Different Mixing Scenario of Quasi-Degenerate Neutrino with Charged Lepton Correction

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ABSTRACT

Theoretically, in order to achieve non-zero $\theta_{13}$ a little deviation from Tribimaximal Mixing (TBM) pattern is needed, especially on $\theta_{13}$ without perturbing the atmospheric and solar mixing angles. In this work we computed the neutrino mixing angles by disturbing the $\theta_{13}$ as well as $\theta_{12}$ in Bimaximal (BM) and Hexagonal mixing (HM) using non-diagonal charged lepton mass. Considering the standard form of mass texture which satisfies TBM we have shown the quasi-degenerate nature of neutrino. This quasi-degenerate type of mass matrix for BM and HM is then used to calculate the deviated mixing pattern which are consistent with recent neutrino oscillation data.

Keywords: Neutrino Mixing, Neutrino Masses, Charged Lepton Correction

1. Introduction

Recent solar, atmospheric, reactor, and accelerator neutrino experiments have provided concrete evidence that neutrinos are massive and they change their flavors during propagation. In the standard neutrino oscillation picture three active neutrinos are involved, with mass-squared differences of order $10^{-3}$ and $10^{-5}$ eV$^2$. The deficit in the neutrino flux from solar and atmospheric neutrinos have confirmed that at least two neutrinos should have non-zero masses. The mixing pattern and the tiny neutrino masses makes the explanation of the origin of neutrino masses and leptonic flavor mixing one of the most prominent problems in the particle physics. The mixing of lepton flavors is described by a $3 \times 3$ unitary matrix, whose nine elements are commonly parametrized in terms of three rotation angles and three CP-violating phases. Defining three unitary rotation matrices in the complex planes one can express neutrino mixing in terms of three rotations:

\[ U_{12} = \begin{pmatrix} c_{12} & s_{12}e^{i\delta_{21}} & 0 \\ -s_{12}e^{i\delta_{21}} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3) \]

where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ with $1 \leq i, j \leq 3$. Using these three rotations, expression of neutrino mixing with standard parameterizations become

\[ U = U_{23}U_{13}P, \quad (4) \]

where $U$ is known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and can be written as

\[ U = U_{PMNS}P, \quad (5) \]

where $P = \text{Diag}(e^{i\delta_1}, e^{i\delta_2}, 1)$ is a diagonal phase matrix which contains two non-trivial Majorana phases of CP violation. This also involves just three irremoveable physical phases $\delta_1$. In this parameterizations the Dirac phase $\delta$ which enters the CP odd part of neutrino oscillation probabilities is given by $\delta = \delta_1 - \delta_2$. The recent global fit [1] to the various neutrino experimental data has given the following mixing angle values and non-zero of $\theta_{13}$.
\begin{align*}
\sin^2 \theta_{23} &= 0.466 \pm 0.073, \\
\sin^2 \theta_{12} &= 0.312 \pm 0.019, \\
\sin^2 \theta_{13} &= 0.016 \pm 0.010.
\end{align*}

