An Experimental Comparison of Quantum Decision Theoretical Models of Intertemporal Choice for Gain and Loss

Taiki Takahashi1, Hiroshi Nishinaka2, Takaki Makino2, Ruokang Han1, Hiroki Fukui2
1Department of Behavioral Science, Center for Experimental Research in Social Science, Hokkaido University, Sapporo, Japan
2Department of Forensic Psychiatry, National Institute of Mental Health National Center of Neurology and Psychiatry, Tokyo, Japan
Email: taikitakahashi@gmail.com
Received September 10, 2012; revised October 13, 2012; accepted October 24, 2012

ABSTRACT
In mathematical physics and psychology, “quantum decision theory” has been proposed to explain anomalies in human decision-making. One of such quantum models has been proposed to explain time inconsistency in human decision over time. In this study, we conducted a behavioral experiment to examine which quantum decision models best account for human intertemporal choice. We observed that a q-exponential model developed in Tsallis’ thermodynamics (based on Takahashi’s (2005) nonlinear time perception theory) best fit human behavioral data for both gain and loss, among other quantum decision models.

Keywords: Discounting; Neuroeconomics; Econophysics; Quantum Decision Theory; Tsallis’ Statistics

1. Introduction
Intertemporal choice and temporal discounting (devaluation of delayed reward as delay until its receipt increases) have been attracting attention in quantum decision theory [1], econophysics [2-4], and neuroeconomics [5]. It is to be noted that intertemporal choice has originally been investigated in economics [6]. In these disciplines, several mathematical models have been proposed to account for human intertemporal choice. Notably, [7] proposed intertemporal choice models based on quantum theoretical foundations (“quantum decision theory”). In mathematical physics and psychology, recent studies developed this type of quantum decision models in order to explain various anomalies in human decision and cognition [7-9]. However, to date, no study has experimentally examined the explanatory powers of the quantum decision theoretical models for human intertemporal choice. This issue is important for future studies in quantum information and decision theory in mathematical physics, econophysics, and neuroeconomics. In this study, we experimentally examined explanatory powers of intertemporal choice models based on quantum decision theory, in relation to psychophysical theory of intertemporal choice [10] based on Tsallis’ thermostatistics [2].

Intertemporal and Probabilistic Choices
In standard economics, the following exponential time-discount model was proposed [11]:

\[ V(D) = V(0) \exp(-k_e D) \]  \hspace{1cm} (1)

where \( V(D) \) is a subjective value of monetary outcome subject receives or pays at delay \( D \) and impulsivity parameter \( k_e \) is an exponential time-discount rate. In behavioral neuroeconomics, a hyperbolic discounting model has been widely employed [5,6]:

\[ V(D) = \frac{V(0)}{1 + k_h D} \]  \hspace{1cm} (2)

where \( V(D) \) is a subjective value of outcome which subject receives/pays at delay \( D \) and impulsivity parameter \( k_h \) is a hyperbolic time-discount rate.

In econophysics, Cajueiro [2] proposed a \( q \)-exponential model based on Tsallis’ statistics [12] to generalize exponential and hyperbolic discounting:

\[ V(D) = \frac{V(0)}{\left(1 + k_q(1 - q) D \right)^{1/[1-q]}} \]  \hspace{1cm} (3)

where \( V(D) \) is a subjective value of outcome which subject receives/pays at delay \( D \) and impulsivity parameter \( k_q \) is a \( q \)-exponential time-discount rate at delay \( D = 0 \). Parameter \( q \) generalizes the exponential function. Equation (3) is equivalent to an exponential model Equation (1) when \( q = 1 \) and equivalent to a hyperbolic model Equation (2) when \( q = 0 \). When \( q \) is less than 1, the
agent’s temporal discounting is referred to as “decreasing impatience”, which may result in preference reversal over time.

