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A New Vectorial Order Approach Based on the Classification of Tuples Attribute and Relative Absolute Adaptive Referent: Applications to Multicomponent Images

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Abstract

In this paper, we are presenting a new vector order, a solution to the open problem of the generalization of mathematical morphology to multicomponent images and multidimensional data. This approach uses the paradigm of P–order. Its primary principle consists, first in partitioning the multi-component image in the attribute space by a classification method in different numbers of classes, and then the vector attributes are ordered within each class (intra-order-class). And finally the classes themselves are ordered in turn from their barycenter (inter-class order). Thus, two attribute vectors (or colors) whatever, belonging to the vector image can be compared. Provided with this relation of order, vectors attributes of a multivariate image define a complete lattice ingredient necessary for the definition of the various morphological operators. In fact, this method creates a strong close similarity between vectors in order to move towards an order of the same principle as defined in the set of real numbers. The more the number of classes increases, the more the colors of the same class are similar and therefore the absolute adaptive referent tends to be optimal. On the other hand, the more the class number decreases or equals two, the more our approach tends towards the hybrid order developed previously. The proposed order has been implemented on different morphological operators through different multicomponent images. The fundamental robustness of our approach and that relating to noise have been tested. The results on the gradient, Laplacian and Median filter operators show the performance of our new order.

1. Introduction

Mathematical morphology (MM) is a non-linear image processing and irreversible approach based on a fundamental structure: Complete lattice $(\mathcal{L}, \leq)$. Hence the construction of morphological operators needs an order between the elements to be treated. Theoretically a partial order is sufficient to construct complete lattices, but total orders are preferred because they do not introduce false colors [1]. Equipped with the complete lattice structure, it is possible to define morphological operators for any type of lattice. Although it is well-defined for binary and grayscale images, there is no admitted absolute order supported extension which enables to perform morphological operations on vector or multimodal images. Thus, the extension of mathematical morphology to multivariate images is a field which is the object of active research. Numerous different approaches have been proposed in the literature for color images and hyper spectral images [2] [3] [4]. Recently, we developed a new hybrid order based on an order reduced to Adaptive absolute reference [5]. We have highlighted the relevance of the morphological operators using this order as well as its robustness to noise. Unfortunately, this approach does not fully manage the proximity between tuples because the comparison between tuples is dependent on a referent and this referent is not necessarily the optimum. In order to take into account the notion of proximity as defined in the set of real, we propose in this article a variant of order of type P-order. It is based on the partition of tuples in different classes. The vectors are ordered in each class and the groups are ordered to obtain a total order. Contrary to the hybrid order [5], which does not support the proximity between tuples since the comparison between tuples is only a referent dependent; this reference is not necessarily the optimum. The order by classification we propose in this article has the advantage of taking into account the notion of proximity between tuples.

The paper is organized as follows: in Section 2, we present the proposed approach then a theoretical and experimental validation of our approach is presented in Section 3. We shall conclude with some perspectives for future works, which will specifically be based on morphological classification and segmentation vector.

2. Research Question

Construction of morphological operators needs a complete lattice structure. In other words, the definition of new orders for the extension of the scalar algorithms vector case. The main issue with this kind of extension is based on the definition of a suitable vector ordering, because there is no natural ordering for

vectors particularly in the case of mathematical morphology that makes it complex for color images application. The main of this paper is to presenting a new vector order, a solution to the open problem of the generalization of mathematical morphology to multicomponent images and multidimensional data [3] [4] [5] [6].

3. Methodology

We present in this section the principle and algorithms for our new order.

3.1. Principle (Vector Order by Classification)

Firstly, the multidimensional compact histogram multicomponent of the image is calculated. Then the image attribute space is partitioned into a number of K-classes. After, we order each class attributes vectors (intraclass order) using the hybrid vector order we proposed recently in [5]. Finally, we propose a strategy for ordering two tuples belonging to different classes. It consists to order the centers of gravity of the relevant classes using the hybrid order. Also, the algorithm responding to this new order by classification is presented in Section 2.2. We emphasize that the partitioning algorithm used is the *K*-means method because of its simplicity and its good performance. In fact, this algorithm was long used on large data sets because of its speed. The work on the *K*-means method was initiated by Hartigan in 1975 [7]. In its algorithmic principle it combines the pixels of an image in separate *K*-classes, *K* being set by the user. An initialization of the *K* centers of gravity representing the *K* clusters is mandatory.

Each pixel of the image is randomly assigned to a cluster and it is iterated as follows: the centers of gravity of the different groups are recalculated and each pixel is again assigned to a group according to the nearest center of gravity. Convergence is achieved when the centers are fixed.

