Risk Migration in Supply Chain Inventory Financing Service

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ABSTRACT

Inventory financing affects the risks of both for banks and supply chain companies. Traditionally, supply chain research focus more on material flow than financial. We construct a supply chain financing risk-information migration model (RMM). In this model, we discussed the preconditions to adopt inventory financing when the enterprises are facing cash constraints. And we simulated the whole operate of supply chain and bank behavior with Matlab. The simulation result shows if loan conditions are satisfied, the total risk value is reduced. Risk migration happens in the financing process. In this process, information-risk proportions are more reasonable.

Keywords: Supply Chain, Inventory Financing, Value of Risk

1. Introduction

Modern corporate finance theory is founded on the proposition that financial capital is supplied to firms by investors who have an “expectation of return”, and that, Cavinato (1991) research show supply chain can reduce cost, improve quality and make lead time shorter [1], thus it can improve competence of the whole supply chain. In traditional supply chain, researchers focus more on material flow than cash flow. It is essential to corporate supply chain research with finance theory. Reciprocally, such expectation represents the firm’s “cost of financial capital” optimization along with materials in supply chain operation. We observe that supply chain theory begins with “irrelevance” pronouncements about a firm’s value being independent of its supply chain optimization. Risk sharing in supply chain financing, which are ignored in most supply chain optimization, in response to these unrealistic assumption, theoretical development has subsequently come to be directed at providing models that are descriptive of the way corporate financial with supply chain [2]. To this end, supply chain financial has increasingly been recognized as first order.

Supply chain structure is defined as the associations among supply chain members [3]; this structure can benefit both vertical and horizontal connected companies

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supported by a great deal of empirical work devoted to the measurement of bond risk premium and the analysis of their dynamics [6]. While financing a firm, although a lender tries to perceive its exposure to default risk by looking into borrower’s accounts, due to lack of proper information, the buying or selling capacities of preceding or following stakeholders, as in manufacturer and its retailer of supply chain remain unknown. Lender’s analysis is then based on certain assumptions. This lack of information is a reality, especially for small firms that are not publicly listed.

The paper is organized as follows. In Section 2 we introduce basic notation, terminology and assumptions. The risk information migration model RMM model and solutions is presented in Section 3. We discuss inference and parameter estimation for RMM and presented experiment results in Section 4. Finally, Section 5 presents our conclusions.

2. Assumptions, Notations

For simplicity we assume that both the firms have no other assets but the cash available with them before they commence their respective activities. Both manufacturer and retailer have no fix asset such as land, buildings, machines etc. Account payable, account receivable, cash, inventory and short term borrowing are considered in the model. \( q_r \) are predicted based on constant elasticity form. The expected quantity of production is equal with mathematical expectation of \( q_r \) \( \hat{q}_r = E(q_r) \). Because of cash constraints and ability to get loan from bank, real quantity is less than expected quantity. Manufacturer firstly predicts the retailer’s purchasing before producing, and decides a price \( p_m \) of finished product. Based on the cash constraints of manufacture, \( L_m \) will be decided. Bank will evaluate the risk and give a max loan available for bank. Retailer predicts a market demand and purchase from manufacture. At any time, cash owned by manufacture and retailer greater than zero, or they will bankrupt.

\( VaR \) is adopted to calculate risk value of manufacture and bank loan. It is equal the maximum loss from the specific confidence level. Based on Jorion, 1997, we can calculate \( VaR \) with Equations (1) and (2) [7].

\[
\begin{align*}
VaR_m &= E(\Pi_m) - \Pi_m^* \\
VaR_M &= E(\Pi_M) - \Pi_M^*,
\end{align*}
\]

A simple two-stage supply chain is considered that consisting of a single manufacturer and a retailer. Manufacture produces goods at a constant rate and ships it to retailer with zero lead time. Retailer is of the classical newsvendor type. Retailer returns the defective quantity to manufacturer who is liable to compensate for it at the end of the period. One bank provides loans to both manufacture and retailer if they applied and passed evaluation of risk level.

