String Cloud and Domain Walls with Quark Matter in Lyra Geometry

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ABSTRACT

We have constructed cosmological models for string cloud and domain walls coupled with quark matter in Lyra geometry. For this purpose we have solved the field equations using anisotropy feature of the universe, special law of variation for Hubble’s parameter proposed by Berman [78] which yields constant deceleration parameter; and time varying displacement field $\beta$. Further some properties of the obtained solutions are discussed.

Keywords: Cosmic Strings; Domain Walls; Quark Matter; Lyra Geometry

1. Introduction

In order to geometrize the whole of gravitation and electromagnetism Weyl [1] proposed a modification of Riemannian manifold. As this theory is physically unsatisfactory, later on Lyra [2] proposed a further modification of Riemannian geometry which bears a close resemblance to Weyl’s geometry. Sen [3] pointed out that the static model with finite density in Lyra manifold is similar to the static model in Einstein theory. Halford [4] showed that the vector field $\varnothing$ in Lyra’s geometry plays a similar role of cosmological constant $\Lambda$ in general theory of relativity. In addition he pointed out that the energy conservation law does not hold in the cosmological theory based on Lyra’s geometry. The scalar-tensor theory of gravitation in Lyra manifold predicts the same effects, within observational limits, as in Einstein theory [5]. Many authors [6-19] constructed different cosmological models and studied various aspects of Lyra’s geometry.

In field theories, topological defects are stable field configurations with spontaneously broken discrete or continuous symmetries [20,21]. Spontaneous symmetry breaking is described within the particle physics context in terms of the Higgs field. The symmetry is said to be spontaneously broken if the ground state is not invariant under the full symmetry of the Lagrangian density. The broken symmetries are restored at very high temperatures in quantum field theories. The topology of the vacuum manifold with $M = Z_2$ is called domain walls [21,22], with $M = S^1$ called strings [23] and one dimensional textures, with $M = S^2$ called monopoles and two dimensional textures, and with $M = S^3$ is called three dimensional textures. The topological defects are called local or global depending on the symmetry is whether local (gauged) or global (rigid). In the early universe these defects are expected to be remnants of phase transitions [24].

String theory attracted the attention of many authors as it possesses the necessary degrees of freedom to describe other interactions, even a mode to describe the graviton. The presence of cosmic strings in the early universe is considered using grand unified theories [25-30]. The study of various aspects of cosmic strings in different theories of relativity is available in the literature [31-52].

The topological defects such as strings, domain walls and monopoles have an important role in the formation of our universe, Hill et al. [53] pointed out that the formation of galaxies are due to domain walls produced during a phase transition after the time of recombination of matter and radiation. Vilenkin [54] and Sikivie and Ipser [55] discussed thin domain wall in Einstein’s theory. Further Schmidt and Wang [56] discussed the same in the context of Brans-Dicke theory. Widrow [57] obtained that a thick domain wall with non-zero stress along a direction perpendicular to the plane of the wall does not allow static metric to be regular throughout entire space. Goetz [58] constructed thick domain wall cosmological model where the scalar field responsible for the symmetry breaking is static while the metric depends on time. Wang [59] obtained a class of exact solution to the Einstein’s field equations representing the gravitational collapse of a thick domain wall. Rahaman et al. [60] found an exact solution of the filed equations for a thick domain wall in a five dimensional Kaluza-Klein
space time within the frame work of Lyra geometry. Pradhan et al. [61] studied plane symmetric domain wall in Lyra geometry. Rahaman and Mukherji [62] constructed two models of domain walls in Lyra geometry. In one of their model the space time is non-singular both in its spatial and temporal behaviour and the gravitational field experienced by a test particle is attractive. In the other model they presented a spherical domain wall with non vanishing stress components in the direction perpendicular to the plane of the wall. Rahaman et al. [63] studied two types of thin domain wall models in Lyra geometry. In their first model the pressure of the domain wall is negligible along perpendicular and transverse direction to the wall. In their second model pressure of the domain wall in the perpendicular direction is negligible but transverse pressures are existed. Further they showed that the thin domain walls have no particle horizon and the gravitational force due to them is attractive. Pradhan et al. [64] obtained general solutions of the field equations for bulk viscous domain walls in Lyra geometry.

