Journal of Modern Physics, 2017, 8, 99-109
http://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

A Detailed Study of the Role of Fermi Energy in Determining Properties of Superconducting NbN

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How to cite this paper: Malik, G.P. (2017) A Detailed Study of the Role of Fermi Energy in Determining Properties of Superconducting NbN. *Journal of Modern Physics*, **8**, 99-109.
http://dx.doi.org/10.4236/jmp.2017.81009

Received: December 27, 2016
Accepted: January 20, 2017
Published: January 23, 2017

Abstract

The recent concern with the role of Fermi energy (E_F) as a determinant of the properties of a superconductor (SC) led us to present new E_F-dependent equations for the effective mass (m^*) of superconducting electrons, their critical velocity, number density, and critical current density, and also the results of the calculations of these parameters for six SCs the T_cs of which vary between 3.72 and 110 K. While this work was based on, besides an idea due to Pines, equations for T_c and the gap at $T = 0$ that are explicitly E_F-dependent, it employed an equation for the dimensionless construct $y = k\theta\sqrt{2m^*}/P_0\sqrt{E_F}$ that depends on E_F only implicitly; k in this equation is the Boltzmann constant, θ is the Debye temperature, and P_0 is the critical momentum of Cooper pairs. To meet the demand of consistency, we give here derivation of an equation for y that is also explicitly E_F-dependent. The resulting framework is employed to (a) review the previous results for the six SCs noted above and (b) carry out a study of NbN which is the simplest composite SC that can shed further light on our approach. The study of NbN is woven around the primary data of Semenov *et al.* For the additional required inputs, we appeal to the empirical data of Roedhammer *et al.* and of Antonova *et al.*

Keywords

E_F-Incorporated Equations for T_c, Δ_0, and j_0 of a Superconductor, NbN

1. Introduction

Some of the recent studies [1]-[7] concerned with high-T_c superconductors (SCs) have been motivated by the belief that Fermi energy (E_F) plays an important role in determining their T_cs and gap-structures. These studies make it natural to ask: why not incorporate E_F (equivalently, chemical potential μ) into the equations for the T_c and the gap Δ of an SC, and then treat it as an independent variable? This is a departure from the usual practice because these parameters are conventionally calculated via equations sans E_F because of the assumption

$$E_F / k\theta \gg 1, \tag{1}$$

where k is the Boltzmann constant and θ is the Debye temperature.

The proposed approach requires, besides the values of T_c and Δ, another property of the SC in order to determine E_F. Upon choosing critical current density j_0 of the SC, new equations for both elemental and composite SCs valid at $T = 0$ were recently presented in [8] for j_0 and the following of their properties: m^*, v_0, and n_s, which denote, respectively, the effective mass of superconducting electrons, their critical velocity at which Δ_0 vanishes, and the density of superconducting electrons. While the results of such a study for Sn, Pb, MgB_2, YBCO, Bi-2212, and Tl-2212 were also reported in [8], it was based on, unlike the equations for Δ_0 and T_c, an equation for the dimensionless construct y, defined below, that is dependent on E_F only implicitly.

$$y = \left(k\theta/P_0\right)\sqrt{2m^*/E_F}, \tag{2}$$

where m^*, P_0, and E_F are in units of electron volts.

To meet the demand of consistency, we present here the derivation of a new equation for y that also contains E_F explicitly—to put it on par with the equations for T_c and Δ_0. While this leads us to review our earlier results, we also undertake here a detailed study of the superconducting properties of NbN because:

(i) It is the simplest composite SC different samples of which (a) have been fabricated by the same method of preparation, (b) are geometrically similar, but (c) differ in size (e.g., film thickness), and for which (d) data in the form $\{T_c, j_0, n_e\}$ are available, where n_e is the density of conduction electrons. This is unlike the composite SCs dealt with earlier, which were not necessarily fabricated by the same method of preparation and for which the values of j_0 and n_e were not available. We were then led to estimate the values of j_0 for these SCs from the data at $T = 4.2$ K. Given the values of T_c and n_e for NbN, we can now also shed light on the ratio n_s/n_e as a function of T_c.

