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An Alternative Demonstration of the Carnot Efficiency "*Without*" Using the Entropy Function

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Abstract

Entropy function is used to demonstrate the Carnot efficiency, even if it is not always easy to understand its bases: the reversible movement or the reversible heat transfer. Here, it is proposed to demonstrate the Carnot efficiency "*without*" using the Entropy function. For this, it is necessary to enhance two concepts: heat transfer based on the source temperature and work transfer based on external pressure. This is achieved through 1) a balance exchanged heat, based on the source temperature and the system temperature, and 2) a balance exchanged work, based on the external pressure and the internal pressure. With these enhanced concepts, Laplace function $\left(p \cdot v^{\gamma} = \text{Constant}\right)$ and Carnot efficiency $\left(\eta_{Carnot} = 1 - \frac{T_{COLD}}{T_{HOT}}\right)$ can be demonstrated without using the Entropy function (S). This is only a new formalism. Usual thermodynamics results are not changed. This new formalism can help to get a better description of realistic phenomena, like the efficiency of a realistic cycle.

Keywords

Entropy, Carnot Efficiency, Laplace Law, Heat, Work, Thermodynamic Engine, Cycle Efficiency

1. Introduction

1.1. Main Paradox about Entropy

"Does Entropy contradict Evolution?" [1]. Creationists point out a serious contradiction or paradox about En-

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tropy. Whereas the world and the Universe should work to chaos according to the second law of Thermodynamics with Entropy, since the Bing Bang and the primordial soap, stars, planets, life, more organized worlds have appeared and not a more chaotic world. For scientist, this paradox would only be apparent, because the Chaos would be created "elsewhere". For Creationist, if there is a contradiction between the two assessments, that means that one is false, and it would be the Evolution one. Here, the purpose is not to debate or to argue about Evolution vs. Creationism. But our purpose will be to recognize this paradox, and between the two assessments, if one is false, it would be the Entropy function.

1.2. History

Let us remind ourselves where Entropy function (noted S) comes from and its role in thermodynamic efficiency and in the increase of disorder.

Sadi Carnot found the engine efficiency of a thermal motor depends only of its temperatures: the higher the difference of temperatures is, the higher the efficiency is.For a thermodynamic cycle, the efficiency η is defined by

$$\eta = \frac{W}{Q_{Hot}} \tag{1}$$

Like water falling from top to bottom to turn the wheel of a mill, Sadi Carnot compared in 1824 the temperature to an invisible fluid dropping from hot to cold which would turn thermodynamic engines [2]. But for his part Joule demonstrated that Heat is not a fluid, and can be proceed from the Work.

Later Rudolf Clausius created in 1854 the Entropy function, which name means "*content transformative*" and sounds like "*Energy*". Because it is not easy to define the entropy itself, difference of entropy (ΔS) of a system is defined as:

$$\Delta S = \int \frac{dQ_{rev}}{T} \tag{2}$$

where T is the absolute temperature of the system, and dQ_{rev} the incremental reversible transfer of heat into that system. Using the equivalence of Heat and Work, efficiency can be written:

$$\eta = 1 - \frac{|Q_{Cold}|}{Q_{Hot}} \tag{3}$$

And thanks to this Entropy function, it can be demonstrated that the engine efficiency will be limited by the temperature of the sources:

$$\eta \le 1 - \frac{T_{Cold}}{T_{Hot}} \tag{4}$$

where T_{Hot} is the temperature of the hot source, and T_{Cold} the temperature of the cold source. $\left(1 - \frac{T_{Cold}}{T_{Hot}}\right)$ is called the Carnot efficiency.

In 1877, Boltzmann linked the Entropy function to the probability of the number of specific ways in which a system may be arranged, especially with irreversible movements of particles.

At the beginning of the 20thcentury, Bergson stated that the entire Universe is changing over time, in a constant direction. And since then, the Entropy function is linked to the Chaos.

An easy way would be to disconnect the Entropy function from the Chaos, but even keeping the original concept of Claudius with reversible transfer, it remains another paradox.

1.3. Another Paradox

Entropy variation is defined by reversible transfer of heat, as seen in Equation (2):

$$\Delta S = \int \frac{dQ_{rev}}{T} \qquad \text{cf Equation (2)}$$

But what does a "reversible" transfer of heat mean? It would mean that heat could go from one side to another, back and forth indefinitely without any change in the environment!… In a way, a perpetual transfer! Let us remember that the reversible transfer of heat was imagined when heat was still considered as a fluid. In fact the heat goes from the hot source to the cold source, and so "any real transformation is irreversible in fact" [3]. Because variation of entropy is defined by reversible heat, that means that demonstrations using entropy are based on impossible transfers!

