Theory of Asymmetric Electrostatic Force

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Abstract

Asymmetric Electrostatic Force is a very interesting phenomenon. The intensity of the electrostatic force that acts on a charged asymmetric shaped conductor changes when the direction of the electric field is reversed. The theory of this phenomenon is not clear until today. Therefore this paper will clearly explain the theory of this phenomenon with Gauss's law. Total ΣES on a closed Gaussian surface around a disk-cylinder shaped charged conductor is the same before and after when the direction of the electric field is reversed. However the distribution of ΣES changes. Many part of total ΣES gather on the front surface of the disk when the electric field is generated. But they transfer to the peripheral side of the cylinder when the direction of the electric field is reversed. This result means that many charges transferred from the surface to the peripheral side. As a result the intensity of the field direction electrostatic force becomes weak.

Keywords

Electrostatic Force, Asymmetric Electrostatic Force,
Asymmetric Shaped Conductor, Charge Distribution, Gauss's Law

1. Introduction

Historically, the main application of electrostatics has been Xerography and powder coating. The main materials used in these technologies are charged powders. The electrostatic force that acts on this powder has been calculated using the following Coulomb equation:

$$F = qE \qquad (1)$$

where F is Electrostatic force, q is Charge quantity, E is Intensity of electric field.

This equation can be applied to a point charge and a sphere-shaped charge carrier [1], and a charged powder can be treated as a point charge. It is apparent from this equation that the intensity of the electrostatic force that acts on the charged powder does not change when the direction of the electric field is re-

versed.

On the contrary the intensity of the electrostatic force that acts on an asymmetric shaped conductor changed when the direction of the electric field was reversed. This interesting new phenomenon was reported depending on simple simulation and experiment. And it was unofficially named as Asymmetric Electrostatic Force [2]. After that, several simulations and experiments have been performed [3] [4] [5] [6]. And finally the existence of the Asymmetric Electrostatic Force was almost confirmed [7].

However the explain of the theory of Asymmetric Electrostatic Force has been not clear until today.

Therefore the purpose of this paper is to explain the theory clearly by Gauss's law.

2. Simulation

2.1. Simulation Method

Figure 1 shows a schematic layout for the simulation. The circular electrodes and the disk-cylinder conductor ere lined up on the Z axis.

The radius of the electrodes was 54.1 mm and the distance between them was 106.9 mm. The radius of the disk was 5.1 mm and its thickness was 0.2 mm. The length of the cylinder was 5.1 mm and the outer and inner radii of the cylinder ware 2.8 mm and 2.6 mm respectively.

When positive high voltage is applied to the right electrode and the left electrode is grounded, an electric field is generated between the electrodes. This electric field is called as the forward electric field from here. On the contrary, when positive high voltage is applied to the left electrode and the right electrode is grounded, another electric field is generated between the electrodes. This
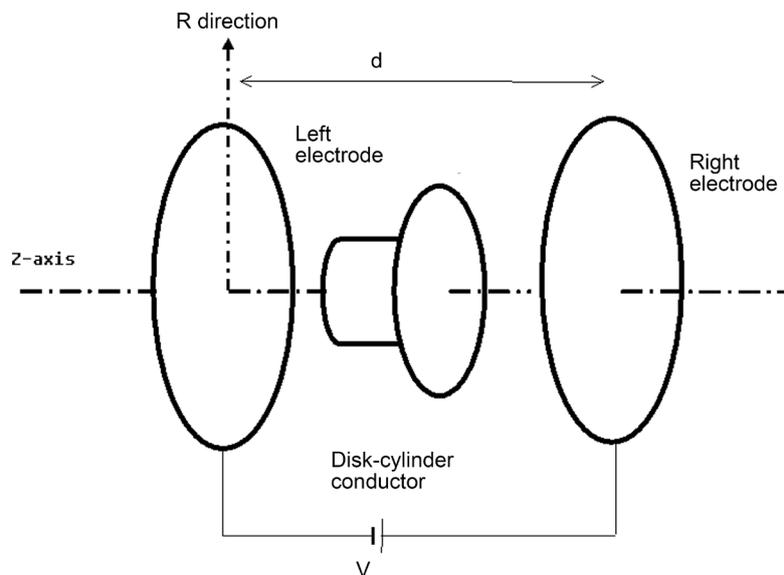

Figure 1. Schematic layout for simulating the electrostatic force acting on a charged disk-cylinder conductor in two different electric field. The directions of them are reversed each other.

electric field is called as the backward electric field from here.