In view of above mixing angles it is clear that the Tribimaximal mixing (TBM) [2-4],
\begin{align*}
U_{\text{PMNS}} &= \begin{pmatrix}
\frac{\sqrt{2}}{3} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix}, 
\end{align*}
\begin{equation}
(9)
\end{equation}
can give the close description to the neutrino oscillation as well with a minor correction in \( \theta_{13} \). The predictions of Equation (9) viz. \( \sin^2 \theta_{23} = \frac{1}{2} \) and \( \sin \theta_{13} = 0 \)
\[ \sin^2 \theta_{12} = \frac{1}{2}, \] are consistent with atmospheric and solar neutrino oscillation with minor correction in \( \theta_{13} \). As the global analysis of neutrino data has provided hints for non-zero \( \theta_{13} \) [5-7]. The first observational hint for non-zero \( \theta_{13} \) has come from the T2K experiment [8]. After T2K experiment MINOS experiment also disfavor the \( \theta_{13} = 0 \) [9]. To achieve non-zero \( \theta_{13} \) theoretically is an interesting topic in neutrino physics. Now, recent analysis on neutrino mixing it is proposed by many papers [10-12] that apart from TBM there are some other mixing pattern like Bimaximal mixing (BM), Hexagonal mixing (HM) and Tetragonal mixing can also give the alternative description of neutrino mixing with correction. Among these, tribimaximal mixing gives very close description of the experimentally found mixing angles when the best fit values are presumed. In the proposed work we give a description of neutrino mixing which gives the framework of bimaximal, tribimaximal and hexagonal mixing with the help of charged lepton correction. In the present paper, we take non-diagonal charged lepton mass in expression of \( m_{LL} \) given by Equation (11). This correction can be realised in the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix in the form of \( U_{\text{PMNS}} = U = U_{\nu}^\dagger U_{\tau} \), where \( U_{\nu} \) diagonalise the charged lepton mass matrix \( m_{eR} \). The matrix \( U_{\nu} \) is considered as Cabibbo like mixing matrix as already been discussed in a recent work [13] for TBM case. This correction will deviate the solar mixing \( \theta_{12} \) and CHOOZ mixing \( \theta_{13} \) to the experimental range in case BM and HM without disturbing the \( \theta_{23} \).

The seesaw mechanism is used to construct neutrino mass models e.g.: Quasi-degenerate, Normal Hierarchical (NH) and Inverted Hierarchical (IH), are discussed in our earlier work [14,15]. Out of these three models which model can give good prediction to the neutrino oscillation is also a topical question in recent neutrino physics. In this work we analyze on Quasi-degenerate neutrino mass model (NH and IH) with different neutrino physics. In this work we analyze on Quasi-degenerate neutrino mass model (NH and IH) with different neutrino physics. In this work we analyze on Quasi-degenerate neutrino mass model (NH and IH) with different neutrino physics.

2. Quasi-Degenerate Neutrino

As mentioned in the introduction neutrino mass eigenvalues can have three kinds of pattern normal, inverted hierarchical and quasi-degenerate. We know, from neutrino oscillations that the neutrino mass pattern is non-degenerate. The pattern is hierarchical, if \( m_1 \ll \Delta m_{23}^2 \), \( m_1 \) is smallest neutrino mass and \( \Delta m_{23}^2 \) is solar mass square difference. When \( m_2 \ll \Delta m_{31}^2 \), the pattern is inverted hierarchical, where \( m_2 \) is the smallest mass. In the quasi-degenerate case both the ordering may be possible.

The most popular neutrino mixing which gives the TBM form is given by Equation (9). This can be generated with two generators \( S \) and \( T \) of \( A_4 \) symmetry, one of which gives charged lepton mass matrix diagonal and other gives the invariant neutrino mass matrix \( m_{\text{TBM}}^\dagger \) [16], where \( m_{\text{TBM}}^\dagger \) is given by
\begin{equation}
(10)
\end{equation}
gives \( m_1 = A + B \), \( m_2 = A - 2B \) and \( m_3 = D \). The mass matrix given by Equation (10) is constructed on the basis of the type I seesaw mechanism,
\begin{equation}
(11)
\end{equation}
where \( m_{eR} \) is diagonal. The present neutrino oscillation data gives the following information for the mass square difference.
\begin{align*}
7.05 \times 10^{-5} \text{eV}^2 &\leq \Delta m_{21}^2 \leq 8.34 \times 10^{-5} \text{eV}^2, \\
2.07 \times 10^{-3} \text{eV}^2 &\leq \Delta m_{31}^2 \leq 2.75 \times 10^{-3} \text{eV}^2,
\end{align*}
with the following best fit values
\[ \Delta m^2_{11} = 7.65 \times 10^{-5} \text{eV}^2, \]
\[ \Delta m^2_{21} = 2.40 \times 10^{-3} \text{eV}^2. \]