(ii) Since the value of the highest T_c reported for it in [9] is 15.25 K, it is the simplest composite SC for which we believe one-phonon exchange mechanism (OPEM) to be operative. This is unlike, e.g., MgB_2 for which, given its T_c, we need to invoke the two-phonon exchange mechanism (TPEM).

(iii) The above features make NbN the simplest testing ground for some key steps of our approach, such as the procedure followed for resolving θ_{NbN} into θ_{Nb} and θ_N.

The paper is organized as follows. In Section 2 are reproduced from [8] those equations that constitute our framework in the OPEM scenario, which may be defined as one in which the T_c of an SC can be accounted for by a value of the interaction parameter λ that satisfies the Bogoliubov constraint, *i.e.*, $\lambda < 0.5$. Section 3 is devoted to derivation of the new equation for y. The study of NbN is taken up in Section 4. A review of our earlier results is taken up in Section 5. The final two sections are devoted to a discussion and conclusions, respectively.

2. E_F-Incorporated Equations for Various Properties of an SC

Recalled below from [8] are some of the equations that we need for NbN. In these equations $|W_0|$ is to be identified with Δ_0. Further, the equations have been written by assuming that μ, E_F, and λ have the same values at $T = 0$ and $T = T_c$, which is in accord with a tenet of the BCS theory. In the following we use μ and E_F interchangeably because they will be seen to differ negligibly. The modified equation for y will be derived in the next section.

Equation for $|W_0|$:

$$\frac{\lambda}{2} I_1\left(\mu, |W_0|\right) - \left[\frac{3}{4} I_2\left(\mu, |W_0|\right)\right]^{1/3} = 0, \tag{3}$$

where

$$I_1\left(\mu, |W_0|\right) = \int_{-k\theta}^{k\theta} \mathrm{d}\xi \sqrt{\xi+\mu} \Big/ \left(|\xi| + |W_0|/2\right), \tag{4}$$

and

$$I_2\left(\mu, |W_0|\right) = \frac{4}{3}(\mu - k\theta)^{3/2} + \int_{-k\theta}^{k\theta} \mathrm{d}\xi \sqrt{\xi+\mu} \left[1 - \xi \Big/ \sqrt{\left(\xi^2 + W_0^2\right)}\right]. \tag{5}$$

Equation for T_c:

$$\frac{\lambda}{2} I_3\left(\mu, T_c\right) - \left[\frac{3}{4} I_4\left(\mu, T_c\right)\right]^{1/3} = 0, \tag{6}$$

where

$$I_3\left(\mu, T_c\right) = \int_{-k\theta}^{k\theta} \mathrm{d}\xi \left[\sqrt{\xi+\mu} \tanh\left(\xi/2kT_c\right)\right] \Big/ \xi, \tag{7}$$

and

$$I_4\left(\mu, T_c\right) = \int_{-\mu_1}^{k\theta} \mathrm{d}\xi \sqrt{\xi+\mu} \left[1 - \tanh\left(\xi/2kT_c\right)\right]. \tag{8}$$

In the above equations

$$\lambda \equiv \left[N(0)V\right] = \left[\left(1/4\pi^2\right)\left(2m^*\right)^{3/2} E_F^{1/2}\right]V, \quad (\hbar = 1) \tag{9}$$

After λ has been determined via (7) with the input of θ, T_c, and any assumed value of μ, the corresponding value of E_F can be determined by the following equation

$$E_F = \left[(\lambda/2) I_3\left(\mu, T_c\right)\right]^2. \tag{10}$$

Equation for y:

$$1 - \lambda \left[y \ln\left\{y/(y-1)\right\} + \ln(y-1)\right] = 0. \tag{11}$$

This equation has been obtained by assuming that

$$E_3/E_1, \; E_2/E_1 \ll 1, \tag{12}$$

where

$$E_1 = k\theta, \; E_2 = P_0\sqrt{E_F/2m^*}, \; E_3 = P_0^2/8m^* \; \left(y = E_1/E_2\right). \tag{13}$$