Let n be the number of color components of the image, $X_i = (x_{i1}, \cdots, x_{in})$ the vector associated with any one color of the image and N is the number of pixels of the image. At each an iteration, the vectors X_i are divided into K groups. Note $(y_1, \cdots, y_K)$ all the centers of gravity of the groups, calculated by averaging the coordinates of group of vectors. An attribute X_i vector pixel is group *j* if:

$$\|X_i - y_j\| = \min_{1 \le u \le K} \|X_i - Y_u\| \tag{1}$$

Each point is assigned to a class with the function defined as follows:

$$f : \begin{cases} \{X_1, \cdots, X_i, \cdots, X_N\} \\ X_i \to f(X_i) \end{cases} \tag{2}$$

Compact groups are then obtained by minimizing the following expression J:

$$J = \sum_{i=1}^{N} \|X_i - C_{f(X_i)}\| \tag{3}$$

where $\{C_1, \cdots, C_K\}$ is the center of gravity of all the classes.

The *K-means* algorithm works by minimizing the Equation (3) on J iteratively.

3.2. Algorithm of Proposed Order

Let's call I the image of the matrix associated with a multicomponent image N of the plan $(N > 1)$. The compact histogram of I is divided into K-classes I_k *where* $I = \bigcup_{K=1}^{K} I_k$ and I_k *the kth partition and* G_k *the barycenter of the I_k class.*

Let X_i^k be the vector I of K-class *and Cardinal* of the *class I_k*.

Consider

$$x_{ref}^k = min\left(x_i^k\right) \quad \text{for } 1 \le i \le |I_k|, \tag{4}$$

the minimum absolute referent *vector.* Where each component corresponds to the minimum per component of all elements of that class and

$$G_{ref} = min\left(G_k\right) \quad \text{for } 1 \le k \le K. \tag{5}$$

The minimum absolute referent vector centers of gravity and *h* is the bit-mixing function defined in [5].

Let's x_i^k and x_j^k be two vectors to be compared (intraclass order):

$$x_i^k \le x_j^k \Leftrightarrow \begin{cases} d\left(x_i^k, x_{ref}^k\right) < d\left(x_j^k, x_{ref}^k\right) ou \\ d\left(x_i^k, x_{ref}^k\right) = d\left(x_j^k, x_{ref}^k\right) \\ et \\ h\left(x_i^k\right) \le h\left(x_j^k\right) \end{cases} \tag{7}$$

To classify the vectors belonging to different classes we will introduce the notion of order between two classes.

Let I_{k1} and I_{k2} be two distinct classes to be compared.

$$I_{k1} \le I_{k2} \Leftrightarrow \begin{cases} d\left(G_{k1}, G_{ref}\right) < d\left(G_{k2'}, G_{ref}\right) ou \\ d\left(G_{k1}, G_{ref}\right) = d\left(G_{k2'}, G_{ref}\right) \\ et \\ h\left(G_{k1}\right) \le h\left(G_{k2}\right) \end{cases} \tag{8}$$

Now, let's consider two vectors x_i^{k1} of I_{k1} and x_j^{k2} of I_{k2}, we admit the following proposal:

$$x_i^{k1} \le x_j^{k2} \Leftrightarrow I_{k1} \le I_{k2}. \tag{9}$$

We can illustrate our algorithm by the flow chart of the Figure 1.

Principal Algorithm

Let *v* be a neighborhood order 3 × 3 around pixel p of image *I with (i,j) as coordinates.* We designate *L* and *M* respectively the number of rows and columns of image *I.*

Let *Hc* be the compact histogram of an image, *K* the number of classes and Classe a struct which contains the colors by classe and the index by classes relatively to the compact histogram, ordred by increasing order.

[*G, Classe*] =**K-MeansBis** (*Hc, K*)

[*HistoOrderlyFinal, IndexOrderlyFinal, ClasseFinal*]

=**OrderByClassification** (*Hc, Classe, K*)

For i=1 to L do

```
┌─────────────────────────────────┐
│           Data :                │
│   - Multicomponent Image        │
│   - Noise                       │
└─────────────────────────────────┘
              │
              ▼
┌─────────────────────────────────┐
│      Partition of Image :       │
│      Function K-MeansBis         │
└─────────────────────────────────┘
              │
              ▼
┌─────────────────────────────────┐
│   Order a Compact Histogram :   │
│      Function HistoOrderly       │
└─────────────────────────────────┘
              │
              ▼
┌─────────────────────────────────┐
│     Principal Algorithm :       │
│      Orderbyclassification       │
└─────────────────────────────────┘
              │
              ▼
┌─────────────────────────────────┐
│           Results :             │
│       -Gradient Image           │
│       -Laplacian Image          │
│       -Median Image             │
│             -                    │
└─────────────────────────────────┘
```
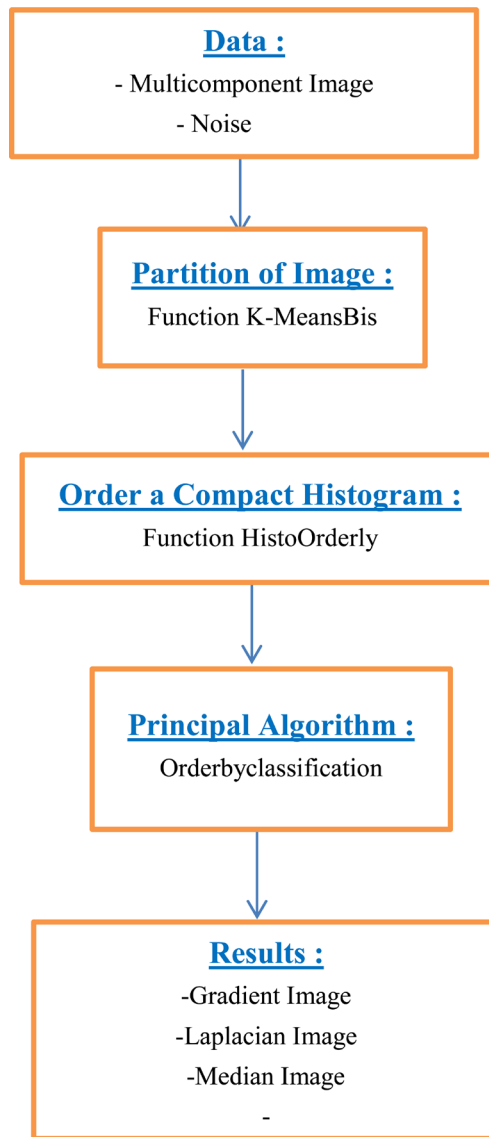
Figure 1. The flow chart of our algorithm.

