

Modified Two-Slit Experiments and Complementarity

Tabish Qureshi

Centre for Theoretical Physics, Jamia Millia Islamia, New Delhi, India
Email: tabish@ctp-jamia.res.in

Received March 17, 2012; revised April 25, 2012; accepted May 5, 2012

ABSTRACT

Some modified two-slit interference experiments claim to demonstrate a violation of Bohr's complementarity principle. A typical such experiment is theoretically analyzed using wave-packet dynamics. The flaw in the analysis of such experiments is pointed out and it is demonstrated that they do not violate complementarity. In addition, it is quite generally proved that if the state of a particle is such that the modulus square of the wave-function yields an interference pattern, then it necessarily loses which-path information.

Keywords: Complementarity; Wave-Particle Duality; Two-Slit Experiment

1. Introduction

It is generally accepted that the wave and particle aspects of quantum systems are mutually exclusive. Niels Bohr elevated this concept, which is probably born out of the uncertainty principle, to the status of a separate principle, the principle of complementarity [1].

Since then, the complementarity principle has been demonstrated in various experiments, the most common of which is the two-slit interference experiment. It has been shown that if the which-way information in a double-slit experiment is stored somewhere, the interference pattern is destroyed, and if one chooses to "erase" the which-way information *after* detecting the particle, the interference pattern comes back. This phenomenon goes by the name of quantum erasure [2,3].

Recently, some interesting experiments were proposed and carried out which claim to violate the complementarity principle [4-6]. A schematic diagram of a typical such experiment is shown in **Figure 1**. Basically, it consists of a standard two-slit experiment, with a converging lens L behind the conventional screen for obtaining the interference pattern. Although the experiments use pin-holes instead of slits, we will continue to refer to them as slits. If the screen is removed, the light passes through the lens and produces two images of the slits, which are captured on two detectors D_A and D_B respectively. Opening only slit A results in only detector D_A clicking, and opening only slit B leads to only D_B clicking. The authors argue that the detectors D_A and D_B yield information about which slit, A or B, the particle initially passed through. If one places a screen before the lens, the interference pattern is visible.

Conventionally, if one tries to observe the interference

pattern, one cannot get the which-way information. The authors have a clever scheme for establishing the existence of the interference pattern without actually observing it. First, the exact locations of the dark fringes are determined by observing the interference pattern. Then, thin wires are placed in the exact locations of the dark fringes. The argument is that if the interference pattern exists, sliding in wires through the dark fringes will not affect the intensity of light on the two detectors. If the interference pattern is not there, some photons are bound to hit the wires, and get scattered, thus reducing the photon count at the two detectors. This way, the existence of interference pattern can be established without actually disturbing the photons in any way. This is similar in spirit to the so called "interaction-free measurements" where the non-observation of a particle along one path establishes that it followed the other possible path, without actually measuring it [7]. The authors carried out the

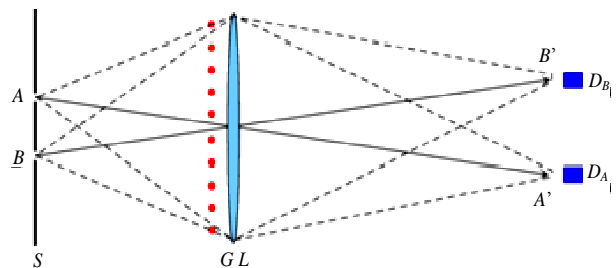


Figure 1. A schematic representation of the modified two-slit experiment. Light emerges from two pin-holes (not slits) A and B and interferes. Thin wires are placed carefully in the exact locations of the dark fringes of the interference pattern. The lens L collects the light and obtains the images of the two slits A' and B', respectively. The detectors D_A and D_B collect the photons for these two images.

experiment and found that sliding in wires in the expected locations of the dark fringes, doesn't lead to any significant reduction of intensity at the detectors. Hence they claim that they demonstrated a violation of complementarity.