Let us now try to demonstrate thermodynamic properties without using this Entropy function, qualified by Bergson [4] as the most "metaphysical" of the physical laws.

2. Definitions and Hypotheses

2.1. The "Internal" Energy

"Internal" energy is not named as such to distinguish it from an external energy, but from the kinetic energy of the system and from the potential energy of the system. From a microscopic point of view, it is the sum of the kinetic energies of the atoms, and it is called U. From a microscopic point of view, it is difficult to define it and in practice, internal energy cannot be measured. It is always surprising to base a theory on a concept we cannot measure!

2.2. The First Law of Thermodynamics

If internal energy cannot be measured, its main property is easy to define and to measure: the variation of internal energy is the sum of the work and the heat exchanged with the outside.

$$\Delta U = W + Q \tag{5}$$

It is called the first law of Thermodynamics

2.3. The Mechanical Work

Traditionally, the infinitesimal work δW exchanged by the system with the outside is calculated thanks to the formula

$$\delta W = -P_{Ext} \cdot dV \tag{6}$$

with dV the volume variation and P_{Ext} the external pressure.

It is surprising that the work received or supplied by the "internal" system is due to the "external" pressure only. A work refers to a volume variation, a movement. According to the mechanical principle of inertia, a movement is due to a force. A force cannot exist if the internal pressure is equal to the external pressure. In fact, a force depends of the "difference" of pressure between the inside and the outside. Let us clarify

- δW_{Ext} is the work supplied (<0) or received (>0) by the outside, the Universe
- δw_{int} is the work supplied (<0) or received (>0) by the inside, the system
- dV_{Ext} the variation of volume of the outside, the Universe
- dv_{int} the variation of volume of the inside, the system

When the volume of the inside system increases (respectively decreases), the volume of the outside decreases (respectively increases):

$$dV_{Ext} = -dv_{int} \tag{7}$$

Taking an axis positive from Left to Right, general expression of the transferred work is:

$$\text{Infinitesimal work} = -\left(P_{Left} - P_{Right}\right) \times \left(\text{Volume variation}\right) \tag{8}$$

which means, if the system is on the Left side, and the outside on the right side (see **Figure 1**):

$$\delta w_{int} = -\left(p_{int} - P_{Ext}\right) \cdot dv_{int} \tag{9}$$

$$\delta W_{Ext} = -\left(p_{int} - P_{Ext}\right) \cdot dV_{Ext} \tag{10}$$

Due to Equation (9), we can check that

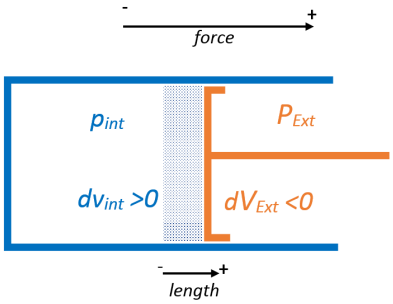

Figure 1. Work exchanged from the inside (in blue) to the outside (in red).

$$\delta W_{Ext} = -\delta w_{int} \tag{11}$$

Note: when we get the same pressure inside and outside, we get equilibrium, there is no movement and so no work:

$$\left[p_{int} = P_{Ext}\right] \Rightarrow \left[\delta W_{Ext} = \delta w_{int} = 0\right] \tag{12}$$

2.4. The Exchanged Heat

The energy radiated $\frac{\delta Q}{dt}$ by a body in vacuum is proportional to its temperature according to Stéphan-Boltzmann law [5] [6]:

$$\frac{\delta Q}{dt} = \sigma T^4 A \tag{13}$$

with the constant $\sigma = 5.67 \times 10^{-8} \left[\mathrm{W \cdot m^{-2} \cdot K^{-4}}\right]$, T the temperature and A the area of the body.

What is true for a single body should be true for several bodies. Let us have an adiabatic (without loss of heat) enclosure with two bodies inside (see **Figure 2**).