For j=1 to M do
Determination of the neighborhood $W=(Vp)$ around the pixel $p(i,j)$

// Determination of V_{min} and V_{max}. //

[Pos]=**FindPositions** (*HistoOrderlyFinal*, *W*); // returns the position of colors in W relatively to *HistoOrderlyFinal*//
V_{min}= HistoOrderlyFinal(Min(Pos));
V_{max}= HistoOrderlyFinal(Max(Pos));

// Determination of the Gradient and the Laplacian of the pixel $p(i,j)$. //

Grad(i,j)=Gradient (p(i,j), V_{min}, V_{max});
Lapl(i,j)=Laplacian (p(i,j), V_{min}, V_{max});
MedFilter_Grad(i,j)=MedianFilter(P(i,j));

MedFilter_Lapl(i,j)=MedianFilter(P(i,j))
End for
End For

Function [V_{max}, V_{min}] = **HybridOrderMaxMin**(*w*)
// *W*: Histogram or matrix of colors //
// Let **I** be the matrix associated with a multicomponent image of *P* plans. We note *Max* and *Min*, the conventional comparison operators that are the maximum and minimum respectively.//
// Let $V_m \in W$ des *N-tuple* define V_{Ref} so as: //

$$V_{Ref\_min}^k = \left(Min\left(V_m^k\right)\right), \forall 1 \le m \le length(w) \ and \ 1 \le k \le P \tag{1}$$

$$V_{Ref\_max}^k = \left(Max\left(V_m^k\right)\right), \forall 1 \le m \le length(w) \ and \ 1 \le k \le P \tag{2}$$

// The dilated V_{max} *and the eroded* V_{min} *of the histogram W* are defined by the following expressions:

$$V_{MAX} = \left\{ X \in V, Min\left\{ dist\left(X, V_{Ref\_max}\right)\right\}\right\} \tag{3}$$

$$V_{Min} = \left\{ X \in V, Min\left\{ dist\left(X, V_{Ref\_min}\right)\right\}\right\} \tag{4}$$

// In the case of no unicity of V_{max} and V_{min}, *the solution vectors are ordered according to the vector order by* bit mixing, see the article [5] //
End

Function [*IndexNewOrderly*, *HistoNewOrderly*] = **HistoOrderly** (*HistoCompact, Index*)
// *HistoCompact*: the compact histogram of an image, see the article [5]. //
// *Index*: is a vector matric containing the position or order of colors in HistoCompact. //
N=length(HistoCompact); // return the total number of colors in the image
IndexNewOrderly$_{min}$=[ ];
IndexNewOrderly$_{max}$=[ ];
HistoNewOrderly$_{min}$=[ ];
HistoNewOrderly$_{max}$=[ ];
HistoNewOrderly=[ ];
j←0
do
j←j+1
[V$_{max}$, V$_{min}$] = ***HybridOrderM$_{ax}$M$_{in}$***(HistoCompact)
Pos$_{Min}$= *FindPosition* (V$_{min}$) // return the position of V$_{min}$ in HistoCompact
Pos$_{Max}$= *FindPosition* (V$_{max}$)
IndexNewOrderly$_{Min}$=[IndexNewOrder$_{min}$, Pos$_{Min}$];
IndexNewOrderly$_{Max}$=[IndexNewOrder$_{Max}$, Pos$_{Max}$];
IndexNewOrderly=[IndexNewOrderly, IndexNewOrderly$_{min}$];//returns the

position of the minimums//

HistoNewOrder =[HistoNewOrder, V_{min}]; // returns colors of the Histo-Compact ordered by increasing order//

HistoCompact(Pos_{Min})= [ ];

Index(Pos_{Min}) =[ ]

While (j<N)

End

Function [G, Classe] =***K-MeansBis*** (HistoCompact, K)

// HistoCompact: The Compact Histogram

// K: The number of classes

// *G*: The matrix of the centers of gravity of the classes ordered by the function HistoOrderly//

// Classe.Index: struct of tree type. It return the list of the positions of colors with regard to HistoCompact by Class //