The complementarity principle is at the heart of quantum mechanics, and its violation will be deeply disturbing to its established understanding. As expected, there has been scepticism towards the modified two-slit experiment, and a heated debate followed [8-16]. Interestingly, different criticisms do not agree with each other on why complementarity is not violated in this experiment. None of the criticisms has been satisfactorily able to point out any flaw in the interpretation of the experiments. Most agree that if the introduction of wires has no effect on the intensity, it shows that interference exists. Almost everybody seems to agree that detecting a photon at (say) D_A means that it came from slit A. However, just because blocking slit B leads to only detector D_A clicking and blocking slit A leads to only detector D_B clicking, quantum mechanics doesn't say that when both slits are open, detector D_A clicking implies that the photon came from slit A. This has also been pointed out by Kastner [9].

Some authors [13,15] have tried to argue along the following line. Blocking dark fringes also blocks out parts of the bright fringes. They believe, when both slits are open, the photons contain full which-slit information, which is partially destroyed by partially blocking the bright fringes. They argue that since without the wires completely blocking the dark fringe, one cannot infer the existence of the interference pattern, when one tries to increase the information about the existence of the interference, the which-slit information is proportionately decreased. However, in these works, the *calculation* shows the existence of full interference, without any blocking wires. Without the blocking wires, the photons are claimed to have full which-slit information (distinguishability = 1, in their language). So, if one were to take their calculation of distinguishability as correct, then the existence of full interference *in the calculation* (in the sense of $|\psi(x)|^2$ yielding an interference pattern) seems to imply that quantum formalism allows existence of interference pattern for photons for which full which-slit information exists. This clearly goes against the spirit of complementarity. In our view, the calculation of distinguishability in these works is fundamentally flawed.

Many authors have fallen back to a more formal interpretation of Bohr's principle, namely that the wave and particle nature cannot be seen in the same experiment, for the same photons. They argue that authors have to do not one, but two experiments to prove their point—one without the wires, one with the wires. Some argue that the existence of fringes has already been assumed, and that the argument of the experiment is circular. However, a reader who respects empirical facts, doesn't see it as

two experiments if putting in the wires is not changing the results. One would say, the interference is out there in the middle, and one can check out that the photons are not passing through certain regions, the dark fringes.

Let us understand what is happening in the experiment slightly better by simplifying the experiment. One might argue that a two-slit experiment is the simplest experiment one can imagine. But this experiment still has a large number of degrees of freedom, and the Hilbert space is large. We will show in the following that the simplest interference (thought) experiment can be just carried out using a spin-1/2 particle.

2. A Simplified "Two-Slit" Experiment

Let there be a spin-1/2 particle traveling along x-axis. For our purpose, its physical motion is unimportant—we will only be interested in the dynamics of its spin. In the two-slit experiment, the particle emerges from the slits in a superposition of two physically separated, localized wave-packets, which are orthogonal to each other. Their physical separation guarantees their orthogonality. Any subsequent unitary evolution will retain their orthogonality. We assume the initial state of the spin to be

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle). \quad (1)$$

Here, the states $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of S_z , and play the role of the two wave-packets that emerge from the double-slit. The time evolution, in the conventional two-slit experiment, spreads the wave-packets so that they overlap. In our thought experiment, we employ a homogeneous magnetic field B , acting along the y-axis, to evolve the two states $|\uparrow\rangle$ and $|\downarrow\rangle$. Thus the Hamiltonian of the system is $H = -BS_y$.

The time evolution operator can be written as

$$\begin{aligned} U(t) &= \exp\left(\frac{i}{\hbar} BS_y t\right) \\ &= \cos\left(\frac{Bt}{2}\right) + i\sigma_y \sin\left(\frac{Bt}{2}\right) \end{aligned} \quad (2)$$

where σ_y is the usual Pauli matrix. The time evolution operator, for a time $\tau = \pi/2B$ has the form

$$U(\tau) = \frac{1}{\sqrt{2}}(1 + i\sigma_y). \quad (3)$$

It is straight forward to see that the time evolution transforms the states $|\uparrow\rangle$ and $|\downarrow\rangle$ as

$$\begin{aligned} U(\tau)|\uparrow\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) \\ U(\tau)|\downarrow\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \end{aligned} \quad (4)$$