It is believed that one of the transitions during the phase transitions of the universe could be Quark Gluon Plasma (QGP) → hadron gas called quark-hadron phase transition when cosmic temperature was T \( \sim 200 \) MeV. The quark-hadron phase transition in the early universe and conversion of neutron stars into strange ones at ultrahigh densities are two ways of formation of strange quark matter [65-67]. In quark bag models based on strong interaction theories it is considered that breaking of physical vacuum takes place inside hadrons. As a result vacuum energy densities inside and outside a hadron become essentially different and the vacuum pressure on the bag wall equilibrates the pressure of quarks and stabilizes the system. It is pointed out that if the hypothesis of the quark matter is true, then some of neutron stars could actually be strange stars built entirely of strange matter [68,69]. The quark matter is modeled with an equation of state (EOS) based on phenomenological bag model of quark matter in which quark confinement is described by an energy term proportional to the volume. In the frame work of this model the quark matter is composed of mass less u, d quarks, massive s quarks and electrons. In the simplified version of the bag model, assuming the quarks are massless and non-interacting we have

$$ P_q = \frac{\rho_q}{3}, $$

where \( \rho_q \) is the quark energy density. The total energy density is given by

$$ \rho_m = \rho_q + B_c, $$

and the total pressure by

$$ P_m = P_q - B_c. $$

Therefore the equation of state for strange quark matter [70,71] is given by

$$ P_m = \frac{1}{3}(\rho_m - 4B_c). $$

Cosmic string is free to vibrate and different vibration modes of the string represent the different particle types since different modes are seen as different masses or spins. Therefore it is plausible to attach quark matter to the string cloud and domain walls. Yilmaz [72] studied rotating cosmological models for domain walls with strange quark matter and normal matter in the non-static and stationary Gödel universes with cosmological constant. Further Yilmaz [73] obtained Kaluza-Klein cosmological solutions of the Einstein’s field equations for quark matter coupled with the string cloud and domain wall. Adhav et al. [74] constructed n-dimensional Kaluza-Klein cosmological models for quark matter coupled with string cloud and domain walls in general relativity. Khadekar et al. [75] studied Kaluza-Klein type Robertson Walker cosmological model by considering variable cosmological term \( \Lambda \) in the presence of strange quark matter with domain wall. Recently Mahanta et al. [76] constructed Bianchi type-III cosmological model with strange quark matter attached to the string cloud in Barber’s second self-creation theory of gravitation.

Motivated by the aforesaid discussion in this paper we consider quark matter coupled to the string cloud and domain walls in the context of Lyra geometry.

2. Field Equations and Their Solutions for the String Cloud with Quark Matter

In this section we consider the metric of the form

$$ ds^2 = dt^2 - A^2 \left( dx^2 + dy^2 \right) - B^2 dz^2 $$

where \( A \) and \( B \) are functions of cosmic time \( t \) only. The energy momentum tensor for string cloud given by Le-telier [32] and Stachel [31] is

$$ T_{ij} = \rho_{u\mu} u_{ij} - \rho_s x_i x_j $$

Here \( \rho \) is the rest energy density for the cloud of strings with particles attached to them, \( \rho_s \) is the string tension density, \( \mu' \) is the four velocity for the cloud of particles, \( x' = \left( 0,0,0,B^{-1} \right) \) is the four vector which represents the strings direction which is the direction of anisotropy and

$$ \rho = \rho_p + \rho_s $$

where \( \rho_p \) is the particle energy density. We know that string is free to vibrate. The different vibrational modes of the string represent different types of particles because these different modes are seen as different masses or spins. Therefore, in this section we take quarks instead of particles in the string cloud. Hence we consider strange quark
matter energy density instead of particle energy density of the string cloud. Thus from (6) we get
\[ \rho = \rho_s + \rho_c + B_c. \]  
(7)