Equation for $j_0(E_F)$:

$$j_0\left(E_F\right) = n_s\left(E_F\right) e^* v_0\left(E_F\right) = A_5 (\theta/y)\left(\gamma/v_g\right)^{2/3} E_F^{2/3}, \; \left(e^* = 2e\right) \tag{14}$$

where

$$A_5 = e\Big/\left(6^{1/3}\pi^2 k^{1/3}\hbar\right) \simeq 3.703\times10^{-4}\ \text{CeV}^{-4/3}\cdot\text{K}^{1/3}\cdot\text{sec}^{-1}. \tag{15}$$

3. The Modified Equation for *y* in the OPEM Scenario

Equation (11) has been derived in [10] (pp. 115-120) by assuming Inequality (1). In order to do away with this inequality, we begin here with the following equation for moving CPs because the present derivation differs from the earlier one only beyond it.

$$1 = \left(V/16\pi^3\right)\int_L^U \mathrm{d}^3 p\left[\tanh\left(\beta C(p)/2\right) + \tanh\left(\beta D(p)/2\right)\right]\Big/\left[C(p) + D(p)\right]. \tag{16}$$

In this equation

$$L = -k\theta + P\alpha x - P^2/8m^* \tag{17}$$

$$U = k\theta - P\alpha x - P^2/8m^* \tag{18}$$

$$\alpha = \sqrt{E_F/2m^*},\quad x = \cos(P,\ p),\quad \beta = 1/kT \tag{19}$$

$$C(p) = E_F + W/2 - P^2/8m^* - P\alpha x - p^2/2m^* \tag{20}$$

$$D(p) = E_F + W/2 - P^2/8m^* + P\alpha x - p^2/2m^* \tag{21}$$

Equation (16) was obtained via a Bethe-Salpeter equation. It seems interesting to point out that when $P = 0$, it reduces to the well known criterion of superconductivity derived by Thouless via the t-matrix approach, as can be seen from [11] and, in greater detail, in [12].

The equation for the critical momentum $P_c(T)$ at any temperature follows from (16) by putting $W = 0$. In terms of $\xi = p^2/2m - E_F$, we then have

$$1 = (\lambda/4)\int_0^1 \mathrm{d}x\left[I_1(x) + I_2(x)\right], \tag{22}$$

where

$$I_1(x) = \int_{-E_1+E_2x}^{E_1-E_2x} \mathrm{d}\xi\varphi(\xi)\tanh\left[(\beta/2)(\xi + E_2x)\right] \tag{23}$$

$$I_2(x) = \int_{-E_1+E_2x}^{E_1-E_2x} \mathrm{d}\xi\varphi(\xi)\tanh\left[(\beta/2)(\xi - E_2x)\right], \tag{24}$$

$$\varphi(\xi) = \sqrt{1+\xi/E_F}\Big/(\xi + E_3), \tag{25}$$

and we have used (9), (13) and (19). Besides, justification to follow, we have dropped E_3 everywhere except in the denominator of (25) in order to avoid the singularity at $\xi = 0$. Compared with the earlier equation for y, the new feature of (22) is that it has the additional factor of $\sqrt{1+\xi/E_F}$ in each of its constituents.

In order to obtain the $T = 0\ (\beta = \infty)$ version of (22), we split both $I_1(x)$ and $I_2(x)$ into two parts: $I_1(x)$ into $I_{11}(x)$ and $I_{12}(x)$ for which the limits of integration are $(-E_1 + E_2x)$ to $-E_2x$ and $-E_2x$ to $(E_1 - E_2x)$, respectively, and $I_2(x)$ into $I_{21}(x)$ and $I_{22}(x)$, where the former is integrated from $(-E_1 + E_2x)$ to E_2x and the latter from E_2x to $(E_1 - E_2x)$. It is then seen that, when $T = 0$, $\tanh(\cdots) = (-1)$ for $I_{11}(x)$ and $I_{21}(x)$ and (+1) for the remaining parts.

