \theta + \kappa$).

education inequality will decrease. Families who are not restricted by birth control ($z_t \geq z_t^*$) will change the fertility rate (as the left in **Figure 3**, n_t^0 becomes $n_t^{0'}$), But the gap between the fertility rates is likely to expand or shrink. Considering the relative income of families in the left figure should satisfy $z_t > z_t^*$, but the difference in fertility rate among families of $z_t \geq z_{1t}^*$ in the figure is increasing. So, in the view of families that are not restricted by birth control, the impact of the expansion of fertility difference is dominant.

Therefore, in this case, the impact of social pension on the next generation of inequality is completely contrary to the situation in the previous section. Increasing social pension will reduce the education inequality of middle and low income families (limited by family planning), and increase the difference of fertility among high income people (not restricted by family planning). When the number of families restricted by birth control is in the majority, the social pension will reduce the income inequality of the next generation by decreasing educational inequality.

4.4. Economic Explanation

In the view of the form of the household disposable income ratio m_t and the price of raising a child p_n improving the part τ_{t+1}^P of Pay-As-You-Go in social pension can increase the income of the family and the price of birth. We define these two effects as income effect and price effect. If all families face the same income and price effects, social pension will not affect the income distribution of the next generation. But for families with different incomes, the relative size of the income and price effects is different. Therefore, how social pension affects the income distribution of the next generation depends on the difference in income and price effects in different income families.

Firstly, taking income effects into consideration, although each family gets the same pension income from the social co ordinate part, the proportion of this part of the income to the family income is different. Low income families have a relatively higher pension income, so their disposable income ratio increases

more. And the increasing degree of the disposable income ratio is inversely proportional to the relative income of the family. $\left(\partial m_t / \partial \tau_{t+1}^P\right) z_t$ is the same for all families in the model.

And for the price effect, social pension is firstly substituted for family pension, and the reduction of family pension will increase the price of fertility. From the perspective of social pension instead of family pension, low income families is more sensitive to social pension, so the effect that social pension reduces family pension is greater. This is reflected in the (7) which determines the negative correlation between the derivative of the family pension degree χ_{t+1} for $\overline{\chi}_{t+1}$ and the income z_t, and this effect depends on the parameter η that determines the degree of family pension differences in different income families. The impact of family support on fertility prices is that the low-income families themselves are more dependent on family support, so the fertility price is increased more. This is because the income of children in low income families is growing faster, and the ratio of pension from their children and their own income is higher. This is reflected by the negative correlation between the derivative of fertility price p_t for the family pension degree χ_{t+1} and the income z_t, his effect depends on the parameter $1-\theta-\kappa$ that determines the degree of human capital growth difference.

Therefore, the degree of price growth has a negative relationship with family income, which may be a convex function of family income, or a concave function. $\left(\partial m_t / \partial \tau_{t+1}^P\right) z_t$ about z_t is possibly increasing or decreasing in the model. When $\eta+(1-\theta-\kappa)>1$, the price effect is convex about income. At this time, the mechanism of social pension instead of family pension has great difference in different income families. When $\eta+(1-\theta-\kappa)<1$, the price effect is concave about income. At this time, the mechanism of social pension instead of family pension has little difference in different income families.

It can be seen that the degree of differentiation of the income effect is not dependent on any parameters, the degree of differentiation of the price effect depends on whether the parameter $\eta-\theta-\kappa$ is more than zero.

When the fertility rate is freely chosen, the income effect makes each family tend to increase the fertility rate. Low income families have a high fertility rate and a greater income effect, so the income effect will make low income families to increase the fertility rate and increase the fertility difference of different income families. The price effect causes every family to reduce the fertility rate, and the fertility price of low income families increase more. Therefore, the price effect will make low income families reduce the fertility rate and reduce the fertility difference of different income families. It can be seen that the influence of income effect and price effect on the difference of fertility rate is opposite. When the difference in the price effect is high, the difference in family fertility will be reduced, as the left in **Figure 2**. When the difference of price effect is low, the difference of family fertility rate will be enlarged, as the left in **Figure 3**.

When fertility is limited by birth control, the income effect encourages every

family to increase education input. And low income families have a relatively low education input and a greater income effect. Therefore, the income effect will help low income families to increase education input and reduce the gap with high income families, so as to reduce the inequality of educational investment of different income families. And the price effect has prompted every family to reduce the investment in education, and the increase in the price of low income families is more. Therefore, the price effect will make low income families less input in education, thus increasing the inequality of the input of different income family education. It can be also seen that the influence of income effect and price effect on educational inequality is opposite. When the difference in the price effect is high, the families' education inequality will enlarge, as the right in **Figure 2**. When the difference of price effect is low, the families' education inequality will reduce, as the right in **Figure 3**.

5. Conclusions

According to the above analysis, the impact of social pension on the inequality of the next generation depends on two factors: the difference of family pension and the limit from population policy. The economic mechanism that social pension instead of family pension has an important role. Low income families are more dependent on the family pension, and the social pension instead of family pension has a greater impact on their fertility and education. Whether this effect changes the difference in fertility and education depends on whether high income families are also dependent on family pension as well.

When there is a significant difference in the degree of dependence on family support between families with different incomes, the inequality of education input between families restricted by birth control will expand, and the difference of the fertility rate between families unrestricted by birth control will reduce. As a result of the increase in education inequality, the income inequality of the next generation will increase. When families with different incomes have little difference in dependence on their children's support, the inequality of education input between families restricted by birth control will reduce, and the difference of the fertility rate between families unrestricted by birth control will expand.

This paper focuses on the analysis of the theoretical model and does not conduct an empirical study. This is the biggest limitation of the paper. The study of the redistribution effect of social security system under birth control can also be developed from the following aspects. We can divide education into public education and family education. We can continue to study the effect of sex preferences of families on the redistribution.

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