From (5) and (7) we get the energy momentum tensor for strange quark matter attached to the string cloud as
\[ T_{ij} = (\rho_s + \rho_c + B_c)u_i u_j - \rho_s x_i x_j. \]  
(8)

Moreover \( u_i \) and \( x_i \) satisfy the standard relations
\[ u_i u_i = -x_i x_i = 1 \quad \text{and} \quad u_i x_i = 0. \]  
(9)

The Einstein’s field equations based on Lyra’s geometry proposed by Sen [3] and Sen and Dunn [16] in normal gauge are
\[ R_{ij} - \frac{1}{2} g_{ij} R + \frac{3}{2} \mathcal{Q}_i \mathcal{Q}_j - \frac{3}{4} g_{ij} \mathcal{Q}_k \mathcal{Q}_k = -\kappa T_{ij}. \]  
(10)

where \( \mathcal{Q}_i = (\beta(t), 0, 0, 0) \) is the displacement vector and other symbols have their usual meaning as in Riemannian geometry.

Using (8) and (9) the field Equations (10) for the metric (4) leads to the following system of equations
\[ \frac{A^2}{A} \frac{\dot{A}'}{A} + 2 \frac{A' B'}{AB} - \frac{3}{4} \beta^2 = \kappa \rho, \]  
(11)
\[ \frac{A'}{A} + \frac{A' B'}{AB} + \frac{B'}{B} + \frac{3}{4} \beta^2 = 0, \]  
(12)
\[ 2 \frac{\dot{A}'}{A} + \left( \frac{A'}{A} \right)^2 + \frac{3}{4} \beta^2 = \kappa \rho_c. \]  
(13)

Here afterwards the dash over the field variable represents ordinary differentiation with respect to \( t \). Now we find that Equations (11)-(13) are there independent equations involving five unknowns \( A, B, \beta, \rho \) and \( \rho_c \). Therefore to obtain exact solution of the field equations we require two more relations. In view of the anisotropy of the space time, we assume that expansion \( \dot{\theta} \) is proportional to the components of shear tensor \( \sigma^2 \) which also represents anisotropy of the universe [77]. This leads to a polynomial relation between the metric coefficients
\[ A = B^n. \]  
(14)

where \( n \) is a non-zero constant.

In addition, with the help of special law of variation of Hubble’s parameter proposed by Berman [78] that yields constant deceleration parameter models of the universe, we consider
\[ q = -\frac{RR^*}{(R')^2} = \text{Constant} \]  
(15)

where \( R = (A^2 B)^{1/3} \) is the overall scale factor. The constant is taken as negative to obtain an accelerating model of the universe. From (15) we obtain
\[ R = \left( at + b + \frac{1}{\sqrt[3]{w}} \right). \]  
(16)

Using (14) in (17) we find
\[ B = (at + b)^{\frac{1}{3}(1+q)(2n+1)}. \]  
(18)

Thus
\[ A = (at + b)^{\frac{3q}{1+q}(2n+1)}. \]  
(19)

Substituting (18) and (19) in (12) we get
\[ \beta^2 = \frac{4a^2}{k^2} \left[ k + kn - 3(n^2 + n + 1) \right] \frac{1}{(at + b)^2} \]  
(20)

where \( k = (1+q)(2n+1) \).