Using \( m_1, m_2 \) and \( m_3 \) from Equation (10), mass square differences and the sum of the three mass eigenvalues \( \{ m_{\text{e,\nu,\mu}} = \sum v_i < 0.61 \text{eV} \} \) can be expressed in terms of \( A, B \) and \( D \). By solving these three equations in terms of \( A, B \) and \( D \) values of \( m_1, m_2 \) and \( m_3 \) are calculated with MATHEMATICA and listed in the Table 1, which are quasi-degenerate in nature.

These values of \( A, B \) and \( D \) are now used to construct the neutrino mass matrices with the help of Equation (10) for NH and IN case are:

- **Normal hierarchical:**
  \[ m_{LL} = \begin{pmatrix} -0.052487 & 0.105217 & 0.105217 \\ 0.105217 & 0.10905 & -0.05632 \\ 0.105217 & -0.05632 & 0.10905 \end{pmatrix} \]  
  \( \text{(12)} \)

- **Inverted hierarchical:**
  \[ m_{LL} = \begin{pmatrix} 0.1063024 & -0.0001196 & -0.0001196 \\ -0.0001197 & 0.100396 & 0.005786 \\ -0.00011966 & 0.005786 & 0.100396 \end{pmatrix} \]  
  \( \text{(13)} \)

After Diagonalising the above mass matrices, calculated mass square differences and mixing angles are listed in the Table 2, which shows TBM property.

### 3. Deviation from Original Mixing Pattern with Charged Lepton Correction

From analysis of recent neutrino oscillation parameter it is observed that neutrino mixing is very close to TBM pattern. Deviation from TBM is recently reported [13, 17], where important corrections are incorporated to make the mixing angle match with the experimental data. Different possible alternatives to this TBM are, e.g. Bimaximal, Trimaximal, Hexagonal mixing as well golden ration angles, discussed in recent work [10]. This alternative mixing pattern can also arrive in the experimental range with corrections. In this section we try to produce a mixing pattern, from Bimaximal, Hexagonal as well as Tribimaximal mixing with charged lepton correction, which is consistent with recent neutrino oscillation data. In general, the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix with charged lepton correction can be expressed as product of two unitary matrices as

\[ U = U^l_{\nu}U^d_{\nu}, \]  
\( \text{(14)} \)

where \( U^l_{\nu} \) diagonalizes the neutrino mass matrix as

\[ m^L \text{(diagonal)} = U^Lm_LL^T, \]  
\( \text{(15)} \)

\( U^d_{\nu} \) diagonalizes the charged lepton mass matrix as

\[ m^{LR} \text{(diagonal)} = U^LM^{LR}U^T, \]  
\( \text{(16)} \)

In our earlier work [14,15], in construction of \( m_{LL} \), using seasaw I we were considered the dirac mass \( M^{LR} \) is diagonal, which can be considered as either charged lepton or up quark type. However, a general form of the Dirac neutrino mass matrix is given by

\[ m^{LR} = \begin{pmatrix} \lambda^m & 0 & 0 \\ 0 & \lambda^n & 0 \\ 0 & 0 & 1 \end{pmatrix} \]  
\( \text{(17)} \)

where \( m^l \) corresponds to \( m, \tan \beta \) for \( (m,n) = (6,2) \) in the case of charged lepton and \( m^l \) for \( (m,n) = (8,4) \) in the case of up-quarks. Here \( \lambda \) can pick value between 0.104 and 0.247 for the Dirac neutrino mass matrix. In this pattern the \( U^\text{PMNS} \) matrix given by Equation (14) does not involve \( U^d_{\nu} \). In the present analysis we use non-diagonal \( m^{LR} \) in seasaw I and due to this reason one has to use the contribution \( U^d_{\nu} \) in the PMNS matrix. We construct a deviated neutrino mixing matrix \( U \) i.e. PMNS matrix using Equation (14), where charged leptons mass matrices are considered to be non-diagonal. This predicts mixing angles, are consistent with recent oscillation data. To construct \( U \) we consider \( U^l_{\nu} \) in three different forms e.g. BM, HM as well as TBM and the