Because the constituents of both $I_1(x)$ and $I_2(x)$ differ from one another only in the matter of limits and an overall sign, we now consider the following indefinite integral:

$$J = \int \mathrm{d}\xi \varphi(\xi) = \int \mathrm{d}z \sqrt{1+z}\big/(z+E_3'), \tag{26}$$

where we have used (25), put $z = \xi/E_F$ and $E_3' = E_3/E_F$, whence

$$J = 2\sqrt{z+1} - 2\sqrt{E_3'-1}\arctan\left[\sqrt{(z+1)/(E_3'-1)}\right]. \tag{27}$$

Therefore, for $E_3' \ll 1$ (as will be seen to be so), we obtain

$$J = 2\sqrt{z+1} + 2i\arctan\left(i\sqrt{z+1}\right) = 2\sqrt{z+1} - \ln\left[\left(1+\sqrt{z+1}\right)\big/\left(1-\sqrt{z+1}\right)\right]. \tag{28}$$

Taking into account the overall sign of $I_{11}(x)$, (28) yields

$$I_{11}(x) = \left[-2\sqrt{1-E_2'x} + 2\sqrt{1-E_1'+E_2'x}\right] + \ln\left[\left\{\left(1+\sqrt{1-E_2'x}\right)\big/\left(1-\sqrt{1-E_2'x}\right)\right\}\left\{\left(1-\sqrt{1-E_1'+E_2'x}\right)\big/\left(1+\sqrt{1-E_1'+E_2'x}\right)\right\}\right] \tag{29}$$

where $E_i' = E_i/E_F$ $(i = 1, 2)$. Since $E_1'/E_2' = E_1/E_2 = y$, we replace E_2' in the above equation by E_1'/y in order to make contact with (11). $I_{12}(x)$, $I_{21}(x)$ and $I_{22}(x)$ can be similarly calculated. For the sake of compactness, we define

$$u_1(E_1',x,y) = \sqrt{1-E_1'x/y},\ u_2(E_1',x,y) = \sqrt{1+E_1'x/y} \tag{30}$$

$$u_3(E_1',x,y) = \sqrt{1-E_1'(1-x/y)},\ u_4(E_1',x,y) = \sqrt{1+E_1'(1-x/y)}. \tag{31}$$

Then substituting $I_1(x)$ and $I_2(x)$ into (22), we obtain

$$1 = (\lambda/4)\int_0^1 \mathrm{d}x \left[T_1(E_1',x,y) + T_2(E_1',x,y)\right] \equiv T(\lambda,E_1',y) \tag{32}$$

where

$$T_1(E_1',x,y) = 4\left\{-u_1(E_1',x,y) - u_2(E_1',x,y) + u_3(E_1',x,y) + u_4(E_1',x,y)\right\},$$

$$T_2(E_1',x,y) = 2\ln\left\{\frac{1+u_1(E_1',x,y)}{1-u_1(E_1',x,y)}\frac{1+u_2(E_1',x,y)}{1-u_2(E_1',x,y)}\frac{1-u_3(E_1',x,y)}{1+u_3(E_1',x,y)}\frac{1-u_4(E_1',x,y)}{1+u_4(E_1',x,y)}\right\},$$

and $T(\lambda,E_1',y)$ has been defined for later convenience. Obtained by retaining the factor $\sqrt{1+\xi/E_F}$ in $I_1(x)$ and $I_2(x)$, (32) for y is the equation we had set out to obtain. It generalizes (11) which was obtained without this factor. While we could earlier solve (11) in the OPEM scenario with the input of λ alone, solution of (32) requires the additional input of θ and E_F. In order to carry out a quick consistency check of (32), we recall that upon solving (6) for Sn (θ = 195 K, T_c = 3.72 K, $\mu/k\theta = 100$), we had earlier obtained λ = 0.2466. The solution of (11) then led to $y = 21.726$. This is precisely the value we now obtain by solving (32) with the same inputs for λ, θ, and $E_1' = E_1/E_F = 1/100$.

4. Study of NbN Based on E_F-Incorporated Equations

4.1. Outline of Procedure

Working in the OPEM scenario, we

(A) Solve. (6) with the input of θ and T_c to determine λ for different assumed values of μ.