// Classe.Histo: struct of tree type. It return the list of colors (Histogram) with regard to HistoCompact by Class //

[G1, Index1]=***kmeans*** (HistoCompact, K);

// kmeans: function which returns the centroids of classes in G1 and the class number which each of the color of HistoCompact belongs to, in Index1 under Matlab //

Index2=[1:K]*;* // Matrix vector containing the numbers in increasing order from 1 to K

[Index2NewOrderly, G]= *HistoOrderly*(*G*1, *Index*2);

For k=1:K

Classe(k).Index=***FindPositions***(Index1, *Index*2*NewOrderly*(k)); // returns the positions of colors of the class number k//

Classe(k).Histo=*HistoCompact*(Classe(*Index*2*NewOrderly*(k)).Index)); // returns the positions of colors of the class number k//

end

End

Function [HistoOrderlyFinal, IndexOrderlyFinal, ClasseFinal] =***OrderByClassification***(Hc, Classe, K)

K:// The number of classes //

Hc:// The Compact Histogram//

HistoOrderlyFinal:// Compact Histogram ordered by classification order//

IndexOrderlyFinal:// Positions of the colors of Compact Histogram ordered by classification order//

ClasseFinal: // struct of tree type. It return the colors and list of the positions of colors ordered by classification order, with regard to HistoCompact by Class //

HistoOrderlyFinal=[ ];

IndexOrderlyFinal=[ ];

[G, Classe] =***K-MeansBis*** (HistoCompact, K)

For k=1:K

       [*IndexNewOrderly,*   *HistoNewOrder*ly]=   ***HistoOrder-ly***(Classe(k).Histo, Classe(k).Index)

       ClasseFinal(k).Histo=*HistoNewOrder*ly;

       ClasseFinal(k).Index= *IndexNewOrderly;*

       HistoOrderlyFinal= [HistoOrderlyFinal, HistoNewOrderly];

       HistoOrderlyFinal=[HistoOrderlyFinal, IndexNewOrderly];

end

End

4. Results and Discussions

In this section, the different parameters below are studied in order to highlight the relevance and robustness of our approach. They are as follows:

- The influence of the number of classes upon the proposed orders;
- Statistical link between our order and other orders;
- The performance of the proposed order through the operators gradient and laplacien as compared to other orders;
- The robustness of the median filter of the proposed order face to noise regard to other orders.

4.1. The Influence of the Number of Classes on the Proposed Orders

We study here the influence of the number of classes on the proposed order. To do this, we first review some mathematical quantities [8], which are useful for our study.

4.1.1. Hamming Distance

Let *A* be an alphabet and *F* the set of sequences of length *n* with values in *A*. **The Hamming distance** between two elements *a* and *b* of *F* is the number of elements of the set of images of *a* that differ from that of *b.*

In other words, if *d*(,) denotes the Hamming distance:

$$\forall U,V \in F \quad U = (u_i)_{i \in [1,n-1]} \text{ and } (v_i)_{i \in [1,n-1]} \quad d(U,V) = \left| \{i : u_i \neq v_i\} \right| \quad (8)$$

where the notation $\left| E \right|$ denotes the cardinality of the set *E.*

4.1.2. Mutual Information

Let (X, Y) be a pair of random variables of joint probability density given by P (X, Y) Let us note the marginal distributions P (x) and P (y). Then the mutual information is in the discrete case:

$$I(X,Y) = \sum_{x,y} p(x,y) \log \frac{P(x,y)}{p(x)p(y)} \quad (9)$$

4.1.3. Correlation Coefficient

The correlation coefficient between two random variables *X* and *Y* each having a

(finite) variance, denoted Cor (*X, Y*) or sometimes ρ_{XY} is defined by

$$Cor(x,y) = \frac{\text{cov}(x,y)}{\rho(x)\rho(y)} \tag{10}$$

where $\text{cov}(x,y)$ denotes the covariance of the variables X and Y and their standard deviations.

4.1.4. The Influence of the Number of Classes upon the Proposed Orders

The proposed order requires a partitioning of the multidimensional histogram of the image and therefore the influence of the corresponding K parameter should be studied.

In fact, the variation of K has been studied on many natural and synthetic color images. To illustrate our approach, an analysis was made on the synthetic image Savoise of **Figure 2** with its compact multidimensional histogram. The tuples or "colors" are ordered according to the lexicographic order indicated by their index of order.

We also presented some results on real images. For fundamental reasons we made K vary between 2 and 6 K as shown in the tables below.

For the proposed new order, (*i.e.*) order by classification, **Table 1** shows the variation of the order of the tuples as a function of the number of classes of the

Synthetic image Savoise	compact histogram (Hc)		
	Index (**lexicographic order**)	*R*	*G*	*B*
	1	63	108	99
	2	70	123	77
	3	86	94	70
	4	107	91	56
	5	121	95	69
	6	187	87	187

Figure 2. Synthetic image Savoise and its compact histogram (ordered according to the lexicographic order), and the index of tuples (position of colors in the compact histogram).

Table 1. Order by classification according to the number of K classes of the image Savoise.