The charged disk-cylinder conductor is electrically floated.

Simulation target space was divided into small 1750 cells. **Figure 2** shows some cells that ware placed around the disk-cylinder conductor. And **Figure 3** shows an expounded picture of the cells that were placed at the center of the disk.

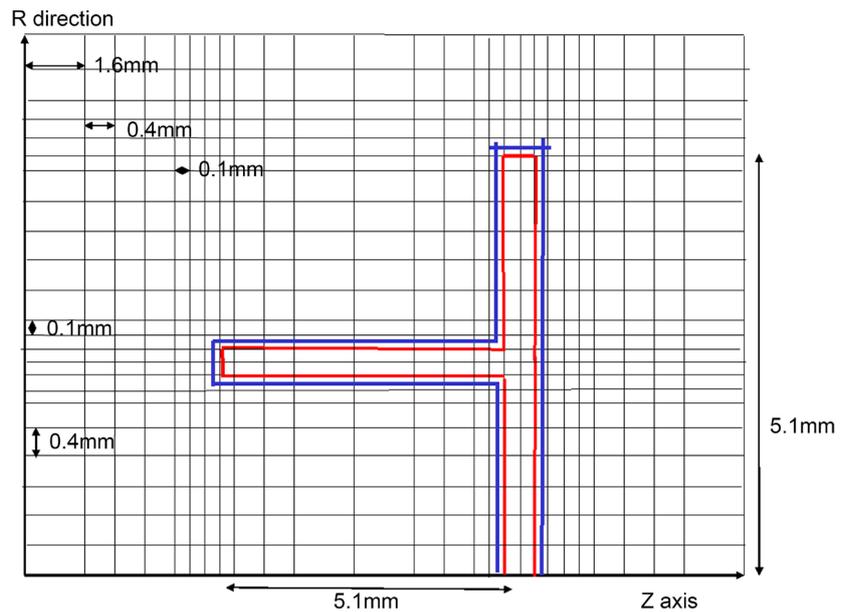In **Figure 2** and **Figure 3**, the red bold lines shows the surface of the disk-cylinder

Figure 2. Cell layout around the disk-cylinder conductor for the axis symmetric finite element method.
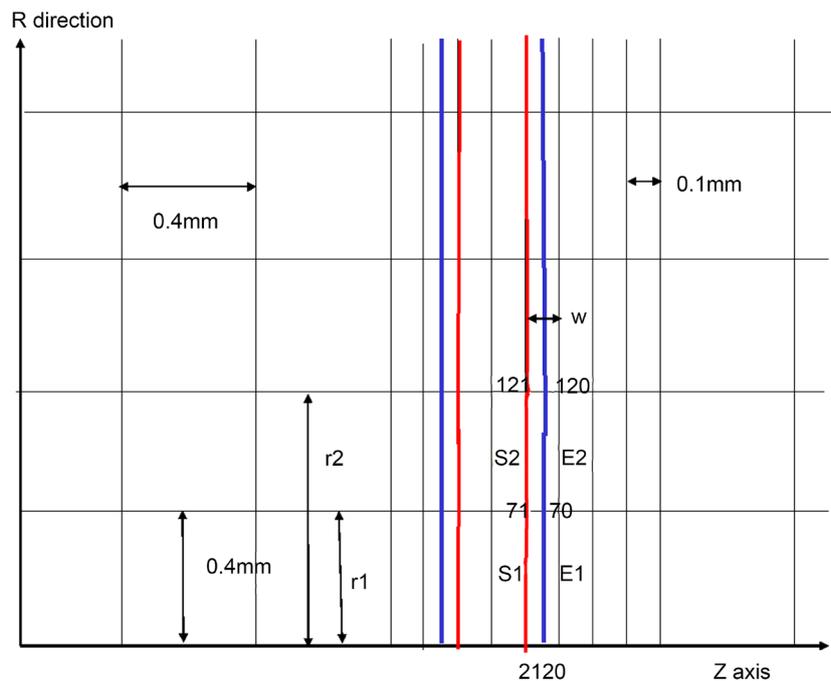

Figure 3. An expounded picture of the cells that were placed at the center of the disk.

conductor. And the blue bold line shows a closed Gaussian surface that surrounds the disk-cylinder conductor. The distance between the surface of the disk-cylinder conductor and the Gaussian surface is 50 μm.