\[
\begin{align*}
i & \quad \text{Interest rate of bank} \\
i' & \quad \text{Interest rate of deposit where } i' < i \\
x_m & \quad \text{Current cash of manufacture} \\
p_m & \quad \text{Price of raw material manufacture bought from supplier} \\
L_m(t) & \quad \text{Raw material inventory of manufacture} \\
c_i & \quad \text{Production cost of manufacture} \\
\hat{\Pi}_m & \quad \text{Expected profit without inventory finance} \\
\Pi_m & \quad \text{Expected profit with inventory finance} \\
\Pi_l & \quad \text{Real profit with inventory finance} \\
q_m & \quad \text{Real quantity need to be produced by manufacture} \\
\hat{q}_m & \quad \text{Expected quantity of product being produced by manufacture} \\
q_r & \quad \text{Retail quantity being sold} \\
p_r & \quad \text{Price of product manufacture selling to retailer} \\
x_r & \quad \text{Current cash of retailer} \\
\alpha & \quad \text{Coefficient of labor changing to product} \\
y_m & \quad \text{Account receivable of manufacture} \\
y_r & \quad \text{Account receivable of retailer} \\
L_m & \quad \text{After risk evaluation, lend available from bank to manufacture} \\
L_m^* & \quad \text{After analysis of market information, load applied from manufacture to bank} \\
z_m & \quad \text{Accounts payable of manufacture} \\
z_r & \quad \text{Accounts payable of retailer} \\
l_m & \quad \text{Finished product inventory of manufacture} \\
\hat{VaR}_L & \quad \text{Value at risk of retailer in borrowing from bank} \\
\hat{VaR}_M & \quad \text{Value at risk of manufacture in borrowing from bank} \\
\theta_L & \quad \text{The portion of } \hat{VaR} \text{ of bank loan after standardized} \\
\theta_M & \quad \text{The portion of } \hat{VaR} \text{ of manufacture earnings after standardized}
\end{align*}
\]

3. The Model

3.1. Bank Profit from Loan

The cost of bank cash is equal \( i' \), thus the cost of manufacture cash is \( i \). Based on maximum expectation of return principle, we can easily get condition of \( i' < i \). The bank can predict the payable of manufacture after one period of product time.

\[
\text{Payable}_m \leq \min \{ \hat{q}_m, q_r \} p_m
\]

If manufacture apply loan from bank without mortgage, the expected profit of bank as follow:

\[
\hat{\Pi}_L = \min \{ L_m (1+i) \cdot \min \{ \hat{q}_m, q_r \} p_m \} - L_m (1+i')
\]

By the profit function 4, when \( L_m (1+i) < \
\[ \min \{ q_m, q_r \} \] bank can get more profit by improve \( L_{\text{fin}} \). When \( L_{\text{fin}}(1+i) > \min \{ q_m, q_r \} \), no matter how much bank loans, bank will loss profit. So the maximize bank profit and exist conditions show as follow:

\[
\left\{ \begin{array}{l}
\max (\tilde{\Pi}_m) = \min \{ q_m, q_r \} \frac{(i-i')}{1+i} \\
L_{\text{fin}} = \min \{ q_m, q_r \} \frac{1}{1+i} \\
L_{\text{fin}}(1+i) \geq \min \{ q_m, q_r \} p_m \geq L_{\text{fin}}(1+i')
\end{array} \right.
\]

When adding inventory mortgage to exist bank profit functions, we can get the object function of bank profit is:

\[
\left\{ \begin{array}{l}
\max (\tilde{\Pi}_m') = \min \{ q_m, q_r \} \frac{(i-i')}{1+i} \\
L_{\text{fin}} = \min \{ q_m, q_r \} \frac{1}{1+i} \\
L_{\text{fin}}(1+i) \geq \min \{ q_m, q_r \} p_m \geq L_{\text{fin}}(1+i')
\end{array} \right.
\]

3.2. Manufacture Profit from Production

Manufacture decides the expected most profit produce quantity \( \tilde{q}_m \). Then manufacture will apply a loan of \( L_{\text{fin}} \), the received loan of manufacture is \( L_{\text{fin}} = \min \{ L_{\text{com}}, L_{\text{fin}} \} \). We can get profit of manufacture as follow:

\[
\tilde{\Pi}_m = \min \{ q_m, q_r \} p_m - (c_i + p_m / \alpha) q_m - iL_{\text{fin}}
\]