Synthetic image: Savoise	*K: Number of classes*				
	2	3	4	5	6
	3	2	2	3	3
	4	1	1	2	4
Order by classification (*Index relating to the Lexico order*)	2	3	3	1	2
	1	4	4	4	1
	5	5	5	5	5
	6	6	6	6	6

synthetic image Savoise.

We see through this table that the order of tuples may change when the number of K classes varies and stabilized itself at a certain rank, here K = 5.

The study of the influence of K was also performed on real images. Direct assessment is not possible because of the number (Nt) of high tuples of the multidimensional histogram. For example, House and Mandrill images have a respective Nt 33,925 and 61,662. However, when K = 1 order by classification coincides with the hybrid reduced order. The same phenomenon is repeated when K tends to Nt.

Therefore, we have introduced in this section similarity measurements such as the generalized Hamming (DH) distance that is to say multi-symbols and the inter-class correlation coefficient (δ) and the mutual information to evaluate the the influence of K parameter when passing a number of classes $K_1 = I$ to a number of classes $K_2 = j$. Thus, the Hamming distance between two tuples of the same length measures the number of symbols or positions for which the two tuples are different. Here DH is expressed as a (%) of the length of the tuples corresponding in practice to an error rate.

We note DH_{ij}, δ_{ij}, IM and the Hamming distance respectively, the coefficient of correlation and mutual information between classes i and j.

We have calculated these similarity measurements on different images. An illustration is made on Savoise photographs House and Lena through Tables 2-4.

On the whole of the images processed and particularly the Tables 2-5 presented shows that:

- The order changes with the variation in the number of K classes. This is due to the colorimetric content of the image;
- The interclass similarity of Hamming varies randomly from an image to another. This is inevitably due to the content of the images;
- When K tends to Nt, the similarity between interclass and orders is higher This has no practical interest for our application.

Indeed, the order of the tuples becomes stable when the number of classes increases.

Regarding the interclass correlation coefficient δ_{ij} the results are not relevant and do not allow to objectively compare the orders because of the inclusion of the number of tuples.

The tests similarity measurement with mutual information IM_{ij} were unsuccessful because constant regardless of the combination of the number of K classes.

In addition, for $i = i_0$ the j variation causes an increase of the Hamming distance interclass orders if the difference between (i_0, j) increases. In other words, the similarity of the interclass orders is greater when the interclass distance is small. As shown in Tables 3-5.

As for the coefficient of linear correlation the interclass orders, it decreases when the deviation of the couple (i, j) increases. This is not the case for images

Table 2. Similarity of order by classification according to the number of classes of the image Savoise by *DH*.

Variables *i* and *j* expressed In number of classes		DH (%)	δ (%)	IM
I	J	DH_{ij}	δ_{ij}	IM_{ij}
2	3	66.67	0.2319	1.5248
2	4	66.67	0.2319	1.5248
2	5	50	0.9275	1.5248
2	6	0	1	1.5248
3	4	0	1	1.5248
3	5	50	0.5028	1.5248
3	6	66.67	0.2319	1.5248
4	5	50	0.5028	1.5248
4	6	66.67	0.2319	1.5248
5	6	50	0.9275	1.5248

Table 3. Similarity of order by classification according to the number of classes of the House image by DH.

Variables *i* and *j* expressed In number of classes		DH (%)	IM
I	J	DH_{ij}	IM_{ij}
2	3	75.7808	12.2584
2	4	76.1805	12.2584
2	5	79.7665	12.2584
2	6	90.4722	12.2584
3	4	51.2883	12.2584
3	5	79.5667	12.2584
3	6	90.8928	12.2584
4	5	74.5294	12.2584
4	6	88.7370	12.2584
5	6	88.6634	12.2584

Table 4. Order similarity by classification as a function of the number of classes of the Mandrill image by DH.

Variables *i* and *j* expressed In number of classes		DH (%)	IM
I	J	DH_{ij}	IM_{ij}
2	3	98.7583	13.9559
2	4	99.7005	13.9559
2	5	99.7192	13.9559
2	6	99.7816	13.9559
3	4	82.9600	13.9559
3	5	88.3106	13.9559
3	6	88.1949	13.9559
4	5	86.0673	13.9559
4	6	88.1575	13.9559
5	6	87.8330	13.9559

Table 5. Order similarity by classification according to the number of classes of the Lena image by DH.

Variables i and j expressed In number of classes		DH (%)	IM
I	J	DH_{ij}	IM_{ij}
2	3	72.8768	13.7068
2	4	79.2956	13.7068
2	5	84.4980	13.7068
2	6	84.5754	13.7068
3	4	79.3166	13.7068
3	5	84.5402	13.7068
3	6	84.4558	13.7068
4	5	84.5894	13.7068
4	6	84.5683	13.7068
5	6	83.8161	13.7068

Savoise and Lena. One might therefore think that its results depend on the colorimetric content of the image.

Also, when the Hamming distance is null, the correlation coefficient equal s 1 and therefore the distributions of the interclass orders are identical (see Table 2).