Now using (18)-(20) in (11) and (13) we obtain string energy density
\[ \kappa \rho = 18a^2 n^2 + 27a^2 n + 9a^2 - 3a^2 nk - 3a^2 k \frac{(at + b)^2}{(at + b)^2} \]  
(21)

and string tension density
\[ \kappa \rho_c = 18a^2 n^2 - 9a^2 n - 9a^2 - 3a^2 nk + 3a^2 k \frac{k^2 (at + b)^2}{k^2 (at + b)^2}. \]  
(22)

Using (18) and (19) the line element (4) is expressed as
\[ ds^2 = dr^2 - (at + b)^{6a(1+q)(2n+1)} \left( dx^2 + dy^2 \right) - (at + b)^{6a(1+q)(2n+1)} dz^2 \]  
(23)

The model (23) represents a string cosmological model with quark matter in Lyra geometry with negative constant deceleration parameter. For the model (23) we have string particle density
\[ \rho_p = \rho - \rho_c = \frac{(k^2 - 1)(18a^2 n^2 - 3na^2 k) + (k^2 + 1)(9a^2 - 3a^2 k) + 9a^2 n(1 + 3k^2)}{\kappa k^2 (at + b)^2}. \]  
(24)
Quark energy density
\[
\rho_q = \rho - B_c = \frac{18a^2n^2 + 27a^2n + 9a^2 - 3a^2nk - 3a^2k - B_c}{\kappa(at + b)^2}
\]  
(25)

Quark pressure
\[
P_q = \frac{\rho_q}{3} = \frac{18a^2n^2 + 27a^2n + 9a^2 - 3a^2nk - 3a^2k - B_c}{3\kappa(at + b)^2}
\]  
(26)

Scalar expansion
\[
\theta = \frac{9a}{(q + 1)(at + b)}
\]  
(27)

Shear scalar
\[
\sigma^2 = \frac{1}{6}\theta^2 = \frac{27a^2}{2(q + 1)^2(at + b)^2}
\]  
(28)

3. Field Equations and Their Solutions for Domain Walls with Quark Matter

The energy momentum tensor of a domain wall [79] in the conventional form is given by
\[
T^\mu_\nu = (P + \rho)u_\mu u_\nu - \rho g^\mu_\nu
\]  
(29)

This perfect fluid form of the domain wall includes quark matter [72] (described by \(\rho_n = \rho_n + B_c\) and \(P_n = P_n - B_c\)) as well as domain wall tension \(\sigma_w\) i.e. \(\rho = \rho_n + \sigma_w\) and \(P = P_n - \sigma_w\). Further \(P_n\) and \(P_m\) are related by the bag model equation of state i.e. Equation (3) and equation of state
\[
P_n = (\gamma - 1)\rho_n
\]  
(30)

where \(1 \leq \gamma \leq 2\) is a constant. Here the four velocity vector \(u^\mu\) is such that \(u^\mu u_\mu = 1\). We use commoving coordinate system \(u^\mu = \delta^\mu_0\). Using the line element (4), the filed equations (10) yield
\[
\left(\frac{A'}{A}\right)^2 + 2\left(\frac{A'B'}{AB}\right)\frac{3}{4}\beta^2 = \kappa \rho
\]  
(31)

\[
\left(\frac{A'}{A}\right)^2 + \left(\frac{A'B'}{AB}\right)\frac{3}{4}\beta^2 = -\kappa P
\]  
(32)

\[
2\left(\frac{A'}{A}\right)^2 + \left(\frac{A'B'}{AB}\right)\frac{3}{4}\beta^2 = -\kappa P
\]  
(33)

where dash over the field variables denote differentiation with respect to \(t\). Here Equations (31)-(33) are three independent equations involving five unknowns \(A, B, \beta, P,\) and \(\rho\). Therefore, in order to obtain exact solution of the field equations two more relations connecting these variables are required. Due to anisotropy of the space-time we assume that scalar of expansion \(\theta\) is proportional to the components of shear tensor \(\sigma^2\) which gives

\[
A = B^a
\]  
(34)

where \(n\) is a non-zero constant.

Further we consider the power law relation between time co-ordinate and displacement field [61,64]
\[
\beta = \beta_d a^r
\]  
(35)

where \(\alpha\) is a constant.