\( L_{\text{fin}} \) is used to conquer the shortage of cash, because \( i' < i \), manufacture will maximize the usage of cash available of its own, at the end of production period, \( x_m = 0 \). The initial cash of manufacture is \( x_{\text{m0}} \). By borrow from bank, manufacture have more opportunities to maximize \( \tilde{\Pi}_m \):

\[
\tilde{\Pi}_m = \tilde{q}_m p_m - \bar{c}_m q_m - iL_{\text{fin}}
\]

\( \bar{c}_m q_m = x_{\text{m0}} + L_{\text{fin}} \)

\( \bar{c}_m = c_i + p_m / \alpha \)

\( \tilde{\Pi}_m = \tilde{q}_m p_m - \bar{c}_m q_m - iL_{\text{fin}} \)

\( = q_m p_m - x_{\text{m0}} - (1+i) L_{\text{fin}} \)

\( \tilde{\Pi}_m = q_m p_m - \bar{c}_m q_m - iL_{\text{fin}} \)

\( = q_m p_m - \bar{c}_m q_m - i(\bar{c}_m q_m - x_{\text{m0}}) \)

\( = q_m [p_m - \bar{c}_m (1+i)] + i x_{\text{m0}} \)

From Equations (10) and (11), we can get the preconditions of loan are \( q_m p_m > x_{\text{m0}} \) and \( p_m > \bar{c}_m (1+i) \).

When the maximize profit is satisfied,

\( L_{\text{fin}} = \max \{ \bar{c}_m q_m - x_{\text{m0}}, 0 \} \).

3.3. RMM Model and Conditions

Based on the analysis both of bank and manufacture decisions, the cash investment and \( \text{VaR} \) value of both bank and manufacture should be balanced, we conclude the following RMM model:

\[
\min \left( \text{VaR}_R / x_{\text{m0}} - \text{VaR}_B / L_{\text{fin}} \right)
\]

Subject to:

\[
\left\{ \begin{array}{l}
L_{\text{fin}} = \min \{ q_m, q_r \} \frac{p_m + s_i I_{\text{fin}}}{1+i} \\
L_{\text{com}}(1+i) \geq \min \{ q_m, q_r \} p_m \geq L_{\text{com}}(1+i')
\end{array} \right.
\]

\( L_{\text{fin}} = \max \{ \bar{c}_m q_m - x_{\text{m0}}, 0 \} \)

\( \tilde{\Pi}_m = \tilde{q}_m p_m - \bar{c}_m q_m - iL_{\text{fin}} \)

\( \tilde{\Pi}_m = \min \{ L_{\text{fin}}(1+i), \min \{ q_m, q_r \} p_m - s_i I_{\text{fin}} \}

\( \text{VaR}_L = E(\tilde{\Pi}_m) - \tilde{\Pi}_m \)

\( \text{VaR}_M = E(\tilde{\Pi}_M) - \tilde{\Pi}_M \)

3.4. Solutions to RMM Model

We use matlab simulate the whole manufacture, retail and loan process with the model built above (Figure 1), the process of simulation shows as Figure 2. The demand for a consumer product and populate the model with \( N \) consumers. The basic demand function for each agent is well behaved:

\( D_i = f_i - P \)

The \( f_i \) are selected at random and represent the heterogeneous tastes of customers. The sum of the individual demands \( \sum_i D_i \) represents the whole market demand. For fashion goods, the size of the market can be assumed small compared to consumers’ incomes and can therefore reasonably ignore the difficulties which the Sonnenschein, Mantel, Debreu theorems (for example Sonnenschein 1972) raise for the shape of aggregate demand functions [8]. The demand curves for each individual customer are well behaved, and we assume that the sum of these is also well-behaved.

We calculate \( \text{VaR} \) follow four steps as follow:

1) Based on Equation (12), calculate the market demand series \( \{ q_1, q_2, q_3 \cdots q_n \} \) and then we can get expectations of \( E(q_i) \) \( q_m = E(q) \) which means the manufacture can predict the market demand in the long
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Figure 1. RMM model in simulation experiment.