A further evolution through a time τ will transform

the states as

$$\begin{aligned} U(2\tau)|\uparrow\rangle &= -|\downarrow\rangle \\ U(2\tau)|\downarrow\rangle &= |\uparrow\rangle \end{aligned} \quad (5)$$

After an evolution through a total time 2τ , if one puts a spin detector, one will either get a “spin-down” or a “spin-up”. In the beginning, if we started out with a state $|\uparrow\rangle$, the detector at the end will register a $|\downarrow\rangle$ state. On the other hand, if we started with a $|\downarrow\rangle$ state, the detector at the end will register a $|\uparrow\rangle$ state. Thus, the detector at the end obtains a which-initial-state information about the spin, exactly as the detectors in the modified two-slit experiment obtain a which-slit information.

So, now we carry out our thought experiment, with the initial state $|\psi\rangle_0$, as given by (1). After a time τ we get into a state which is equivalent to the interference region in the modified two-slit experiment:

$$U(\tau)|\psi\rangle_0 = \frac{1}{2}(|\uparrow\rangle - |\downarrow\rangle) + \frac{1}{2}(|\uparrow\rangle + |\downarrow\rangle). \quad (6)$$

Equation (6) represents an interference pattern, because the “down-spins” cancel out to give destructive interference, while the “up-spins” add up to give constructive interference. So, in our two-state interference experiment, there is one dark fringe and one bright fringe. After parts of the states have been destroyed by the “destructive interference”, what is left is just

$$U(\tau)|\psi\rangle_0 = \frac{1}{2}|\uparrow\rangle + \frac{1}{2}|\uparrow\rangle. \quad (7)$$

Now, these two contributions from the two initially orthogonal states are identical. States which are not orthogonal, are naturally not distinguishable. Thus the which-way or which-initial-state information is lost at this stage. It might serve some useful purpose by keeping the two contributions separate and evolving them for a further time to see what they yield at the spin detector.

$$U(2\tau)|\psi\rangle_0 = \frac{1}{2\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) + \frac{1}{2\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle) \quad (8)$$

So, we see that each part of the initial state leads to a superposition of $|\uparrow\rangle$ and $|\downarrow\rangle$ states. So, although the spin-detector at the end still gives either a “spin-up” or a “spin-down”, it gives no information about whether the initial state was a $|\uparrow\rangle$ or $|\downarrow\rangle$. Yet, if one started out with either a $|\uparrow\rangle$ or $|\downarrow\rangle$ initial state, which is equivalent of closing one slit, (5) tell us that each will go to only its corresponding detector, thus giving the which-initial-state information.

Taken in isolation, this over-simplified thought experiment may not prove anything substantial, but it does provide a clue to where the problem lies in the modified two-slit experiments. It appears that the very existence of interference destroys which-way information.

3. Interference and Which-Way Information

3.1. Which-Way Information Destroys Interference

There have been some recent developments in understanding the origins of complementarity. It has been argued, and demonstrated, that one need not actually carry out a which-way measurement in order that the interference disappears. Mere existence of which-way information in the state is sufficient to destroy any potential interference [3,17]. In other words, one can say that mere existence of the possibility of getting which-way information is enough to destroy the interference pattern. This can be simply demonstrated as follows. Suppose that the state of a particle having passed through a double-slit, just before hitting the screen, is given by

$$\psi(x) = \psi_A(x) + \psi_B(x) \quad (9)$$

where $\psi_A(x)$ and $\psi_B(x)$ represent the amplitude of the particle passing through slit 1 and 2, respectively. Probability of finding the particle at a point x on the screen, is given by

$$\begin{aligned} |\psi(x)|^2 &= |\psi_A(x)|^2 + |\psi_B(x)|^2 \\ &+ \psi_A^*(x)\psi_B(x) + \psi_B^*(x)\psi_A(x) \end{aligned} \quad (10)$$