(B) Solve (32) to obtain the values of y corresponding to each pair of (μ,λ) values obtained above.

(C) Calculate j_0 via (14) for each triplet of $\{\mu, \lambda, y\}$ values till it is found to agree with its experimental value.

As predictions, this process also yields the values of m^*, n_s, and v_0 via equations derived in [8] and noted in **Table 3**. As a further check, we calculate $|W_0|$ via (3) by employing the values of μ and λ that led in (C) to the experimental value of j_0.

Before we can proceed as above, we need to fix the Debye temperature of the ions that cause pairing in NbN, *i.e.*, θ_{Nb}.

4.2. Debye Temperature of Nb Ions in NbN

θ_{NbN} is not quoted in [9]. The reported values for it vary in the range 250 - 335 K [13] [14] [15] [16]. We begin by adopting [13]

$$\theta_{NbN} = 335\text{ K}. \tag{33}$$

We now need to resolve θ_{NbN} into θ_{Nb} and θ_N, which *must* be different because masses of Nb and N ions are different. As in [8], we do so via the following equations

$$\theta_{NbN} = 0.5\theta_{Nb} + 0.5\theta_N \tag{34}$$

$$\theta_{Nb}/\theta_N = \left[\left\{1+\sqrt{m_N/(m_N+m_{Nb})}\right\}\Big/\left\{1-\sqrt{m_N/(m_N+m_{Nb})}\right\}\right]^{1/2}, \tag{35}$$

where m_N (m_{Nb}) is the atomic mass of N (Nb). While the first of the above equations has been routinely used for binaries, the second equation has been derived [10] by assuming that the constituents of the binary simulate weakly coupled oscillations of a double pendulum. The equations above have been written by assuming that Nb is the upper bob of the double pendulum. With $m_{Nb} = 92.91$, $m_N = 14.007$, and θ_{NbN} as in (33), the solutions of these equations yield

$$\theta_{Nb} = 397.8\text{ K (Nb as the upper bob)} \tag{36}$$

$$\theta_{Nb} = 105.7\text{ K (Nb as the lower bob)}, \tag{37}$$

the corresponding values for θ_N being 272.2 and 564.3 K (which we do not need). In the following we shall perform all calculations with both the above values of θ_{Nb}.

4.3. Choosing the Values of T_c for Which the Data in [9] Are Addressed

In [9], while values of T_c varying between 9.87 and 15.25 K have been reported for 13 samples of NbN for which the values of j_0 lie in range 2.92 - 13.30 MA·cm^{-2}, the values of n_e have been reported at only three values of T_c, which are 10.72, 14.02, and 15.17 K. Hence we limit the scope of this paper to these values of T_c only.

4.4. A Consistency Check of (6)

If we solve the usual BCS equation for T_c (*i.e.*, the equation sans E_F) with θ = 105.7 (397.8 K) and T_c = 10.72 K, we obtain λ = 0.4142 (0.2682). These are precisely the values we obtain via (6) for the same values of T_c and θ and the additional input of μ (or E_F) = 100 $k\theta$ for each value of θ being considered. Note that $\mu/k\theta = 100$ manifestly satisfies constraint (1). It is hence seen that (6) incorporating μ is a valid generalization of the usual equation sans μ, and may therefore be used for arbitrary values of μ.

4.5. Fixing Additional Required Inputs

Having fixed the values of θ_{Nb} and T_c, we can carry out steps (A) and (B) spelled out in Section 4.1; to carry out step (C) we additionally need the values of γ and the cell parameters of different samples of NbN, which are not given in [9]. We fix these by appealing to the data in [13]. A summary of all the inputs required for this study is given in Table 1. Based on the data in [17], this table includes the estimated values of Δ_0 at each of the $T_c s$ under consideration