First one will emit (so it will have a negative value) the energy

$$\delta q_1 = -\sigma T_1^4 A dt \tag{14}$$

and this emitted energy will be received (so it will have a positive value) by the second body. The second body will emit the energy

$$\delta q_2 = -\sigma T_2^4 A dt \tag{15}$$

and this emitted energy will be received by the first body. When we do the balance, the energy emitted and received by each body is
for the first body

$$\delta Q_1 = \delta q_2 - \delta q_1 \tag{16}$$

$$\delta Q_1 = \sigma\left(T_2^4 - T_1^4\right) A dt \tag{17}$$

and for the second body

$$\delta Q_2 = \delta q_1 - \delta q_2 \tag{18}$$

$$\delta Q_2 = \sigma\left(T_1^4 - T_2^4\right) A dt \tag{19}$$

We can check at the equilibrium where the temperatures are equal that each body emits and receives the same amount of energy, thus the balance for each body is nil:

$$\left[T_1 = T_2\right] \Rightarrow \left[\delta Q_1 = \delta Q_2 = 0\right] \tag{20}$$

Note: mathematically, the difference of temperature is equal to

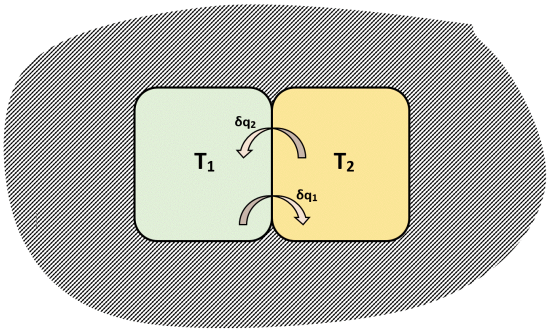

Figure 2. Heat exchanged between two bodies.

$$T_2^4 - T_1^4 = (T_2 - T_1)\left(T_2^3 + T_2^2 T_1^1 + T_2^1 T_1^2 + T_1^3\right) \tag{21}$$

So the energy emitted and received by a body can also be written as

$$\delta Q_1 = s \cdot (T_2 - T_1) \cdot dt \tag{22}$$

where T_2 is the outside temperature (of the source), T_1 is the inside temperature (of the system) and s is a factor depending on the temperature:

$$s = \sigma \cdot \left(T_2^3 + T_2^2 T_1^1 + T_2^1 T_1^2 + T_1^3\right) \cdot A \tag{23}$$

Why should we use this group? Because at the usual temperature, for example at 10˚C and 30˚C, the mathematical difference of s with the average temperature (20˚C) will be about 0.1%, which is insignificant:

$$s(10^\circ\text{C}; 30^\circ\text{C}) = 100732228 \cdot \sigma \cdot A$$

and

$$s(20^\circ\text{C}) = 100615028 \cdot \sigma \cdot A$$

And with regards to the difference with the entropy, s is only a calculation, it is not a *state* "*function*".

To conclude, we do not need to use the Entropy function to explain that Heat goes from hot to cold. It can be explained by a hot body emitting more heat than a cold body, and so the difference of heat looks like a heat transfer from the hot body to the cold body.

2.5. The Writing Convention

To simplify the reading (and the writing), when it is possible or easier we will use:

- Small letters for the internal system.
- Capital letters for the external Universe, or sources.
- $\delta Q_{cold} = s \cdot (T_{COLD} - T_{cold}) \cdot dt$ is the energy emitted and received from the cold inside system.
- $\delta Q_{COLD} = s \cdot (T_{cold} - T_{COLD}) \cdot dt$ is the energy emitted and received from the cold outside source.

Another application (from Equations (10) and (11).

- $dw_{int} = -(p_{int} - P_{Ext}) \cdot dv_{int}$ is the work received or supplied by the internal system (small letters).
- $dW_{Ext} = -(p_{int} - P_{Ext}) \cdot dV_{Ext}$ is the work received or supplied by the external Universe (Capital letters).

Let us now demonstrate the first "isentropic" law without using the entropy function.

3. Laplace's Law Demonstration

3.1. Traditional Demonstration

Traditionally the Laplace law is demonstrated by using the two equations hereafter

$$p_{int} \cdot V = R \cdot T \tag{24}$$

and

$$\delta W = -P_{Ext} \cdot dV \qquad \text{cf. Equation (6)}$$

Then, it is done the hypothesis of a reversible way (let us remember that a reversible way does not exist) in order to write the equality:

$$p_{int} = P_{Ext} \tag{25}$$

which is clearly excessive, because when the internal pressure and the external pressure are equal, there is no movement!

