Optimum Inclination Angles of Booster Mirrors and Solar Radiation Availability on the Horizontal and Inclined Box Type Solar Cookers

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ABSTRACT

Mathematical relations are developed to compute optimum inclination angle of booster mirror for horizontally placed cooker (λ) and for optimally inclined cooker (ψ) during all months (selected day) of the year at 30°N latitude for maximizing the reflected component of solar intensity onto the absorber plate of the cooker. A solar radiation model is also developed and used to compute the ratio of various solar intensities on horizontal, inclined and normal surface of the absorber plate for all months at 30°N latitude. These ratios give a clear indication of greater solar radiation availability on the optimally inclined cooker as compared to the horizontally placed cooker for faster cooking especially during winter months when solar radiation capture is small. Experimental validations have also been performed to access the accuracy of the developed relations and model.

Keywords: Solar Radiation; Solar Cooker; Optimum Inclination; Booster Mirror

1. Introduction

Many important modular studies have been performed on box type solar cookers till date to optimize their performance. Transient analysis was performed to get the overall thermal performance of the box type solar cooker by [1]. A box type solar cooker which could perform well in clear sunny days was developed [2]. An oblique pan was also designed for putting the food material in tilted position. Parametric study of box type solar cooker was also performed with and without booster mirrors called reflectors [3]. Thermal performance of box type solar cooker was tested and a test procedure was developed to test its performance using two figures of merit F₁ and F₂ [4].

An improved box type solar cooker with tilted absorbing surface was developed but the problem of placing the food material could not be solved [5]. Methods of testing and evaluating the advanced version of the box-type solar cooker were also developed [6]. A model for prediction of the cooking power of a solar cooker based on three controlled parameters (solar intercept area, overall heat loss coefficient, and absorber plate thermal conductivity) and three uncontrolled variables (insolation, temperature difference, and load distribution) was developed [7]. The performance of solar cookers by analyzing the previously collected data was also evaluated [8]. The role of cooking vessel inside the cooker was established taking into consideration its lid and the bottom surface [9, 10]. Cooking vessel design in cylindrical shape of box type was also improved and its heat transfer from the lid was also made faster to the food material placed inside the vessel [11, 12]. A comparative experimental study of a box type solar cooker with two different cooking vessels was made [13]. Fins are shown to improve the heat transfer from the internal hot air of the cooker towards the interior of the vessel where the food to be cooked is placed. An optimally inclined box type solar cooker with modified cooking vessel design was presented [14]. The performance of optimally inclined cooker was consistently better as compared to conventional horizontally placed cooker in terms of higher solar radiation availability, absorber plate temperature, higher chamber temperature and lesser cooking time.

The review reveals that various researchers around the world have developed many improved solar cooker designs either by increasing the aperture (solar interception area), by using multiple reflectors, by changing the inclination of the solar cookers, by reducing the overall heat transfer coefficient, by increasing the conductive heat transfer of absorber plate and by modifying the cooking
vessel design. In this study, a mathematical model is developed to compute the optimum inclination angle of the booster mirror for horizontally placed and inclined cooker for maximizing the reflected component of the solar radiation for better performance during all months and selected latitudes.

2. Description of Horizontal and Inclined Box Type Solar Cookers

Two identical box type solar cookers of length 580 mm, width 300 mm and height 155 mm each were fabricated using galvanized iron sheet of 0.8 mm thickness. One cooker was kept in horizontal position on ground (Figure 1) while the other cooker was kept on an optimally inclined frame (Figure 2) in inclined position. Each cooker has two top glass covers of 4 mm thickness and a booster mirror with a provision of altering the angle of inclination. A round shape cylindrical cooking vessel of 175 mm diameter and 52 mm depth was placed inside the horizontally placed cooker. The designed parallelepiped cooking vessel was placed horizontally inside the inclined cooker for greater heat transfer to the food material.