4.6. Results

For each of the three values of T_c and both the values of θ_{Nb} noted above, we carried out steps (A)-(C) noted in Section (4.1) for $100 \le \mu/k\theta_{\text{Nb}} \le 1$. For the sake of brevity, presented in Table 2 are the results corresponding to θ_{Nb} = 105.7 K for only those values of $\mu/k\theta_{\text{Nb}}$ for which the calculated values of j_0 are in close agreement with their experimental values noted in Table 1. In obtaining these results we have assumed that θ_{NbN} and hence θ_{Nb} does not change significantly with T_c—as is seen from the data in [13]. Thus, up to this stage, having fixed the value of θ_{Nb} as 105.7 K, we have shown that each subset of the $\{T_c,\ j_0\}$ experimental values can be accounted for by a corresponding set of $\{\mu,\ \lambda\}$ values. Since it is pertinent to ask if we could have achieved similar agreement by adopting a different value of θ_{Nb}, we observe that (i) (3) and (6) can be employed only for values of $\mu/k\theta \ge 1$ —otherwise we run into complex values because of the factor $\sqrt{\xi + \mu}$; (ii) for μ as any multiple of $k\theta_{\text{Nb}}$, the value of λ calculated via either of these equations must be less than 0.5 in order to satisfy the Bogoliubov constraint, and (iii) for any value of T_0, j_0 *increases* as μ is increased.

Table 1. Experimental values of $\{T_c,\ j_0,\ n_e\}$ [9], $\{\gamma,\ a_0\}$ [13], and Δ_0 [17] employed for the study of NbN in this paper.

T_c (K)	j_0 $(\text{MA}\cdot\text{cm}^{-2})$	n_e $(10^{23}\ \text{cm}^{-3})$	$\gamma(\text{mJ}\cdot\text{K}^{-2}\cdot\text{gat}^{-1})$	Cell parameter a_0 $(10^{-8}\ \text{cm})$	Δ_0 (meV)
10.72	3.81	2.59	2.61	4.032	2.06
14.02	11.49	1.26	3.20	4.297	2.38
15.17	13.38	1.26	3.41	4.389	2.31

Table 2. Results of calculations for θ_{Nb} = 105.7 K. The value of $\mu/k\theta_{\text{Nb}}$ against each T_c is the one that led—via the values of E_F, λ, y, and v_g (the gram-atomic volume of NbN)—to a value of j_0 in close agreement with its experimental value noted in Table 1. v_g was calculated with the input of a_0 from Table 1 and the atomic masses of the N_b and N, as in [8].

T_c (K)	$\mu/k\theta_{\text{Nb}}$	μ (meV)	E_F (meV)	λ	y	$v_g\,(\text{cm}^3\cdot\text{gat}^{-1})$	$j_0\,(\text{MA}\cdot\text{cm}^{-2})$	Δ_0 (meV)
10.72	1	9.11	9.19	0.4300	4.496	13.158	3.65	1.79
14.02	4	36.4	36.5	0.4670	3.653	15.926	11.4	2.41
15.17	4.75	43.3	43.3	0.4845	3.418	16.971	13.6	2.64

We now take up the results following from θ_{Nb} = 397.8 K. The least permissible value of μ corresponding to it, *i.e.*, $\mu = k\theta_{Nb}$, led to $j_0 = 9.39$ MA/cm^2 for T_c = 10.72 K and $j_0 = 12.3$ MA/cm^2 for T_c = 14.02 K. Since both these j_0 values are greater than their experimental counterparts, in the light of observation (iii) above, one might attempt to employ *lower* values of μ —which is ruled out because of (i). In fact the value 105.7 K seems like the *upper limit* for θ_{Nb} because we had to employ the least value of μ corresponding to it in order to achieve agreement between the calculated and the experimental values of j_0 at T_c = 10.72 K. As a concrete example in support of this statement, we note that θ_{Nb} = 125 K, led via the least permissible value of μ corresponding to it to the following results:

$$\lambda = 0.4008,\ E_F = 10.84\ \text{meV},\ y = 5.244,\ j_0 = 4.14\ \text{MA/ cm}^2.$$

Since this value of j_0 exceeds the experimental value, we need to employ a lower value of μ —which is impermissible because we have already employed for it the lowest allowed value.