3.2. Traditional Formulas

We will use the perfect gas formula and definitions hereafter:

$$p_{int} \cdot v_{int} = R \cdot T_{int} \tag{26}$$

$$H = U + p_{int} v_{int} \tag{27}$$

$$C_v = \left(\frac{\partial U}{\partial T_{int}} \right)_{v_{int}} \tag{28}$$

$$C_p = \left(\frac{\partial H}{\partial T_{int}} \right)_{p_{int}} \tag{29}$$

$$\gamma = \frac{C_p}{C_v} \tag{30}$$

With previous equations, it is traditionally demonstrated that

$$C_v = \frac{R}{\gamma - 1} \tag{31}$$

3.3. Alternative Demonstration of Laplace Law

According to the first law of thermodynamic, with the new expression defined earlier:

$$\Delta U = w + Q \qquad \text{cf Equation (5)}$$

which can also be written as:

$$dU = \delta w + \delta Q \tag{32}$$

In the case of an adiabatic transformation (and so irreversible), there is no heat exchange:

$$\delta Q = 0 \tag{33}$$

$$dU = \delta w \tag{34}$$

and using Equations (10) and (26):

$$C_v \cdot dT_{int} = -\left(p_{int} - P_{Ext} \right) \cdot dv_{int} \tag{35}$$

Dividing by the temperature T_{int}:

$$C_v \cdot \frac{dT_{int}}{T_{int}} = \frac{P_{ext}}{T_{int}} \cdot dv_{int} - \frac{p_{int}}{T_{int}} \cdot dv_{int} \tag{36}$$

Using Equation (7) on the variation of internal and external volume

$$C_v \cdot \frac{dT_{int}}{T_{int}} = -\frac{P_{ext}}{T_{int}} \cdot dV_{ext} - \frac{p_{int}}{T_{int}} \cdot dv_{int} \tag{37}$$

And using Equation (26) of perfect gas, for internal gas and for external gas

$$C_v \frac{dT_{int}}{T_{int}} = -\left(\frac{R \cdot T_{Ext}}{V_{Ext}}\right)\frac{1}{T_{int}} dV_{Ext} - \left(\frac{R \cdot T_{int}}{v_{int}}\right)\frac{1}{T_{int}} dv_{int} \tag{38}$$

$$C_v \frac{dT_{int}}{T_{int}} = -\frac{R \cdot T_{Ext}}{V_{Ext} \cdot T_{int}} dV_{Ext} - \frac{R \cdot T_{int}}{v_{int} \cdot T_{int}} dv_{int} \tag{39}$$

$$C_v \frac{dT_{int}}{T_{int}} = -R\frac{T_{Ext}}{T_{int}}\frac{dV_{Ext}}{V_{Ext}} - R\frac{dv_{int}}{v_{int}} \tag{40}$$

For the outside Universe, it is considered as infinite,

$$V_{Ext} \approx \infty \tag{41}$$

So

$$\lim_{V_{Ext}\to\infty} \frac{dV_{Ext}}{V_{Ext}} = 0 \tag{42}$$

And so

$$\lim_{V_{Ext}\to\infty} \left(-R\frac{T_{Ext}}{T_{int}}\frac{dV_{Ext}}{V_{Ext}}\right) = 0 \tag{43}$$

Then

$$C_v \frac{dT_{int}}{T_{int}} = -R\frac{dv_{int}}{v_{int}} \tag{44}$$

Integrating it:

$$C_v \cdot \ln\left(\frac{T'_{int}}{T^0_{int}}\right) = -R \cdot \ln\left(\frac{v'_{int}}{v^0_{int}}\right) \tag{45}$$

Then using Equation (31) on the property of C_v:

$$\frac{T'_{int}}{T^0_{int}} = \left(\frac{v^0_{int}}{v'_{int}}\right)^{\gamma-1} \tag{46}$$

Which is equivalent, using Equation (26) of perfect gas

$$p'_{int} v'^{\gamma}_{int} = p^0_{int} \left(v^0_{int}\right)^{\gamma} \tag{47}$$

It is nothing else than the Laplace law:

$$p_{int} \cdot v^{\gamma}_{int} = \text{Constant} \tag{48}$$

The difference with the classical demonstration is that it has not been confused p_{int} and P_{Ext}. Only the adiabatic property (and not the "reversible" property) was used. That means Laplace law has been demonstrated without using neither the Entropy function nor the Entropy property.

4. Carnot's Efficiency Demonstration

4.1. Traditional Demonstration

Traditionally to demonstrate the Carnot efficiency, the in-equation hereafter is used, which links the sources temperatures to the exchanged energies:

$$\frac{\delta Q_{hot}}{T_{HOT}} + \frac{\delta Q_{cold}}{T_{COLD}} \le 0 \tag{49}$$

where δQ_{hot} (>0) is the energy received from the HOT source and δQ_{cold} (<0) the energy supplied to the

COLD source. This equation comes from the Entropy function.