Figure 2. Flow of simulation experiment.

Figure 3. Relation curves between product quantity and VaR.

Table 1. Typical data results of several experiments.

<table>
<thead>
<tr>
<th>q_m</th>
<th>p_m</th>
<th>τ_m</th>
<th>s_f</th>
<th>VaR_M</th>
<th>VaR_M'</th>
<th>VaR_L</th>
<th>VaR_L'</th>
<th>ω_L</th>
<th>ω_L'</th>
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</thead>
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<tr>
<td>40000</td>
<td>40</td>
<td>28</td>
<td>20</td>
<td>3256.23</td>
<td>5675.20</td>
<td>7846.01</td>
<td>1102.30</td>
<td>0.71</td>
<td>0.16</td>
</tr>
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<td>40000</td>
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<td>50</td>
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<td>1340.35</td>
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<td>5769.20</td>
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<td>2456.43</td>
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<td>1263.28</td>
<td>4320.43</td>
<td>5472.21</td>
<td>702.16</td>
<td>0.81</td>
<td>0.14</td>
</tr>
</tbody>
</table>

2) Select data basing on cash constrained conditions of \( q_m p_m > x_m \) and \( p_m > \tau_m (1+i) \).

3) Calculate \( \Pi_L \) and \( \Pi_m \) based on Equations (5), (6), (7) and (8).

4) Calculate \( VaR_m \) and \( VaR_b \) based on Equations (1) and (2).

Necessary and sufficient conditions on the bivariate utility function vary according to the conditions imposed on the joint distribution of the risks. If only independent risks are considered, then any utility function which is concave in its first argument will satisfy the condition of risk aversion. If risk aversion is required for all possible pairs of risks, then the bivariate utility function has to be additively separable.

4. Experiments Results (Table 1)

The operating of supply chain is divided into a certain number of periods and the model with suitable demand forecasts is solved to yield scheduling/planning decisions for each period, and only those belonging to the first period are implemented. At the end of the first period, the state of the system, including inventory levels, is updated and the cycle is repeated with the horizon advanced by run;
one period considering the demand forecast for the new period, which is now available. Therefore, the deterministic formulation next described comprises a set of planning periods, and only the first one includes the detailed scheduling decisions with shorter time increments. Such detail period moves as the model is solved in time, thus the term rolling horizon.

Calculate VaR of both manufacture and bank, then standardize VaR to $[0,1]$, then we can get 
$$\theta = \frac{VaR_m}{(VaR_m + VaR_u)}$$
basing on Equation (1) and (2). Get $\theta_m$ by $\theta_m = 1 - \theta$, Table 1 shows the typical data results of several experiments in computer simulation when $x_{m0} = 1000$, $i = 0.10$ and $i' = 0.06$. With different initial variables, the bank $VaR$ will decrease when adopt inventory mortgage, the potential profit is growing. For the manufacture, after use inventory mortgage, $VaR$ is larger than before. The potential income is growing because bank can offer more loans which reduce the manufacture shortage of cash, so the manufacture can produce more to maximum profit.

When the manufacture satisfies $q_m p_m > x_{m0}$ and $p_m > \tau_m (1+i)$, accompany with market demand increasing, the VaR of bank decrease because manufacture’s capability of making profit. If using inventory mortgage, the VaR value for manufacture is increasing because more cash are put in producing and inventory (Figure 3).

5. Conclusions

We discussed manufacture and retail supply chain structure which both facing cash-constrain and a bank that finances the manufacturer. Supply chain inventory mortgage must satisfy preconditions of $q_m p_m > x_{m0}$ and $p_m > \tau_m (1+i)$, that is member of supply chain will use self-owned capital before using inventory mortgage, and the cost of loan must less than the profit rate. In inventory mortgage, both bank and manufacture are benefit because the risk migration. After migration of risk, it is more compatible with the information shared between supply chain member and bank. For supply chain members, they have more market information than bank in production operate process, after sharing inventory information with bank, this reduce the bank shortage information. So the migration of risk can help optimize the whole supply chain and bank.

REFERENCES