Adding Equation (31) with (32) and (33) we get
\[
-\kappa(P + \rho) = \frac{A''}{A} + 3\left(\frac{A'B'}{AB}\right)\frac{B''}{B} + \left(\frac{A'}{A}\right)^2
\]  
(36)

and
\[
-\kappa(P + \rho) = 2\frac{A''}{A} + \frac{2}{4}\left(\frac{A'}{A}\right)^2 + 2\frac{A'B'}{AB}
\]  
(37)

From (34), (36), and (37) we obtain
\[
\frac{B''}{B} + 2n\left(\frac{B'}{B}\right)^2 = 0, n \neq 1
\]  
(38)

Integrating Equation (38), we find
\[
B = (ct + d)^{\frac{1}{2n+1}}
\]  
(39)

where \(c \neq 0\) and \(d\) are constants of integration. From (34) and (39) we have
\[
A = (ct + d)^{\frac{\alpha}{2n+1}}
\]  
(40)

Substituting (35), (39) and (40) in (31)-(33) we obtain
\[
\rho = \rho_n + \sigma_w = \frac{1}{\kappa}\left[\frac{n^2\epsilon^2 + 2nc^2}{(2n+1)^2(\epsilon + d)^2} - \frac{3}{4}\beta_d^2 a^{2r}\right]
\]  
(41)

and
\[
P = P_n - \sigma_w = \frac{1}{\kappa}\left[\frac{n^2\epsilon^2 + 2nc^2}{(2n+1)^2(\epsilon + d)^2} - \frac{3}{4}\beta_d^2 a^{2r}\right]
\]  
(42)

Further we find the scalar expansion \(\theta = \frac{3c}{ct + d}\) and shear scalar \(\sigma^2 = \frac{3c^2}{2(\epsilon + d)^2}\).

From (41) and (42) we observe that the solutions represent stiff domain walls. To determine the tension of the domain walls \(\sigma_w\) and density and pressure of the quark matter, we will use the equations of state given by (3) and (30) separately.

3.1. Case 1

By using equation of state for quark matter i.e. Equation (3) we get
\[ \rho_n = \frac{3}{2\kappa} \left[ \frac{n^2 c^2 + 2nc^2}{(2n+1)^2 (ct + d)^2} - \frac{3}{4} \beta_0^2 t^{2\alpha} \right] + B_c \]  
(43)

and

\[ P_n = \frac{1}{2\kappa} \left[ \frac{n^2 c^2 + 2nc^2}{(2n+1)^2 (ct + d)^2} - \frac{3}{4} \beta_0^2 t^{2\alpha} \right] - B_c \]  
(44)

Now from equations (1) and (2) we have

\[ \rho_q = \frac{3}{2\kappa} \left[ \frac{n^2 c^2 + 2nc^2}{(2n+1)^2 (ct + d)^2} - \frac{3}{4} \beta_0^2 t^{2\alpha} \right] \]  
(45)

and

\[ P_q = \frac{1}{2\kappa} \left[ \frac{n^2 c^2 + 2nc^2}{(2n+1)^2 (ct + d)^2} - \frac{3}{4} \beta_0^2 t^{2\alpha} \right] \]  
(46)

Again with the help of (41) and (43) we obtain

\[ P_q = \frac{1}{2\kappa} \left[ \frac{n^2 c^2 + 2nc^2}{(2n+1)^2 (ct + d)^2} - \frac{3}{4} \beta_0^2 t^{2\alpha} \right] \]  
(47)

In this case domain walls behave like invisible matter due to their negative tension. Further we find \( P_q = \frac{P_n}{3} \) as proposed by Bodmer [66] and Witten [67].