The trouble is it is forgotten that when two bodies (the system and the source) are at the same temperature, there is no exchange of heat (see **Figure 3**):

$$\left[T_{COLD\ source} = T_{AB}\right] \Rightarrow \left[\delta Q_{cold} = 0\right] \tag{50}$$

$$\left[T_{HOT\ source} = T_{CD}\right] \Rightarrow \left[\delta Q_{hot} = 0\right] \tag{51}$$

Starting again from the definition of the cycle efficiency with Equation (1) and (3),

$$\eta = \frac{W}{Q_{hot}} = \frac{W}{0} = \text{undefined} \tag{52}$$

$$\eta = 1 - \frac{\left|Q_{cold}\right|}{Q_{hot}} = 1 - \frac{0}{0} = \text{undefined} \tag{53}$$

the Carnot efficiency is mathematically undefined!

Let us check now the situation without using the Entropy function or its property.

4.2. Preliminaries Demonstrations

4.2.1. A Realistic Cycle

Let us have a cycle as shown in **Figure 4**:

- From A to B, in contact with the COLD source (with a source temperature lower than the system temperature), the system is quite isotherm, cools down but with $T_{AB} \neq T_{COLDsource}$. Let us call the average temperature of the system between *A* and *B*:

$$T_{cold} = T_{AB} \approx T_A \approx T_B \tag{54}$$

- From *B* to *C*, the system is pressed without adding or loosing heat (it is an adiabatic process):

$$Q_{BC} = 0 \tag{55}$$

- From *C* to *D*, in contact with the HOT source (with a source temperature higher than the system temperature), the system is quite isotherm, worms up but with $T_{CD} \neq T_{HOTsource}$. Let us call the average temperature of the system between *C* and *D*:

$$T_{hot} = T_{CD} \approx T_C \approx T_D \tag{56}$$

- From *D* to *A*, the system expands without adding or loosing heat (it is an adiabatic process):

$$Q_{DA} = 0 \tag{57}$$

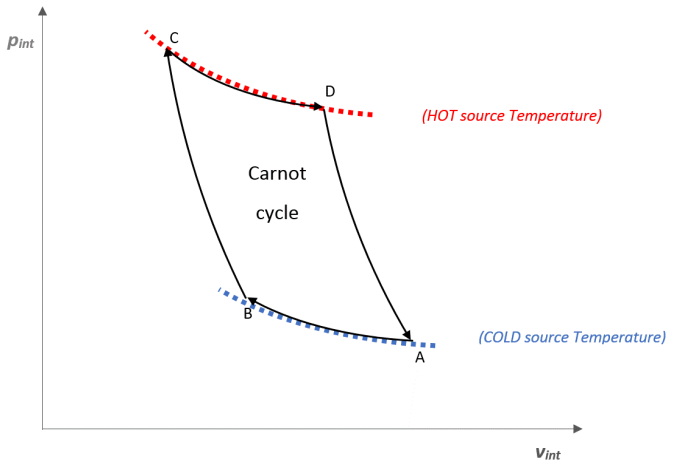

Figure 3. Carnot cycle.

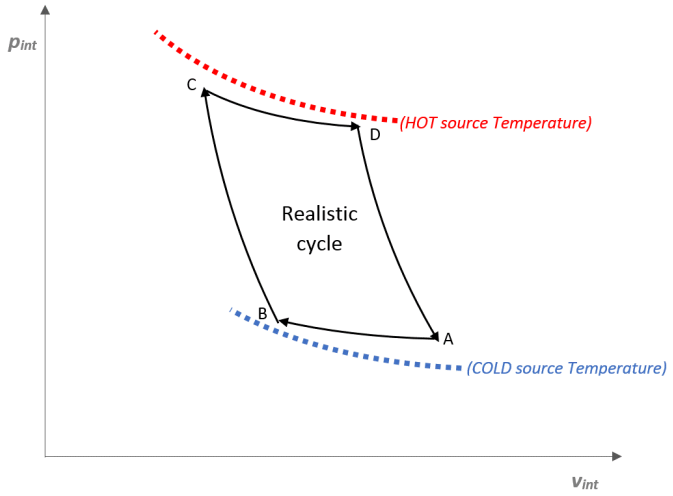

Figure 4. Realistic cycle.