3. Mathematical Model for Optimum Tilt Angle of the Booster Mirrors

3.1. Horizontally Placed Cooker

Optimum tilt angle of the booster mirror is computed for all months and given latitude for maximizing the reflected radiation component onto the absorber plate. In Figure 1, at the top edge A of the booster mirror

\[
\lambda + \theta_z + i = 90^\circ
\]

(1)

also

\[
\angle CAB + r = 90^\circ
\]

(2)

from Equations (1) and (2)

\[
\angle CAB = \lambda + \theta_z
\]

(3)

If ray \( I_{1r} \) strikes the top of the inclined reflector mirror at an angle \( \lambda + \theta_z \), then the line AC formed by reflected ray \( R_{1r} \) reaching the edge of the upper glass cover also has the same internal angle \( \lambda + \theta_z \), then in triangle ABC;

\[
\angle CAB = \theta_z + \lambda, \angle ABC = 90^\circ \pm \lambda
\]

and

\[
\angle ACB = 90^\circ - 2\lambda - \theta_z
\]

Also mathematically we know that

\[
\frac{BC}{\sin(\theta_z + \lambda)} = \frac{AB}{\sin(90^\circ - 2\lambda - \theta_z)} = \frac{AC}{\sin(90^\circ \pm \lambda)}
\]

(4)

Since \( AB = BC = W \) (width of the absorber plate)

\[
W = \frac{W}{\sin(\theta_z + \lambda)} = \frac{W}{\sin(90^\circ - 2\lambda - \theta_z)}
\]

(5)

Equation (5) gives the value of optimum tilt angle of the booster mirror \( \lambda \) for horizontally placed cooker;

\[
\lambda = \frac{90^\circ - 2\theta_z}{3}
\]

(6)

3.2. Optimum Tilt Angle of the Booster Mirror for Inclined Cooker

Optimum tilt of the booster mirror (\( \psi \)) in case of north facing reflector (Figure 2) of inclined solar water heater was given by [15] and is used for inclined cooker as

\[
\psi = (\pi - \beta - 2\phi + 2\delta)/3
\]

(7)

The declination angle “\( \delta \)” is computed using the given Equation (8)

\[
\delta = 23.45 \sin \left( \frac{360(284 + n)}{365} \right)
\]

(8)
where “n” is the number of days starting from January 1.

4. Solar Radiation Model

Variation of effective width of the sun rays intercepted by the absorber plate in horizontal and inclined position of the cooker is shown in Figures 3(a) and (b).

In ΔABC

\[ \angle A = 90 - \alpha_i \]

\[ W_h = W \times \cos(90 - \alpha_s) \]  

(9)

Where \( \alpha \) can be computed using Equation (15).

Hourly solar radiation incident on the inclined surface of the absorber plate depends upon the time of the day \( i.e. \) the hour angle \( \omega \) (zero at noon, negative in the morning and positive in afternoon, varies by 15° after each hour), \( n^{th} \) day of the year (starts from January 1) \( i.e. \) declination angle \( \delta \), altitude angle \( \alpha_s \) with horizontal or zenith angle \( \theta_z \) with vertical and surface azimuth angle \( \gamma \) (in northern hemisphere, it is zero for south facing surfaces, 180° for north facing surfaces, –90° for east facing and +90 for west facing surfaces), latitude angle \( \phi \) of a place and tilt angle \( \beta \) of the surface with horizontal.

Zenith angle of sun on the inclined absorber plate \( \theta_i \) is given by [16]

\[ \theta_i = \cos^{-1} \left[ \sin f (\sin \delta \cos \beta + \cos \delta \cos g \cos \omega \sin \beta) + \cos f (\cos \delta \cos \omega \cos \beta - \sin \delta \cos g \sin \beta) + \cos \delta \sin g \sin \omega \sin \beta \right] \]  

(10)

Zenith angle of the sun with vertical \( (\theta_z) \) and solar altitude angle \( (\alpha_s) \) with horizontal at any time of the day and for any day of the year can be determined at any specific latitude location is given by [16].

\[ \theta_z = \cos^{-1}(\cos \phi \cos \delta \cos \omega + \sin \delta \sin \phi) \]  

\[ \omega = 15^\circ \left( \text{Solar} - 12 \right) \]  

(11)

\[ \omega = \frac{15^n}{360} \]  

(12)

The hour angle is 15° times the number of hours from solar noon. It is negative before noon, zero at noon and positive after 12.