### 3.2. Case 2

If we use (30) in (41) and (42) we get

\[ \rho_n = \frac{1}{\gamma \kappa} \left[ \frac{2n^2 c^2 + 4nc^2}{(2n+1)^2 (ct + d)^2} - \frac{3}{2} \beta_0^2 t^{2\alpha} \right] \]  
(48)

\[ P_n = \frac{\gamma - 1}{\gamma \kappa} \left[ \frac{2n^2 c^2 + 4nc^2}{(2n+1)^2 (ct + d)^2} - \frac{3}{2} \beta_0^2 t^{2\alpha} \right] - B_c \]  
(49)

\[ \rho_q = \frac{1}{\gamma \kappa} \left[ \frac{2n^2 c^2 + 4nc^2}{(2n+1)^2 (ct + d)^2} - \frac{3}{2} \beta_0^2 t^{2\alpha} \right] - B_c \]  
(50)

\[ P_q = \frac{\gamma - 1}{\gamma \kappa} \left[ \frac{2n^2 c^2 + 4nc^2}{(2n+1)^2 (ct + d)^2} - \frac{3}{2} \beta_0^2 t^{2\alpha} \right] + B_c \]  
(51)

and

\[ \sigma_w = \frac{\gamma (n^2 c^2 + 2nc^2) - 2n^2 c^2 - 4nc^2}{\kappa (2n+1)^2 (ct + d)^2} + \frac{6\beta_0^2 t^{2\alpha} - 3\gamma \beta_0^2 t^{2\alpha}}{4\kappa} \]  
(52)

In this case, when \( \gamma = 1 \) we get domain walls solutions with negative tension and dust quark matter. When \( \gamma = \frac{4}{3} \), we have domain walls solutions with negative tension and quark matter solution like radiation. When \( \gamma = 2 \), we have stiff quark matter solution and domain walls disappear.

### 4. Conclusions

In this paper we obtained exact solutions of the field equations for string cloud and domain walls with quark matter in Lyra geometry. In our solutions we observe the following properties:

1) In the case of string cloud with quark matter for \( n = 1 \), we get dust quark matter solution i.e. \( \rho_q = 0 \). In this model the universe starts at an initial epoch \( t = -\frac{b}{a} \).

At initial epoch the physical parameters \( \theta \) and \( \sigma^2 \) diverge. As cosmic time \( t \) gradually increases \( \theta \) and \( \sigma^2 \) decrease and finally they vanish when \( t \to \infty \). This is consistent with the results of Brook-Havent national laboratory [80, 81]. Here we find \( \frac{\sigma}{\theta} \approx 0.408 \). The present upper limit of \( \frac{\sigma}{\theta} \) is $10^{-15}$ obtained from indirect arguments concerning the anisotropy of the primordial black body radiation [82]. The greater value of \( \frac{\sigma}{\theta} \) for our model than the aforesaid limit indicates that the model represents the early stages of the evolution of the universe.

At initial epoch \( t = -\frac{b}{a} \), the gauge function \( \beta \) diverges.

With the increase in cosmic time \( t \), gauge function \( \beta \) decreases and disappears as \( t \to \infty \). Hence this theory leads to Einstein general theory of relativity at infinite time.

2) In the case of domain walls with quark matter we obtained stiff domain wall solution. Since in the more realistic case in which the domain walls interact with the primordial plasma, the equation of state for domain walls is expected to be stiffer than that of a radiation, our solutions correspond to the early stages of evaluation of the universe. Here we note that the universe starts at an initial epoch \( t = -\frac{d}{c} \). At the initial epoch the physical parameters \( \theta \) and \( \sigma^2 \) diverge. With the increase in cosmic time \( t \) scalar expansion \( \theta \) and shear scalar \( \sigma^2 \) decrease and finally they vanish as \( t \to \infty \).

In case 1 of strange quark matter coupled to domain walls we get \( P_q = \frac{P_n}{3} \) as proposed by Bodmer [66] and Witten [67]. In this case domain walls behave like invisible matter due to their negative tension.

In case 2 of aforesaid model when \( \gamma = 2 \), we get domain walls with negative tension and dust quark matter.
When $\gamma = \frac{4}{3}$, we have domain walls with negative tension and quark matter like radiation. When $\gamma = 2$, domain walls disappear and we have stiff quark matter.

Here we observe that most of the properties of our models are similar to the Kaluza-Klein cosmological models (in string cloud and domain walls coupled with quark matter) obtained earlier by Yilmaz [73] and Adhav et al. [74].

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