The electric potentials of all 1750 cell points were calculated by Axis symmetric Finite Element Method (FEM). This method can simulate the three-dimensional world. This is mathematically a two-dimensional program, but this is physically a three-dimensional program. Of course, this program is limited for simulating an axis-symmetric material. This method is well known in science and technology fields. Therefore the detail of the calculation is not explained in this paper.

FEM was introduced to Japan by Dr. Matsubara [8] [9] [10]. Then a FEM program for simulation of this research was made from his papers. The reliability of this program was confirmed by simulating a well-known subject that can be solved analytically [7].

The closed Gaussian surface shown by bold blue lines in **Figure 2** consist of 56 different areas. They have different area S_k and different electric field E_k each other. However total of $E_k * S_k$ becomes constant always. And it can be calculated by the following equation (Gauss's law) [11].

$$\sum_{k=1}^{56} E_k S_k = q/\varepsilon_0 \tag{2}$$

where E_k is the intensity of the electric field that through a small area of the Gaussian surface, S_k is area of a small area of the Gaussian surface, q is quantity of charge that is given to the disk-cylinder conductor, ε_0 is Permittivity constant.

For example area S_1 and S_2 shown in **Figure 3** are calculated by the following equations.

$$S_1 = \pi \times r_1^2 \tag{3}$$

$$S_2 = \pi \times r_2^2 - \pi \times r_1^2 \tag{4}$$

where r_k is outer radius of area k.

The intensity of the electric field E_1 and E_2 are calculated by the following equations.

$$E_1 = \frac{\dfrac{(V_{20}+V_{70})}{2} - \dfrac{(V_{21}+V_{71})}{2}}{w} \tag{5}$$

$$E_2 = \frac{\left(\dfrac{(V_{70}+V_{120})}{2} - \dfrac{(V_{71}+V_{121})}{2}\right)}{w} \tag{6}$$

where V_k is simulated voltages of each cell points, w is width of the cell.

The front surface of the disk has 18 different areas as shown in **Figure 2**. Therefore $\sum ES_{dfs}$ of the front surface of the disk is calculated by the following equation.

$$\sum ES_{dfs} = \sum_{k=1}^{18} E_k S_k \tag{7}$$

where E_k is the intensity of the electric field that through a small area of the Gaussian surface, S_k is area of a small area of the Gaussian surface, k is number of the small area of the disk-cylinder conductor.

And ΣES of other parts can be calculated with the same method.

2.2. Simulation Result of the Disk Conductor and the Cylinder Conductor

Before simulating ΣES of the disk-cylinder conductor, ΣES of the lonely disk and ΣES of the lonely cylinder were simulated separately. −0.8854 nC was selected as the Charge quantity of them. Because the value of total ΣES ($=q/\varepsilon_0$) became just 100 always. And +40622 volts was selected as the high voltage that was applied to right or left electrode. The intensity of the electric field became 380000 V/m.

Figure 4 shows the simulation results of the lonely disk conductor. Total ΣES did not change, they were just 100 in the forward electric field and the backward electric field. However, 76% of them was occupied by the front surface in the forward electric field. On the contrary, 76% of them was occupied by the back surface in the backward electric field. And 12% of them was occupied by the back surface in the forward electric field and 12% of them was occupied by the front surface in the backward electric field. And 12% of them remained on the circumference side surface in the both electric field. As a result, the distribution pattern of ΣES was right and left symmetric.