<table>
<thead>
<tr>
<th>Type</th>
<th>( A )</th>
<th>( B )</th>
<th>( D )</th>
<th>( m_1 )</th>
<th>( m_2 )</th>
<th>( m_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH</td>
<td>-0.052487</td>
<td>0.105217</td>
<td>0.16537</td>
<td>0.157704</td>
<td>0.157947</td>
<td>0.16537</td>
</tr>
<tr>
<td>IH</td>
<td>0.106302</td>
<td>-0.0001197</td>
<td>0.09461</td>
<td>0.106182</td>
<td>0.106541</td>
<td>0.09461</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Type</th>
<th>( \Delta m^2_{13}[10^{-3}\text{eV}^2] )</th>
<th>( \Delta m^2_{23}[10^{-3}\text{eV}^2] )</th>
<th>( \sin^2 \theta_{12} )</th>
<th>( \sin^2 \theta_{23} )</th>
<th>( \sin^2 \theta_{13} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NH</td>
<td>7.67</td>
<td>2.39</td>
<td>0.37</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>IH</td>
<td>7.68</td>
<td>2.40</td>
<td>0.33</td>
<td>0.5</td>
<td>0</td>
</tr>
</tbody>
</table>
elements of matrix $U_{ij}$ are calculated considering it is a Cabibbo-Kobayashi-Maskawa like mixing i.e. [13]:

$$\sin \theta_{12} = \lambda, \quad \sin \theta_{13} = a \lambda^2, \quad \sin \theta_{23} = b \lambda^2,$$  \hspace{1cm} (18)

where $\lambda$ varies between 0.104 to 0.247 and $a$, $b$ varies between 0.2 to 5. From Equation (18) calculated $\theta_i$ are used to construct the elements of matrix $U_{ij}$.

### 3.1. Hexagonal Mixing

The standard form of hexagonal mixing matrix is

$$H_{2} = \begin{pmatrix}
\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\
-\frac{1}{2} & \frac{\sqrt{3}}{2} & -1 \\
-\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\
\end{pmatrix}$$  \hspace{1cm} (19)

which can predict $\sin^2 \theta_{12} = \frac{1}{4}$, $\sin^2 \theta_{23} = \frac{1}{2}$ and $\sin^2 \theta_{13} = 0$. Using Equation (19) one can also construct $m_{LL}$ for HM as

$$m_{LL}^{HM} = \begin{pmatrix}
\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\
-\frac{1}{2} & \frac{\sqrt{3}}{2} & -1 \\
-\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\
\end{pmatrix}^T \begin{pmatrix} m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3 \end{pmatrix}$$  \hspace{1cm} (20)

and has a texture

$$m_{LL}^{HM} = \begin{pmatrix}
A & B & B \\
B & A + \frac{8}{3}B + D & A + \frac{8}{3}B - D \\
B & A + \frac{8}{3}B - D & A + \frac{8}{3}B + D \\
\end{pmatrix},$$  \hspace{1cm} (21)

where mass eigenvalues are

$$m_1 = A - \frac{2}{3}B, \quad m_2 = A + \sqrt{6}B, \quad m_3 = D.$$  \hspace{1cm} (22)