4.2. Statistical Link between the Different Levels

We study in this section the statistical relationship between the classification by order (marked OD_C) proposed in this article and other existing orders in the literature that we deemed relevant [5] such orders as bit mixing (OD_E), The reduced order (OD_R) and the hybrid order (OD_H). Only the Hamming distance is used here to evaluate the similarity between the different orders.

As before, for each image, the compact multidimensional histogram was calculated. The rank of tuples in the histogram is its index. For the different orders we have calculated each multidimensional histogram.

Using the multi-symbol Hamming distance, we have calculated the similarity rate of index ranks between orders two by two for a multitude of images for some results are recorded in the tables below.

Note that we took several different values of k for order by classification.

For each given order and image the inter order mutual information is identical and of the same value as that of the orders by interclass classification.

No law allows us to appreciate the distance of Hamming inter order when the class K number varies. This could be justified by the colorimetric content of the images.

When K = 2, the Hamming distance between the hybrid order and the classification order is the smallest for each image, as shown in Table 7 and Table 8. However, for the synthetic savoise image, the minimum Hamming distance is reached for the values of K = 2 and K = 6, as shown in Table 6. We can see that

when the number of class is equal to the number of color in the image, the order by classification coincides with the hybrid order.

5. Performance Analysis of the Proposed Order through the Operators Gradient, Laplacian

Morphological Gradient and Laplacian

In image processing, the Gradient and the Laplacian are operators allowing highlighting the high frequency information in an image. Indeed, they perform a function of detecting contours. In discrete functional mathematical morphology,

Table 6. Similarity between orders: Case of the synthetic image Savoise.

Variables p and q express The choice of an order		DH (%)										IM
		DH_{pk}					Pq					
P	Q	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$	IM_{pk}
OD_L	ODE	33.33	33.33	33.33	33.33	33.33	0.6923	0.6923	0.6923	0.6923	0.6923	1.5248
OD_L	OD_R	66.67	66.	66.67	66.67	66.67	0.2486	0.2486	0.2486	0.2486	0.2486	1.5248
OD_L	OD_H	66.67	66.67	66.67	66.67	66.67	0.2615	0.2615	0.2615	0.2615	0.2615	1.5248
OD_L	Od_c	66.67	33.33	33.33	33.33	66.67	0.9149	0.9149	0.9149	0.9149	0.9149	1.5248
ODE	OD_R	33.33	33.33	33.33	33.33	33.33	0.6179	0.6179	0.6179	0.6179	0.6179	1.5248
ODE	OD_H	50	50	50	50	50	0.4722	0.4722	0.4722	0.4722	0.4722	1.5248
ODE	OD_C	50	50	50	66.67	50	0.4722	0.3982	0.3983	0.4801	0.4722	1.5248
OD_R	OD_H	33.33	33.33	33.33	33.33	33.33	0.9572	0.9572	0.9572	0.9572	0.9572	1.5248
OD_R	OD_C	33.33	66.67	66.67	33.33	33.33	0.9572	0.1036	0.1036	0.8602	0.9572	1.5248
OD_H	OD_C	0	66.67	66.67	50	0	1	0.2319	0.2319	0.9275	1	1.5248

Table 7. Similarity between orders: Case of the natural image House.

Variables p and q express The choice of an order		DH (%)										IM
		DH_{pk}					Pq					
P	Q	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$	$K=2$	$K=3$	$K=4$	$K=5$	$K=6$	IM_{pk}
OD_L	ODE	99.6950	99.6950	99.6950	99.6950	99.6950	0.7314	0.7314	0.7314	0.7341	0.7314	12.2584
OD_L	OD_R	99.8212	99.8212	99.8212	99.8212	99.8212	0.6936	0.6936	0.6936	0.6936	0.2486	12.2584
OD_L	OD_H	99.8317	99.8317	99.8317	99.8317	99.8317	0.7037	0.7037	0.7037	0.7037	0.7037	12.2584
OD_L	OD_C	99.8528	99.8528	99.8738	99.8528	99.8423	0.6916	0.6886	0.6979	0.7240	0.7354	12.2584
ODE	OD_R	99.6004	99.6004	99.6004	99.6004	99.6004	0.9828	0.9828	0.9828	0.9828	0.9828	12.2584
ODE	OD_H	99.6319	99.6319	99.6319	99.6319	99.6319	0.9856	0.9856	0.9856	0.9856	0.9856	12.2584
ODE	OD_C	99.5583	99.5688	99.5688	99.5077	99.5077	0.9857	0.9753	0.9755	0.9734	0.0969	12.2584
OD_R	OD_H	98.1176	98.1176	98.1176	98.1176	98.1176	0.9572	0.9572	0.9572	0.9572	0.9572	12.2584
OD_R	OD_C	98.7170	99.2323	99.1061	99.1061	99.1061	0.9947	0.9910	0.9913	0.9824	0.9824	12,2584
OD_H	OD_C	38.3426	76.2541	76.3803	80.0189	90.8297	0.9671	0.9636	0.9840	0.9551	0.9551	12.2584

Table 8. Similarity between orders: Case of the natural image Mandrill.