The last two terms represent interference. Now, let us suppose that in a slightly modified version of this experiment, the state is given by

$$\psi(x) = \psi_A(x)|1\rangle + \psi_B(x)|2\rangle \quad (11)$$

where $|1\rangle$ and $|2\rangle$ are certain orthonormal eigenstates of a suitable observable, which get entangled with the particle, and hence carry which-way information. A measurement of that observable yielding $|1\rangle$ will lead to a definitive conclusion that the particle passed through slit 1, and likewise for $|2\rangle$. In this case the probability of finding the particle at a point x , is given by

$$|\psi(x)|^2 = |\psi_A(x)|^2 + |\psi_B(x)|^2. \quad (12)$$

The two terms which would have given interference, are killed by the orthogonality of $|1\rangle$ and $|2\rangle$. One would notice that a which-way measurement is not even needed here—mere existence of which-way information, or mere possibility of a which-way measurement, is enough to destroy interference.

3.2. Interference Destroys Which-Way Information

In the following we will prove that the converse of which-way information necessarily destroys interference is also true. This would mean that a quantum state which has a form required to yield interference, cannot contain any

which-way information.

In any variant of the conventional two-slit experiment, the initial state has to be in a superposition of two orthogonal states. This follows simply from the fact that the two slits are distinguishable. In the case of a Mach-Zehnder interferometer, a half-silvered mirror splits the beam into superposition of two spatially separated beams. Thus the initial (unnormalized) state $|\psi\rangle_0$ can be written as

$$|\psi\rangle_0 = |\psi_A\rangle_0 + |\psi_B\rangle_0. \quad (13)$$

These states then evolve and reach the region where they spatially overlap. $|\psi_A\rangle_0$ and $|\psi_B\rangle_0$ evolve to $|\psi_A\rangle$ and $|\psi_B\rangle$ respectively, so that in the region of overlap, the state looks like

$$|\psi\rangle = |\psi_A\rangle + |\psi_B\rangle. \quad (14)$$

The time evolution being unitary, these two parts retain their orthogonality, so that $\langle\psi_A|\psi_B\rangle = 0$. Let us assume that in the region where they interfere, they have the form:

$$|\psi_A\rangle = |\alpha\rangle + |\gamma\rangle, \quad |\psi_B\rangle = |\beta\rangle - |\gamma\rangle, \quad (15)$$

where $|\gamma\rangle$ is chosen to be normalized, keeping $|\psi_A\rangle$ and $|\psi_B\rangle$ unnormalized, which can always be done. Here, $|\alpha\rangle$, $|\beta\rangle$ and $|\gamma\rangle$ need not be simple states—each may involve a multitude of degrees of freedom. However, $|\psi_A\rangle$ and $|\psi_B\rangle$ have to have this form, so that there are parts from the two which cancel out exactly, to give the so-called dark-fringes. In the case of the conventional two-slit experiment, $|\gamma\rangle$ would constitute the pattern involving all the dark fringes (a specific example of this appears in the next section). For the case of a Mach-Zehnder interferometer, $|\gamma\rangle$ represents the part of the state from one beam, reaching one of the two detectors. Interference happens when $|\gamma\rangle$ parts reaching one detector, from both the beams, cancel out. The result is, that particular detector does not detect any particles (dark fringe). Thus, if a state described by (14), (15) has a non-zero $|\gamma\rangle$, one can be sure that a measurement will lead to an interference pattern. One can thus associate a non-zero $|\gamma\rangle$ with the existence of interference, even without doing an actual measurement.

If complementary could indeed be violated, the parts of the two initial states which are left (after the destructive interference), namely $|\alpha\rangle$ and $|\beta\rangle$, should be orthogonal, so that they can contain a which-way information. States which are not orthogonal, cannot lead to distinguishable outcomes in a measurement. In order that $|\gamma\rangle$ describes the pattern of dark fringes, it should be orthogonal to $|\alpha\rangle$ and $|\beta\rangle$, so that it is distinguishable from the other parts. However, the following analysis goes through even without that restriction.