Intensity of extra terrestrial radiation \( I_{ext} \) measured on a plane normal to the radiation on the \( n^{th} \) day of the year,

\[ I_{ext} = I_n \left[ 1 + 0.034 \cos \left( \frac{360n}{365} \right) \right] \]  

(13)

Value of direct normal solar radiation in terrestrial region depends upon turbidity factor \( T_r \) of atmosphere [17].

\[ I_n = T_r \times \exp \left[ \frac{-T_r}{0.9 + 9.4 \sin \alpha_s} \right] \]  

(14)

\( T_r \) is known for different months and for different regions [18].

\[ \alpha_s = 90 - \theta_z \]  

(15)

\[ I_{bh} = I_n \cos \theta_z \]  

(16a)

\[ I_{bi} = I_n \cos \theta_s \]  

(16b)

The amount of diffuse radiation available on the inclined surface can be known as (Tiwari, 2006)

\[ I_d = \frac{1}{3} \left( I_{ext} - I_n \right) \cos \theta_z \]  

(17)

The reflected component of total radiation is then computed as

\[ I_R = r \times I_{gh} \times \sin^2 \beta / 2 \]  

(18)

Where “r” is the ground reflectivity (0.3)

\[ I_{gh} = I_{bh} + I_d \]  

(19)

On the horizontal surface of the absorber plate at solar noon.

Total solar radiation falling on the inclined surface of the absorber plate at solar noon is given by

\[ I = I_{bi} + I_{bh} + I_R \]  

(20)

A computer program in C++ is developed and used for computing the solar radiation flux on various surfaces.

5. Results and Discussion

5.1. Optimum Tilt Angles

Optimum tilt angles computed for winter and summer months at 30°N latitude are shown in Table 1. It shows

Figure 3. Variation of effective width of the sun rays intercepted by the absorber plate in (a) horizontal and (b) inclined position.
that for horizontal cooker, optimum inclination angle $\lambda$ is 3.3°, 1.2° and 8.2° outwards (+) from the vertical position ($\lambda = 0$°) in October, February and March. It becomes 2.08°, 5.6° and 4.2° inwards (−) from the vertical position during November, December and January due to greater solar zenith angle ($\theta_z$). During summer, optimum inclination angle $\lambda$ is 20.8°, 11.2°, 6.8°, 8.9°, 16.7° and 23.3°. In the case of inclined cooker, the optimum inclination angle $\psi$ in winter months is 18.5°, 12.1°, 9.4°, 11.0° 16.3° and 27.6°. This variation is much higher in summer months as 41.5°, 47.6°, 50.6°, 49.2°, 44° and 36.2° for maximization of reflected component.

5.2. Solar Radiation Capture Ratios

The ratio $W_w/W_i$ is 0.62, 0.73, 0.84, 0.76, 0.65 and 0.60 during January, February, March, October, November and December which shows that intercepted width for solar radiation capture is much smaller for horizontal cooker as compared to inclined cooker. The ratio of solar radiation intensity available on inclined to horizontal cooker ($I_i/I_{gh}$) is 1.51, 1.33, 1.15, 1.26, 1.45 and 1.66 during January, February, March, October, November and December which shows that inclined cooker receives much higher solar radiation intensity as compared to horizontal cooker in winter months. The variation of horizontal versus normal surface and inclined versus normal surface also shows that inclined cooker receives almost maximum possible radiation during all months of the year as the ratio is always close to one. Whereas for horizontal cooker this variation is 0.59 to 1.00 during winter and summer months.

5.3. Experimental Validation

The Figure 4 shows that the measured and predicted values of solar radiation intensities match well for horizontal as well as for inclined surface of absorber plate within 5% of the standard deviation. It shows that the developed model is accurate enough for making correct predictions for solar radiation availabilities at optimized inclination angles.

Finally it can be concluded that inclined cooker has much better solar radiation capture during winter months for efficient cooking as compared to horizontally placed cooker at 30°N latitude.

Table 1. Optimum tilt angles computed for horizontal ($\lambda$) and inclined cooker ($\psi$) at 30°N latitude during all months of the year.