Recently, on the other hand, Yukalov and Sornett [1] proposed several types of intertemporal choice models based on quantum decision theory (QDT). In their models, it is basically assumed that

$$\frac{d}{dt} \pi(t, t_0) = -k(t, t_0) \pi(t, t_0)$$

(4)

where $\pi(t, t_0)$ is a “prospect state” and $k(t, t_0)$ is a time-dependent time-decay (discount) rate. It is to be noted that in Yukalov and Sornett’s temporal discounting model, temporal discounting is assumed to occur due to a decrease in probability as delay increases, in line with Sozou’s evolutionary theory of hyperbolic discounting [13,14]. Based on this assumption, Yukalov and Sornett [1] derived several intertemporal choice models (i.e., discount functions, in terms of economic theory). The general form for the Yukalov-Sornett time-discount functions is

$$\gamma(t, t_0) = \gamma\left(1 + \left(t - t_0\right)^n\right)$$

(5)

where $\gamma$ and $n$ are free parameters and the time-scale is set at 1 day. [1] further derived several simplified temporal discounting models (Models 1, 2, 2’, 3, 4, see [12], for explicit functional forms of the QDT-based models) based on reasonable approximations. However, little is known regarding which model is the best for explaining human intertemporal decision-making. Therefore, we experimentally investigated the explanatory powers of the proposed models based on QDT and the q-exponential time-discount model based on Tsallis’ thermostatistics and Takahashi’s logarithmic time perception theory of “hyperbolic” discounting [10,15,16].

2. Method

2.1. Participants

Forty-one (22 male and 19 female) students were recruited from Chuo University in Japan to take part in the experiment. The mean age was 23.31 years old (standard deviation = 3.20).

2.2. Procedure

The participants were asked to perform time discounting tasks of both gain and loss. They were seated individually in a quiet room, facing the experimenter across a table. Then they received a simple instruction that they were asked to choose from a series of alternatives of monetary amounts either gain or loss with certain probabilities and imagine them, though hypothetical, as real money in this experiment. The employed time discounting tasks were the same as our previous studies [15,16].

2.3. Data Analysis

The procedures of data analysis in the present study is similar to our previous studies [4,14]. Switching points of the time discounting tasks were defined as the means of the largest adjusting amount in which the standard alternative choice and the smallest adjusting amount in which the adjusting alternative choice. Indifference points of individuals were calculated by averaging the switching point in the ascending and descending adjusting conditions. The indifference points of the group data were obtained by calculating the medians of individuals’ indifference points in order to compare the goodness-of-fit among the models based on QDT and Tsallis’ statistics at the group level. The fitness of each equation was estimated with AIC (Akaike Information Criterion) values, which is the most standard criterion for the fitness of mathematical model to observed data. All statistical procedures were conducted with R statistical language.

3. Result

We present fitted parameters and goodness-of-fit (AIC) in Tables 1 (gain) and 2 (loss). From these analyses, we can see that the q-exponential time-discount model fit the behavioral data best for both gain and loss, consistent with logarithmic time-perception theory of intertemporal choice proposed by [10]. It is also to be noted that q values in the q-exponential discounting were smaller than 1 for both gain and loss, indicating decreasing impatience (“hyperbolic” discounting) for both signed outcomes. Furthermore, we can see that Model 2 and 4 are mathematically equivalent (Tables 1 and 2).

4. Discussion

To our knowledge, this study is the first to experimentally examine time discount models for gain and loss proposed in the Quantum Decision Theory and the q-exponential discounting model based on deformed algebra developed in Tsallis’ thermostatistics. Our results demonstrated that 1) Model 2 and 4 (temporal discounting with non-zero limit at large delays) best fit among the quantum decision theoretic intertemporal choice models, and 2) the q-exponential model best fitted human temporal discounting behavior for both gain and loss. It is to be emphasized that, even when novel temporal discounting models proposed in QDT, the q-exponential time-discount model best accounted for human intertemporal choice for both gain and loss, consistent with our previous studies [10,15,16]. Therefore, future studies in econophysics, QDT, and neuroeconomics should model.
human intertemporal choice with the $q$-exponential discount model, which can be derived from Takahashi’s nonlinear time perception theory of hyperbolic discounting [10].

5. Acknowledgements

The research reported in this paper was supported by a grant from the Grant-in-Aid for Scientific Research (Global COE and on Innovative Areas, 23118001; Adolescent Mind & Self-Regulation) from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

REFERENCES