Variables *p* and *q* express The choice of an order		DH (%)										IM
		DH_{pk}					Pq					IM_{pk}
P	*Q*	*K* = 2	*K* = 3	*K* = 4	*K* = 5	*K* = 6	*K* = 2	*K* = 3	*K* = 4	*K* = 5	*K* = 6	
OD_L	ODE	99.9376	99.9376	99.9376	99.9376	99.9376	0.7910	0.7910	0.7910	0.7910	0.7910	13.9559
OD_L	OD_R	99.9314	99.9314	99.9314	99.9314	99.9314	0.7603	0.7603	0.7603	0.7603	0.7603	13.9559
OD_L	OD_H	99.9688	99.9688	99.9688	99.9688	99.9688	0.7476	0.7476	0.7476	0.7476	0.7476	13.9559
OD_L	OD_C	99.9750	99.9563	99.9563	99.9563	99.9526	0.6880	0.6879	0.7691	0.7675	0.7796	13.9559
ODE	OD_R	99.8939	99.8939	99.8939	99.8939	99.8939	0.8850	0.8850	0.8850	0.8850	0.8850	13.9559
ODE	OD_H	99.9251	99.9251	99.9251	99.9251	99.9251	0.8736	0.8736	0.8736	0.8736	0.8736	13.9559
ODE	OD_C	99.9064	99.8503	99.8565	99.8939	99.8877	0.8753	0.8873	0.8347	0.8234	0.8283	13.9559
OD_R	OD_H	99.4572	99.4572	99.4572	99.4572	99.4572	0.9940	0.9940	0.9940	0.9940	0.9940	13.9559
OD_R	OD_C	99.4821	99.1140	99.2887	99.4322	99.4135	0.9789	0.0.9728	0.9735	0.9650	0.9638	13.9559
OD_H	OD_C	66.5128	99.6880	99.7879	99.7754	99.8253	0.9756	0.9681	0.9687	0.9610	0.9585	13.9559

they rely on two basic operators: dilation and erosion, which correspond to the maximum and minimum in a neighborhood of a pixel of the image commonly called a vectorial structuring element. We used the Symmetric Laplacian in this work. The following formulas describe these operators:

Let I be a multicomponent image with n (n> = 1) components and a B a structuring element. It is noted $\delta_B(I)$ expands I versus *B* and $\delta_B(I)$ the eroded *I*. Let *Max* and *Min* respectively be the maximum and minimum elements of a set.

$$\delta_B(I) = \max\{I_B(p)\}, \tag{11}$$

$$\varepsilon_B(I) = \min\{I_B(p)\}, \tag{12}$$

Hence the following expressions of the Gradient and the Laplacian are noted respectively Grad and Lapl:

$$\boldsymbol{Grad}_B(I) = \delta_B(I) - \varepsilon_B(I) \tag{13}$$

$$\boldsymbol{Lapl}_B(I) = \delta_B(I) + \varepsilon_B(I) - 2I \tag{14}$$

The objective in this part is to evaluate quantitatively our gradient and Laplacian operators through the order by proposed classification and the hybrid order. Due to the lack of direct methods for evaluating the results of these operators, we used binary segmentation or thresholding to obtain contour segmentation images. For this we used the Otsu thresholding method [9] for this purpose. At this stage, we used several methods of evaluating contour segmentation [10] to indirectly evaluate our gradient and Laplacian results. Also, in the literature the proposed evaluation methods are supervised, i.e. they require field truths or expert segmentations.

In fact, the BSDS300 [10] Benchmark database with an image base with expert segmentations of contours allowed us to evaluate our contour detection opera-

tors through the two vector orders. **Figure 3** shows two (02) images of the benchmark database with their ground truths.

From a quantitative point of view, different measurements of contour segmentation evaluation exist to evaluate the relevance of our contour detection operators. In our context we took into account distance from Pratt as well as Vinet measure.

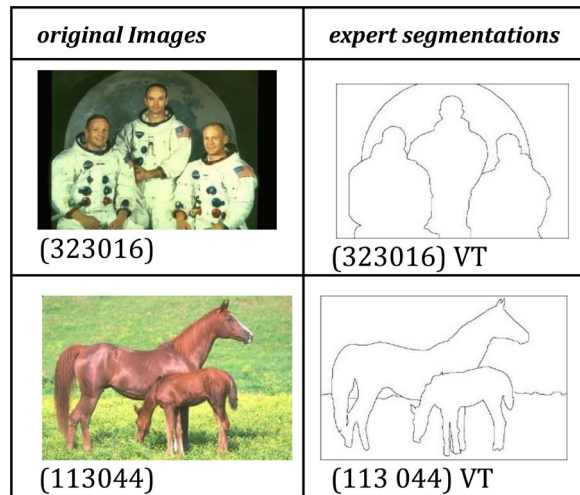The results are shown in **Table 9(a)** and **Table 9(b)**. From the results

original Images	expert segmentations
(323016)	(323016) VT
(113044)	(113 044) VT

Figure 3. Images of the BSDS300 Benchmark database with their expert segmentations.

Table 9. (a) Performance of gradient operators and Laplacian (Measure of Pratt); (b) Performance of gradient and Laplace operators in relation to the different vector orders (Measure of Vinet).