<table>
<thead>
<tr>
<th>Winter Month and date, (n)</th>
<th>Latitude (°N) 30</th>
<th>$\theta_z$</th>
<th>$\lambda$</th>
<th>$\psi$</th>
<th>Summer Month and date, (n)</th>
<th>Latitude (°N) 30</th>
<th>$\theta_z$</th>
<th>$\lambda$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct. 15, (288)</td>
<td>40.1</td>
<td>0.3</td>
<td>18.5</td>
<td>3.3</td>
<td>Apl. 15, (105)</td>
<td>20.8</td>
<td>16.1</td>
<td>41.5</td>
<td></td>
</tr>
<tr>
<td>Nov. 15, (319)</td>
<td>49.3</td>
<td>−2.08</td>
<td>12.1</td>
<td>1.2</td>
<td>May 15, (135)</td>
<td>11.2</td>
<td>22.5</td>
<td>47.6</td>
<td></td>
</tr>
<tr>
<td>Dec. 15, (349)</td>
<td>53.5</td>
<td>−5.6</td>
<td>9.4</td>
<td>16.3</td>
<td>June 15, (166)</td>
<td>6.8</td>
<td>25.5</td>
<td>50.6</td>
<td></td>
</tr>
<tr>
<td>Jan. 15, (15)</td>
<td>51.3</td>
<td>−4.2</td>
<td>11.0</td>
<td>41.5</td>
<td>July 15, (196)</td>
<td>8.9</td>
<td>24.0</td>
<td>49.2</td>
<td></td>
</tr>
<tr>
<td>Feb. 15, (46)</td>
<td>43.2</td>
<td>1.2</td>
<td>16.3</td>
<td>11.4</td>
<td>Aug. 15, (227)</td>
<td>16.7</td>
<td>18.8</td>
<td>44.0</td>
<td></td>
</tr>
<tr>
<td>Mar. 15, (74)</td>
<td>32.7</td>
<td>8.2</td>
<td>23.3</td>
<td>36.2</td>
<td>Sept. 15, (258)</td>
<td>27.6</td>
<td>11.4</td>
<td>36.2</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Shows the ratios of intercepted widths for horizontal ($W_w$) and inclined cooker ($W_i$) and solar radiation intensities on Horizontal ($I_{gh}$), Inclined ($I_i$) and normal ($I_n$) surfaces for horizontal and inclined cookers at 30°N latitude during all months of year.

<table>
<thead>
<tr>
<th>Month and date, ($T_r$)</th>
<th>Latitude 30°N</th>
<th>$W_w/W_i$</th>
<th>$I_i/I_{gh}$</th>
<th>$I_w/I_n$</th>
<th>$I_i/I_n$</th>
<th>$W_w/W_i$</th>
<th>$I_i/I_{gh}$</th>
<th>$I_w/I_n$</th>
<th>$I_i/I_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan. 15, (3.1)</td>
<td>0.62</td>
<td>1.51</td>
<td>0.62</td>
<td>0.99</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Feb. 15, (3.2)</td>
<td>0.73</td>
<td>1.33</td>
<td>0.73</td>
<td>1.00</td>
<td>0.95</td>
<td>1.04</td>
<td>0.96</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Mar. 15, (3.5)</td>
<td>0.84</td>
<td>1.15</td>
<td>0.84</td>
<td>1.00</td>
<td>0.88</td>
<td>1.08</td>
<td>0.88</td>
<td>0.97</td>
<td>0.97</td>
</tr>
<tr>
<td>Apr. 15, (3.9)</td>
<td>0.93</td>
<td>1.05</td>
<td>0.93</td>
<td>1.00</td>
<td>0.76</td>
<td>1.26</td>
<td>0.76</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>May 15, (4.1)</td>
<td>0.98</td>
<td>1.02</td>
<td>0.98</td>
<td>1.00</td>
<td>0.65</td>
<td>1.45</td>
<td>0.65</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>June 15, (4.2)</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.99</td>
<td>0.60</td>
<td>1.66</td>
<td>0.59</td>
<td>0.99</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Figure 4. Measured and predicted solar radiation intensities on the horizontal and inclined surfaces of absorber plate on 15th March, 2012.

REFERENCES