### 3.2. Bimaximal Mixing

The standard form of bimaximal mixing is written as:

$$U_{BM} = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{2} & \frac{1}{2} & -1 \\
-\frac{1}{2} & \frac{1}{2} & 1 \\
\end{pmatrix}$$  \hspace{1cm} (23)

which predicts $\sin^2 \theta_{12} = \frac{1}{2}$, $\sin^2 \theta_{23} = \frac{1}{2}$ and $\sin^2 \theta_{13} = 0$. Using Equation (23) for the quasi-degenerate case one can construct neutrino mass model for BM case as:

$$m_{LL}^{BM} = \begin{pmatrix}
(1 - 2\delta_1 - 2\delta_2) & -\delta_1 & -\delta_1 \\
-\delta_1 & 1 - \delta_1 - 2\delta_2 & 2\delta_2 \\
-\delta_1 & 2\delta_2 & 1 - \delta_1 - \delta_2 \\
\end{pmatrix} m_o,$$  \hspace{1cm} (25)

where mass eigenvalues are

$$m_1 = (1 - \delta_1 - \sqrt{3}\delta_1 - 2\delta_2) m_o, \quad m_2 = (1 - \delta_1 + \sqrt{3}\delta_1 - 2\delta_2) m_o, \quad m_3 = m_o.$$  \hspace{1cm} (26)

The $m_{1,2,3}$ values can be calculated for HM as well as BM case, which are quasi-degenerate in nature using similar procedure adopted in Section 2 for TBM case. For HM $A = 0.1044669$, $B = -0.128304$, $D = 0.210183$ and for BM $\delta_1 = 7.2 \times 10^{-5}$, $\delta_2 = 3.9 \times 10^{-3}$ are calculated using MATHEMATICA. Values of mass square differences and mixing angles for BM & HM are given in Table 3. The expression for $m_o$ is defined as [15] $m_o = m_i^2 / v_e$, where $v_e = 6.6 \times 10^{12}$.

Now elements of $U_{ij}$ are taken from Equation (19) and (23) for HM and BM respectively. Then using Equation (18) choosing the suitable value for $\lambda$, $a$ and $b$ elements of $U_{ij}$ have been calculated. Finally deviated matrix $U$ is constructed using the following equation:
In this work we have tried to show quasi-degenerate, \( \theta_{13} \) level with a small correction in charged \( \theta_{13} \) coming from analysis of global neutrino PMNS matrix to get which can predicts recent neutrino oscillation parameters accurately in \( 3 \sigma \) level with a small correction in charged lepton part. There is a good scope for extension of this work with CP violating phases. By using \( \Delta m^2_{21}, \Delta m^2_{31} \) and \( \sum m_1 \) it is also possible to get non-zero \( \theta_{15} \) and solar and atmospheric mixing angle directly in the range of experimental values.

### 4. Conclusion

Tribimaximal mixing provides a very close description of neutrino mixing angles. However the present hint of non-zero \( \theta_{13} \) coming from analysis of global neutrino oscillation data may indicate that it is broken. Here in this work, we try to use the charge lepton correction in PMNS matrix to get which can predicts recent neutrino oscillation parameters. Different neutrino mixings e.g. TBM, BM, HM as well as tetragonal mixing are well established and can explain the different pattern of neutrino in context of their mass. Analysis on these mixing are very mass important to give comments on two important aspects e.g. neutrino mass hierarchy and non-zero \( \theta_{13} \). In this work we have tried to show quasi-degenerate (NH, IH) property of neutrino by parametrized standard form of neutrino mass matrix using \( \Delta m^2_{21}, \Delta m^2_{31} \) and \( \sum m_1 \) considering neutrino is tribimaximally mix. Then the deviated mixing pattern from TBM, BM and HM have been constructed using charged lepton corrections. From this analysis it can be conclude that neutrino can mix tribimaximally, hexagonally, and bimaximally which can predicts the neutrino oscillation parameters accurately in \( 3 \sigma \) level with a small correction in charged lepton part. There is a good scope for extension of this work with CP violating phases. By using \( \Delta m^2_{21}, \Delta m^2_{31} \) and \( \sum m_1 \) it is also possible to get non-zero \( \theta_{15} \) and solar and atmospheric mixing angle directly in the range of experimental values.

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### REFERENCES


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