(a)

Benchmark Images	323016		113044	
Contour Operators	Gradient	Laplacian	Gradient	Laplacian
Hybridorder	0.9146	0.3657	0.8497	0.5111
Classification order (K = 2)	0.9145	0.3662	0.8178	0.5152
Classification order (K = 3)	0.9148	0.3672	0.8760	0.5161
Classification order (K = 4)	0.9145	0.3680	0.8110	0.5153
Classification order (K = 5)	0.9141	0.3659	0.8269	0.5133
Classification order (K = 6)	0.9144	0.3659	0.8754	0.5160

(b)

Benchmark Images	323016.*jpg*		113044.*jpg*	
Contour Operators	Gradient	Laplacian	Gradient	Laplacian
Hybridorder	0.3790	0.2095	0.2855	0.2586
Classification order (K = 2)	0.3781	0.2099	0.2938	0.2603
Classification order (K = 3)	0.3790	0.2108	0.2990	0.2581
Classification order (K = 4)	0.3790	0.2099	0.3016	0.2595
Classification order (K = 5)	0.379	0.2103	0.2894	0.2590
Classification order (K = 6)	0.3790	0.2095	0.2981	0.2599

obtained we can make the following analysises:

The Vinet measurement generally gives constant values for the different values of K in the case of order by proposed classification; therefore it does not discriminate their performance.

- However, Pratt's measurement shows the performance of the different operators.

- The set of results from the different images of the database show that the proposed classification approach is generally efficient than the hybrid order for the gradient operator.

On the contrary, the hydric order is more efficient than the order by classification for the laplacian operator.

Note that the lower the measurement, the better the contour detection operator.

6. Robustness of the Order Provides Noise through the Median Filter

To test the effectiveness of our proposal to noise, we added impulsive noise to different images of the Benchmark BSDS300 Images database to study the robustness of the proposed approach and the median filter. We chose two images here for illustration (113044.jpg, 323016.jpg) and different noise powers were added to the images described by the parameter p.

For each noisy image we applied the median filter using the proposed order and the hybrid order, and then we calculated the error rates generated by each order. The results are shown in Table 10 and Table 11.

Table 10. Comparison of median filters for different vector orders studied picture 113044.jpg.

Image 113044.jpg noise	$P = 0.001$	$P = 0.007$	$P = 0.01$	$P = 0.03$	$P = 0.05$
Hybridorder	0.8488	0.8366	0.8353	0.8488	0.8409
Classification order (K = 2)	0.9996	0.9991	0.9983	0.9996	0.9987
Classification order (K = 3)	0.9996	0.9974	0.9983	0.9996	0.9996
Classification order (K = 4)	0.9987	0.9996	0.9991	0.9987	0.9991
Classification order (K = 5)	0.9983	0.9991	0.774	0.9983	0.9991
Classification order (K = 6)	0.9991	1.0000	0.9996	0.9991	0.9996

Table 11. Comparison of median filters for different vector orders Studied we 323016.jpg picture.

Image 323016.jpg noise	$P = 0.001$	$P = 0.007$	$P = 0.01$	$P = 0.03$	$P = 0.05$
Hybridorder	0.8162	0.8170	0.8053	0.8275	0.8288
Classification order (K = 2)	0.9883	0.9922	0.9930	0.9909	0.9887
Classification order (K = 3)	0.9848	0.9904	0.9839	0.9891	0.9913
Classification order (K = 4)	0.9900	0.9865	0.997	0.9861	0.9961
Classification order (K = 5)	0.9822	0.9857	0.9865	0.9857	0.9870
Classification order (K = 6)	0.9900	0.9917	0.9909	0.9957	0.99

We noted that the results obtained have the same order of magnitude for the set of images.

However the error rate median filter is lower than the proposed hybrid order. We can conclude that our proposal is less robust to noise compared to the hybrid order. This could be explained by the fact that the classification algorithm used, in particular K-means, is sensitive to noise.

7. Conclusions and Perspectives

In this paper, we have presented a new vector order, a solution to the open problem of the generalization of mathematical morphology to multicomponent images and multidimensional data. Our proposal is a P-order. Indeed, it first parses the multicomponent image in the attribute space by the K-means classification method into different numbers of classes. Then the attribute vectors are ordered within each class (intra class order). And finally the classes themselves are ordered in turn from their barycenter (interclass order). Equipped with this order the space attribute of the image is a complete lattice.

We can conclude that on all images tested, the proposed order by classification gives better results than the hybrid order on the gradient operator, specifically for edge detection. However, our order resists less noise than the hybrid order.

In our future work we intend to improve the noise resistance of our order by using a classification method rather than the K-means algorithm. Then we know that K varies theoretically from 1 to Nt. Indeed, the choice or the implementation of an unsupervised evaluation criterion should automatically allow us to obtain the value K = Kopt producing for each morphological operator (Gradient, Laplacian, Median filter, etc.), the optimal solution whatever the vector order may be.

What is more, we shall also focus our research on the development of new methods of Vector morphological segmentation based on scalar approaches or grayscale.

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