4.2.2. Comment about the "Quasi-Isotherm"

In previous demonstrations, some "quasi"-isotherm parts were noticed. Let us look at a realistic example to check the value. The Stirling engine is an engine with outside sources.

From Solo Kleinmotorem [7], 10 kW output, 650˚C (923 K) Hot temperature, 150 bars, 1500 rpm, Helium (5183 J/kg.K; 0.05 kg/m^3 at 1 bar), for 161 Liters of engine displacement:

$$\text{Power} = \text{Flow} \times \text{Capacity} \times \text{Temperature difference} \tag{58}$$

Numerical application:

$$10000 = \left(\frac{161}{1000} \frac{1500}{60} 0.05 \times 150 \right) \times 5183 \times 0.06$$

So the temperature difference will be of (0.06/923=) 0.01%, which is negligible. The temperature T_{hot} of the system can effectively be considered as quasi-constant.

$$T_A = T_B = T_{hot} \tag{59}$$

4.2.3. Equality of Work

Starting again from the first law of Thermodynamics Equation (5),

$$\Delta U_{BC} = w_{BC} + Q_{BC} \tag{60}$$

$$\Delta U_{DA} = w_{DA} + Q_{DA} \tag{61}$$

Because these two parts are adiabatic, $Q_{BC} = Q_{DA} = 0$ as previously seen Equation (55) and Equation (57),

$$\Delta U_{BC} = w_{BC} \tag{62}$$

$$\Delta U_{DA} = w_{DA} \tag{63}$$

Then, using the definition of C_v from Equation (28), we can write for perfect gas:

$$\Delta U_{BC} = C_v \left(T_C - T_B \right) \tag{63}$$

$$\Delta U_{DA} = C_v \left(T_A - T_D \right) \tag{64}$$

And so from the construction of the cycle and the Equations (62) to (64):

$$w_{BC} = C_v \left(T_{hot} - T_{cold} \right) \tag{65}$$

$$w_{DA} = C_v \left(T_{cold} - T_{hot} \right) \tag{66}$$

The first result is:

$$w_{BC} = -w_{DA} \tag{67}$$

4.2.4. Equality of Pressure

In §3, without using the Entropy function, we have demonstrated

$$p_{int} v_{int}'^{\gamma} = p_{int}^{0} \left(v_{int}^{0}\right)^{\gamma} \quad \text{cf. Equation (47)}$$

and so using the Equation (26) of the property of perfect gas, we can get the general equation:

$$\frac{T_{int}'}{T_{int}^{0}} = \left(\frac{p_{int}'}{p_{int}^{0}}\right)^{\frac{\gamma-1}{\gamma}} \tag{68}$$

and in particular, for the adiabatic parts (BC) and (DA):

$$\frac{T_C}{T_B} = \left(\frac{P_C}{P_B}\right)^{\frac{\gamma-1}{\gamma}} \tag{69}$$

$$\frac{T_D}{T_A} = \left(\frac{P_D}{P_A}\right)^{\frac{\gamma-1}{\gamma}} \tag{70}$$

Because we have quite isotherm part with (AB) and (CD):

$$\frac{T_C}{T_B} \approx \frac{T_{hot}}{T_{cold}} \tag{71}$$

$$\frac{T_D}{T_A} \approx \frac{T_{hot}}{T_{cold}} \tag{72}$$

and so

$$\left(\frac{P_C}{P_B}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{P_D}{P_A}\right)^{\frac{\gamma-1}{\gamma}} \tag{73}$$

$$\frac{P_C}{P_B} = \frac{P_D}{P_A} \tag{74}$$

The second result is:

$$\frac{P_B}{P_A} = \frac{P_C}{P_D} \tag{75}$$

4.2.5. Equality of Temperature

For an isotherm part of a cycle, for a perfect gas:

$$\delta w = -\left(p_{int} - P_{Ext}\right) \cdot dv_{int} \quad \text{cf. Equation (9)}$$

Including the perfect gas property of Equation (26)

$$\delta w_{AB} = -\left(\frac{R \cdot T_{AB}}{v_{int}} - P_{ext}\right) \cdot dv_{int} \tag{76}$$

$$\delta w_{CD} = -\left(\frac{R \cdot T_{CD}}{v_{int}} - P_{ext}\right) \cdot dv_{int} \tag{77}$$