Our considerations so far have been based on the derived values of θ_{Nb} from θ_{NbN} = 335 K. In order to find if there is a lower limit on the value of θ_{Nb}, we now report our findings based on the values of θ_{Nb} derived from the lowest value of θ_{NbN} that was noted above, *i.e.*, 250 K. This value leads via (34) and (35) to θ_{Nb} = 296.8 (Nb as the upper bob) and θ_{Nb} = 78.9 K (Nb as the lower bob). Since the former of these values exceeds the upper limit noted above, we did not pursue it any further. For the latter value, we obtained for any assumed value of $\mu/k\theta \geq 1$,

$$\lambda \geq 0.5392(T_c = 14.02) \text{ and } \lambda \geq 0.5630(T_c = 15.17).$$

Because both these values of λ are in conflict with the Bogoliubov constraint, we conclude that θ_{Nb} cannot be as low as 78.9 K. The value closest to it that yields values of λ satisfying the Bogoliubov constraint at both the T_c*s* is θ_{Nb} = 100 K, for which, e.g., $\lambda = 0.4971\ (T_c = 15.17)$.

Above considerations raise the question: Could $\mu/k\theta_{Nb} < 1$ for NbN? If so, it would put NbN in the category of heavy-fermion SCs [18]. Since there is no compelling reason to believe that this may be so, we did not pursue this idea.

Given in **Table 3** are the predicted values of various parameters concomitant with

Table 3. With θ_{Nb} = 105.7 K, predicted values of various parameters of NbN that are concomitant with the calculated values of j_0 given against each T_c in **Table 2**.

T_c	$s(E_F) = m^*(E_F)/m_e$	$n_s(E_F)(10^{20}\ \text{cm}^{-3})$	$n_s(E_F)/n_e\,(10^{-3})$	P_0(eV)	$v_0\,(10^4\,\text{cm}\cdot\text{sec}^{-1})$
10.72	18.2	3.11	1.20	91.13	7.35
14.02	11.6	12.5	9.92	44.91	5.69
15.17	10.95	14.8	14.8	42.84	5.74

Notes: (i) The equations employed for the calculation of the above parameters have been derived in [8] and are as follows: $s(E_F) = A_1(\gamma/v_g)^{2/3} E_F^{-1/3}\,(A_1 \simeq 3.305\times10^{-10}\ \text{eV}^{-1/3}\cdot\text{cm}^2\cdot\text{K}^{4/3}), n_s(E_F) = A_2(\gamma/v_g)E_F, (A_2 \simeq 2.729\times10^7\ \text{eV}^{-2}\cdot\text{K}^2)$, $P_0(E_F) = A_3(\theta/y)(\gamma/v_g)^{1/3} E_F^{-2/3}, (A_3 \simeq 1.584\times10^{-6}\ \text{eV}^{4/3}\cdot\text{cm}\cdot\text{K}^{-1/3})$, $v_0(E_F) = A_4(\theta/y)(\gamma/v_g)^{-1/3} E_F^{-1/3},\ (A_4 \simeq 1.406\times10^8\ \text{eV}^{2/3}\cdot\text{sec}^{-1}\cdot\text{K}^{-5/3})$. (ii) The product [$n_s(E_F)$ e v_0] at each T_c yields the same value for j_0 as was calculated via (14) and given in **Table 2**. (ii) The values of $E_3' = E_3/E_F = P_0^2/(8sm_eE_F)$ are 1.12×10^{-2}, 1.2×10^{-3} and 9.5×10^{-4} for T_c = 10.72, 14.02 and 15.17 K, respectively, which justify the approximation made in obtaining (32).

the experimental values of T_c and j_0 of NbN at the three T_cs. Among these, the values of Δ_0 are in reasonably good agreement with their experimental counterparts (see Table 1), considering that the latter are estimated values based on the data of [17].