The norm of $|\psi\rangle$ is given by

$$\begin{aligned} \langle\psi|\psi\rangle &= \langle\psi_A|\psi_A\rangle + \langle\psi_B|\psi_B\rangle \\ &= \langle\alpha|\alpha\rangle + \langle\alpha|\gamma\rangle + \langle\gamma|\alpha\rangle + \langle\gamma|\gamma\rangle \\ &\quad + \langle\beta|\beta\rangle - \langle\beta|\gamma\rangle - \langle\gamma|\beta\rangle + \langle\gamma|\gamma\rangle \\ &= \langle\alpha|\alpha\rangle + \langle\alpha|\gamma\rangle + \langle\gamma|\alpha\rangle + \langle\beta|\beta\rangle \\ &\quad - \langle\beta|\gamma\rangle - \langle\gamma|\beta\rangle + 2 \end{aligned} \quad (16)$$

Using orthogonality of $|\psi_A\rangle$ and $|\psi_B\rangle$, we get

$$\langle\alpha|\gamma\rangle - \langle\gamma|\beta\rangle = 1 - \langle\alpha|\beta\rangle. \quad (17)$$

Substituting this in (16) yields

$$\langle\psi|\psi\rangle = \langle\alpha|\alpha\rangle + \langle\beta|\beta\rangle - \langle\alpha|\beta\rangle - \langle\beta|\alpha\rangle + 4. \quad (18)$$

In the region of overlap the state has the actual form $|\psi\rangle = |\alpha\rangle + |\beta\rangle$. Using this, the norm of $|\psi\rangle$ can be written as

$$\langle\psi|\psi\rangle = \langle\alpha|\alpha\rangle + \langle\beta|\beta\rangle + \langle\alpha|\beta\rangle + \langle\beta|\alpha\rangle. \quad (19)$$

Combining (18) and (19), we get

$$\langle\alpha|\beta\rangle + \langle\beta|\alpha\rangle = 2. \quad (20)$$

From the above equation it is obvious that $|\alpha\rangle$ and $|\beta\rangle$ can never be orthogonal. Hence $|\psi\rangle$ in overlap region contains no which-way information.

This general analysis shows that if the state has a form which could yield interference, it cannot contain any which-way information. In other words, existence of interference necessarily destroys which-way information.

So, complementarity is robust, and cannot be violated in any such interference experiment where one tries to look for which-way information after interference.

4. Two-Slit Experiment with Wave-Packets

Coming back to the modified two-slit experiment, let us see what implication the preceding analysis has on it. Clearly, in the modified two-slit experiment, after the particle passes through the interference region, the which-way information is lost. The detector D_A clicking doesn't mean that the particle came from slit A. It may appear hard to visualize that a wave-packet which travels in a straight line from slit A can contribute to a click on detector D_B , which is not in its direct path. However, the argument of momentum conservation will hold only if we knew that the particle started out from slit A—in that situation it would never reach detector D_B . In order to demonstrate the validity of these arguments for the modified two-slit experiment, let us look at the two-slit experiment in more detail.

We carry out the analysis for massive particles, to show that the argument hold not just for photons, but for any quantum particle. Consider the particle to be moving along the x-axis, and the slit plane parallel to the y-axis. We will only be interested in the dynamics of the state in

the y-direction, whereas the x-axis motion just serves the purpose of transporting the particle from the slits to the detectors. Let us assume that when the particle emerges from the double-slit, its state is given by a superposition of two distinct, spatially localized wave-packets. The state at this time, which we choose to call $t = 0$, can be written as

$$\begin{aligned} \psi(y, 0) &= \frac{1}{(\pi/2)^{1/4} \sqrt{\cdot}} \left(a e^{-\frac{(y-y_0)^2}{2}} + b e^{-\frac{(y+y_0)^2}{2}} \right), \end{aligned} \quad (21)$$

where \cdot is the width of the wave-packets, $2y_0$ is the slit separation, and a and b are the amplitudes of the two wave-packets.