Then with the same logic that we argued for Equation (44)

$$w_{AB} = -R \cdot T_{AB} \cdot \ln\left(\frac{V_B}{V_A}\right) + 0 \tag{78}$$

$$w_{CD} = -R \cdot T_{CD} \cdot \ln\left(\frac{V_D}{V_C}\right) + 0 \tag{79}$$

Or with the perfect gas property, the third results are

$$w_{AB} \approx R \cdot T_{cold} \cdot \ln\left(\frac{P_A}{P_B}\right) \tag{80}$$

$$w_{CD} \approx R \cdot T_{hot} \cdot \ln\left(\frac{P_C}{P_D}\right) \tag{81}$$

4.2.6. Equality of Energy

For an isotherm part, for a perfect gas:

$$\Delta U = 0 \tag{82}$$

For the quasi isotherm part (CD), for a perfect gas

$$\Delta U_{CD} = w_{CD} + Q_{CD} \approx 0 \tag{83}$$

The fourth result or quasi-isotherm work is:

$$w_{CD} \approx -Q_{CD} \tag{84}$$

$$w_{AB} \approx -Q_{AB} \tag{85}$$

4.3. Alternative Demonstration of Carnot Efficiency

4.3.1. Inequality

In **Figure 5** in P-V, the pressure is represented as a function of the volume, so the area is proportional to the work according to the formula:

$$w = \int p(v) \cdot dv \tag{86}$$

Because the hot temperature T_{hot} of the system is lower than the HOT temperature T_{HOT} of the source, and the cold temperature T_{cold} of the system is higher than the COLD temperature T_{COLD} of the second source, the area of the system is always lower than the area delimited by the source temperatures.

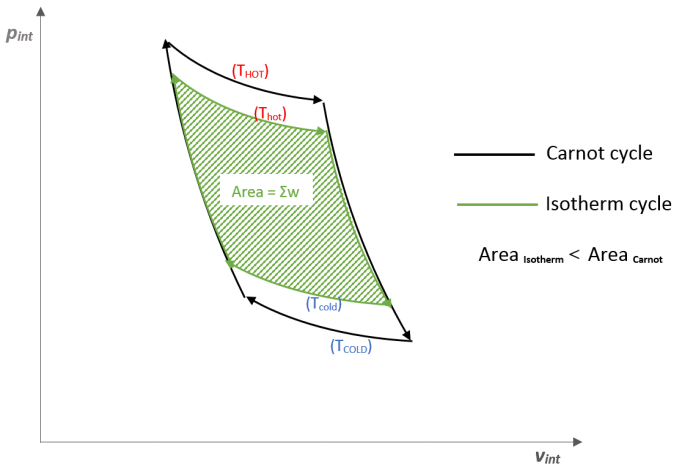

Figure 5. Isotherm cycle vs. Carnot cycle.

$$\int_{T_{cold}}^{T_{hot}} p(v)\cdot dv < \int_{T_{COLD}}^{T_{HOT}} p(v)\cdot dv \tag{87}$$

So for a same heat transferred (cf Equation (1) on the efficiency definition):

$$\eta_{isotherm\ cycle} < \eta_{Carnot} \tag{88}$$

4.3.2. Carnot Efficiency

Let us now demonstrate cycle efficiency without using the Entropy function.

The total work of the cycle is the sum of the supplied and received works:

$$w = w_{AB} + w_{BC} + w_{CD} + w_{DA} \tag{89}$$

As previously demonstrated with the first result:

$$w_{BC} = -w_{DA} \quad \text{cf. Equation (67)}$$

So

$$w = w_{AB} + w_{CD} \tag{90}$$

As previously demonstrated with the fourth result:

$$w_{CD} \approx -Q_{CD} \quad \text{cf. Equation (84)}$$

So the efficiency definition of Equation (1):

$$\eta = \left|\frac{w_{AB} + w_{CD}}{-w_{CD}}\right| \tag{91}$$

or

$$\eta = \left|1 + \frac{w_{AB}}{w_{CD}}\right| \tag{92}$$

Due to the third results

$$\eta = \left|1 - \frac{R\cdot T_{cold}\cdot \ln\left(\frac{P_B}{P_A}\right)}{R\cdot T_{hot}\cdot \ln\left(\frac{P_C}{P_D}\right)}\right| \tag{93}$$

And due to the second result

$$\eta = 1 - \frac{T_{cold}}{T_{hot}} \tag{94}$$

And of course if we extrapolate to the limits with $T_{cold} = T_{COLD}$ and $T_{hot} = T_{HOT}$, we could get

$$\eta_{Carnot} = 1 - \frac{T_{COLD}}{T_{HOT}} \tag{95}$$

4.4. Comments

Sadi Carnot considered the temperature as a fluid: the higher the temperature difference is, the more efficient it is (as for mill, the higher the water level difference is, the more efficient it is). He did not used the entropy function (which was invented by Rudolf Clausius).