5. A Review of the Results Obtained in [8] in View of the Modified Equation for *y*

For Sn and Pb, all our earlier results remain unchanged because solution of (32) for these elements yields the same values for y that were obtained via (11). Since the values of $\mu/k\theta$ that were needed for these elements are rather large, 55 for Sn and 108 for Pb, this result was to be expected; it also establishes that (32) is a valid generalization of (11). To bring out the extent to which the solutions of the two equations differ for *low* values of $\mu/k\theta$, we note that if we *erroneously* employ (11) for Sn for $\mu/k\theta = 1$, θ = 195 K and λ = 0.2516 (these values are consistent with Δ_0 of the SC), then we obtain y = 20.083 [8]; employment of (11) in this case is erroneous because the equation was obtained by assuming that $\mu/k\theta \gg 1$. On the other hand, solution of (32) for this case leads to $y = 21.613$.

For each of the high-T_c SCs dealt with in [8], there are two θs—say, θ_1 and θ_2—and two λs in the problem. The E_F-dependent equation for y that we now need to employ is

$$1 = T\left(\lambda_1, \Sigma_1', r_1 y\right) + T\left(\lambda_2, \Sigma_2', r_2 y\right), \tag{38}$$

where $T(\cdots,\cdots,\cdots)$ was defined in (32), $\Sigma_i' = k\theta_i/E_F$, $r_i = \theta_i/\theta$ $(i = 1,2)$, $y = k\theta/E_2$, θ being the Debye temperature of the SC, and E_2 was defined in (13). It is hence seen that $r_i y = k\theta_i/E_2$—as it ought to be. Without the multipliers r_1 and r_2, y would denote $k\theta_1/E_2$ in the first term on the RHS of (38) and $k\theta_2/E_2$ in second term, whereas with the multipliers y has the same definition (*i.e.*, $k\theta/E_2$) for both the terms.

Equation (38) generalizes (32) to the TPEM scenario; because it explicitly contains E_F as a variable, it is also a generalized version of equation (30) in [8]. Upon solving (38) with the input of θ, θ_1, θ_2, λ_1, λ_2, and E_F as given in [8] for any of the high-T_c SCs, we obtain the same value for y that we had obtained earlier. Notwithstanding the fact that *all* our results reported in [8] remain unchanged is fortuitous—for lower values of $\mu/k\theta$ than were required in [8], we ought to employ the more accurate (38) rather than equation (30) in [8].

6. Discussion

In connection with fixing θ_{Nb}, we recall that Debye temperature is just another way to specify Debye frequency; it is not to be confused with thermodynamic temperature. We now note that, based on neutron powder diffraction experiments, different values of Debye temperature for the constituents of anisotropic LCO have been reported [19]. This lends support to the idea that the Debye temperature of a composite SC needs to be "resolved." The results reported here depend only on the value of θ_{Nb}, for the identification of which we have simply employed (34) and (35) as a vehicle.

Among the five variables that determine j_0—see Equation (14)—v_g seems to stand alone. We draw attention to a discussion of this variable in [8].

7. Conclusions

The main results of this paper are: (i) a new E_F-dependent equation for the dimensionless construct y defined in (2) has been derived, (ii) it has been shown that the experimental values of T_c, j_0, and Δ_0 of NbN are explicable in the OPEM scenario by a value of θ_{Nb} in the range 100 - 106 K, (iii) predictions have been made about the values of m^*, n_s, and v_0 that are concomitant with the T_c and j_0 values of NbN, (iv) the greater the value of the ratio n_s/n_e, the greater is the value of T_c, and (v) it has been pointed out that we need to employ the new equations for y presented here when $\mu \approx k\theta$.

The work reported here is continuation of an attempt to find via theory tangible clues about raising the T_cs of composite SCs. The role of experiment in this quest can hardly be over-emphasized. While huge amounts of such data about hundreds of SCs are now available, we have not come across a single composite SC for which *all* the relevant parameters identified here, *i.e.*, θ, T_c, Δ_0, j_0, m^*, v_0, n_e, n_s, γ, and v_g, have been reported.

We conclude by noting that the derivations of most of the equations employed in this paper and the concepts on which they are based, e.g., multiple Debye temperatures, superpropagator, and the Bogoliubov constraint, can be found at one place in [10].

Acknowledgements

The author thanks Dr. A. Semenov for kindly responding to his queries concerned with the experimental data reported in [9], and Professor D.C. Mattis for encouragement.

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