The wave-packets evolve in time, during which they spread, and reach the region where they overlap, and thus, interfere. This state can just be obtained by evolving (21), using the Hamiltonian $H = p^2/2m$. Hence, the state at time t can be written as

$$\begin{aligned} \psi(y, t) &= aC(t) \exp\left(-\frac{(y-y_0)^2}{2 + \frac{2i}{m}t}\right) \\ &+ bC(t) \exp\left(-\frac{(y+y_0)^2}{2 + \frac{2i}{m}t}\right), \end{aligned} \quad (22)$$

where

$$C(t) = \frac{1}{(\pi/2)^{1/4} \sqrt{2 + \frac{2i}{m}t}}. \quad (23)$$

Now, this state can also be rewritten as

$$\begin{aligned} \psi(y, t) &= aC(t) e^{-\frac{y^2+y_0^2}{\Omega(t)}} \left[\cosh\left(\frac{2yy_0}{\Omega(t)}\right) + \sinh\left(\frac{2yy_0}{\Omega(t)}\right) \right] \\ &+ bC(t) e^{-\frac{y^2+y_0^2}{\Omega(t)}} \left[\cosh\left(\frac{2yy_0}{\Omega(t)}\right) - \sinh\left(\frac{2yy_0}{\Omega(t)}\right) \right] \end{aligned} \quad (24)$$

where $\Omega(t) = 2 + \frac{2i}{m}t$. The term involving

$\sinh\left(\frac{2yy_0}{\Omega(t)}\right)$ in this expression is an example of $|\gamma\rangle$

introduced in the preceding section.

In the usual case of a two-slit experiment, the amplitudes coming from the two slits are approximately equal, i.e., $a = b = 1/\sqrt{2}$. In this case, the sinh terms cancel out to give the dark fringes, and what is left is

$$\begin{aligned} \psi(y, t) &= \frac{1}{2} aC(t) \left(e^{-\frac{(y-y_0)^2}{\Omega(t)}} + e^{-\frac{(y+y_0)^2}{\Omega(t)}} \right) \\ &+ \frac{1}{2} bC(t) \left(e^{-\frac{(y-y_0)^2}{\Omega(t)}} + e^{-\frac{(y+y_0)^2}{\Omega(t)}} \right) \end{aligned} \quad (25)$$

In this state, the parts of the state coming from the two slits are equal. So, there is no which-way information in the state any more. If a lens is used after this stage, which takes the part $e^{-(y-y_0)^2/\Omega(t)}$ to one detector and the other part to the other detector, one can easily see from (25) that a part coming from one slit, becomes a superposition of two parts going to the two detectors. So, each detector receives equal contribution from the two slits, and registering a particle gives no information on which slit the particle came from. To a mind used to classical way of thinking, it might appear that the particle received at a particular detector came from a particular slit, but the preceding analysis shows that this presumption is erroneous.

Note however, that if one of the slits is closed, say, if $a = 1$ and $b = 0$ the state at time t will be $C(t) e^{-(y-y_0)^2/\Omega(t)}$, which goes to only one detector when a lens is used.

A critic might argue that instead of cancelling out some parts of the two wave-packets, one might just evolve them separately, and because they were initially orthogonal, they will give distinct result at the end. The answer to that is, if one really looks for interference by putting thin wires, one is blocking those very parts of the wave-packets which are cancelling out. So, those parts will not reach the detectors, even if one insists on not cancelling them out. On the other hand, the argument of bringing in wires is not really needed. The state (25) as such, has no which path information.

5. Conclusion

Although the modified two-slit experiments do have genuine interference, as shown by introducing thin wires, the detectors detecting the photons behind the converging lens, do not yield any which-way information. It was not easy to see this without mathematically rigorous arguments. The wave-packet analysis makes this fact trivially obvious. Many earlier analysis of complementarity have concentrated on showing how existence of which-way information destroys interference. We have taken the reverse approach, as demanded by the modified two-slit experiment. We have shown that if a state is such that the modulus square of its wave-function yields an interference pattern, it cannot contain any which-way information. The complementarity principle is thus robust and cannot be violated in any experiment which is a variant

of the two-slit experiment. We see yet again that in dealing with quantum systems, trusting classical intuition can easily lead one to wrong conclusions.

6. Acknowledgements

The author thanks R. Srikanth for very useful discussions. The author thanks the Centre for Philosophy and Foundations of Science, New Delhi, for organizing annual meetings on foundations of quantum mechanics, which provide a platform for exchange of ideas.

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