Here, we still have the same conclusion—the higher the temperature difference is, the more efficient it is—but for other reasons:

- Because they are isenthalpic (without heat exchange), the work received on BC is equal to the work delivered in DA: balance is nil.

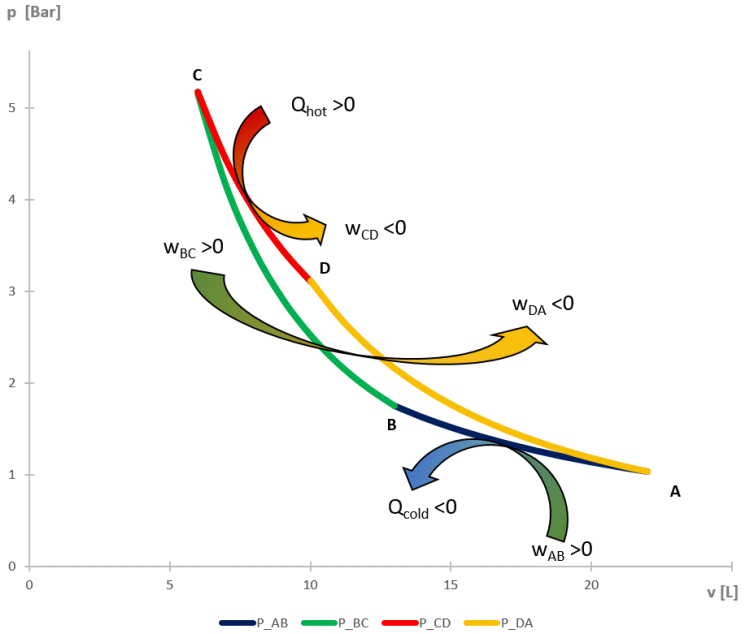

Figure 6. Heat and work balance.

- Because they are quite isotherm, the heat received in CD is transformed into work, and the work received in AB is transformed into "heat" to the cold source. But in the cycle, the heat received is proportional to the temperature of the hot source (cf Equation (81)), and the heat delivered is proportional to the cold one. So the system receives more heat than it delivers, and so the balance is delivered in work. See **Figure 6**.

5. Conclusions

Traditional thermodynamics was elaborated two centuries ago, when heat was understood as an invisible fluid! The main concepts were formalised during this period, like the Carnot's efficiency and then the Entropy function.

The purpose of this paper is to demonstrate the Carnot's efficiency without using the Entropy function. For this, we have generalised both concepts:

- The balance exchanged work, which includes the external pressure and the internal pressure.
- The balance exchanged heat, which includes the temperature of the source and the temperature of the system.

On these two general concepts, we have demonstrated first the Laplace's law, or adiabatic compression/depression, and then the inequality of Carnot efficiency.

In my previous essay [8], the work variation δw had not yet been generalised. So the comparison chart of this other article has been updated in **Appendix** (**Chart 1**).

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Appendix

Chart 1. Traditional form vs. proposed form.

Thermodynamics	Traditional form	Proposed form
Energy variation	$dU = \delta Q + \delta W$	$dU = \delta Q + \delta w$
Exchanged work	$\delta W_{rev} = -P_{ext} \cdot dv$	$\delta w_{int} = -\left(p_{int} - P_{Ext}\right) \cdot dv_{int}$ $\delta W_{Ext} = -\left(p_{int} - P_{Ext}\right) \cdot dV_{Ext}$
Exchanged heat	$\delta Q_{rev} = T \cdot dS$	$\delta Q = s \cdot \Delta T \cdot dt$ $\delta Q = \delta q_2 - \delta q_1$
Reversible movement	Valid	Not valid
Reversible heat transfert	Valid	Not valid
Entropy	Valid	Not valid
Isotherm movement	Valid	Not valid (it is quasi-isotherm)
Heat goes from hot to cold	A consequence of dS	A consequence of δQ
Laplace law	A consequence of dS	A consequence of δQ
Carnot efficiency	$\eta_{Carnot} = 1 - \frac{T_{COLD}}{T_{HOT}}$	Undefined (it is a limit)
Cycle efficiency	Undefined	$\eta_{Cycle} = 1 - \frac{T_{cold} - T_{COLD}}{T_{HOT} - T_{hot}}$