Figure 5 shows the simulation result of the lonely cylinder. Total ΣES was just 100 in the both electric field. And only 11% of them was occupied by the circular front edge in the forward electric field and only 11% was occupied by the circular back edge in the backward electric field. And ΣES of the circular back edge in the forward electric field and ΣES of the front circular edge in the backward electric field were 0%. As a result, 89% of them remained in the circular side surface in the both electric field. Then the distribution pattern of ΣES was right and left symmetric.
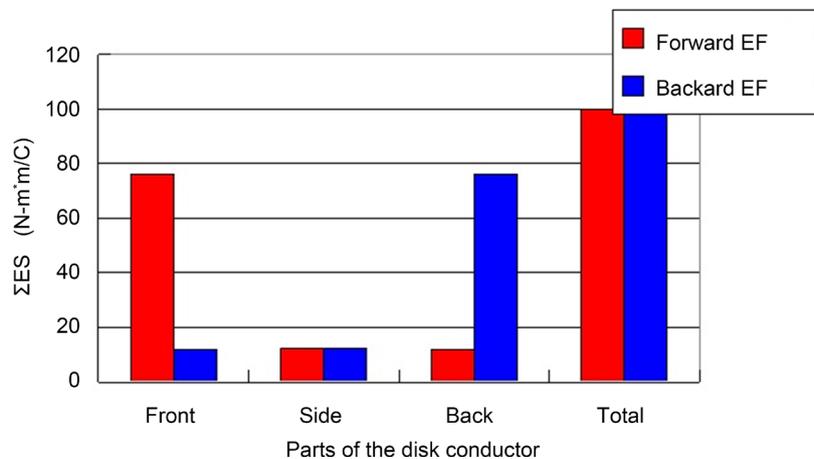

Figure 4. ΣES of each parts of the lonely disk shaped charged conductor in the forward and backward electric field.
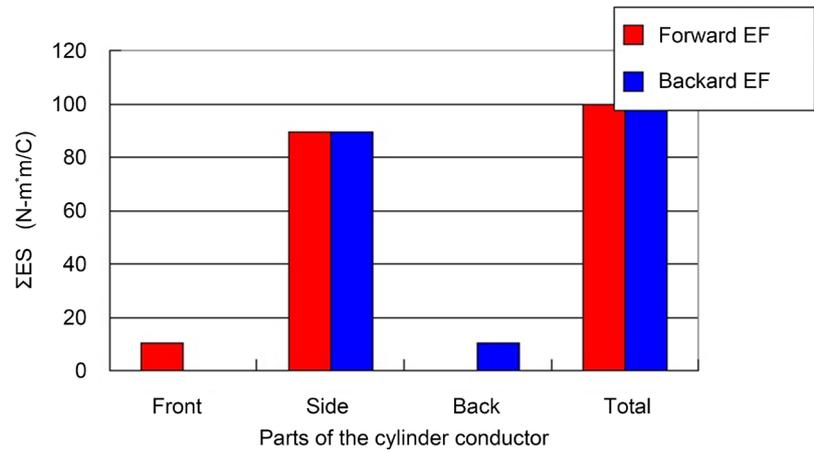
Figure 5. ΣES of each parts of the lonely cylinder shaped charged conductor in the forward and backward electric field.

2.3. Simulation Result of the Disk-Cylinder Conductor

Figure 6 shows the simulation result of the disk-cylinder conductor. Total ΣES was 100 too in the both electric field. However ΣES of the disk parts was +110 and ΣES of the cylinder parts was −10 in the forward electric field. This unexpected result will be explained in the next consideration chapter. And ΣES of the disk parts reduced to 16% and ΣES of the cylinder parts increase to 84% in the backward electric field. And ΣES of the disk parts in the forward electric field was mainly consist of the front surface (80%). And ΣES of the cylinder parts in the backward electric field was mainly consist of the circular side surface (74%). Therefore the distribution pattern of ΣES was not right and left symmetric.

Those three simulation results means that if the shape of the conductor is symmetric, the distribution pattern of ΣES becomes symmetric and if the shape of the conductor is not symmetric, the distribution pattern of ΣES does not become symmetric.

3. Consideration

3.1. Charge Distribution Change

Usually, ΣES is not a familiar physical units for us. Therefore it must be changed to a quantity of electric charge by the following equation [11] for a clear considering.

$$q_k = \sigma_k S_k = \varepsilon_0 E_k S_k \tag{8}$$

where σ_k is surface charge density on the small area of the disk-cylinder conductor, E_k is the intensity of the electric field that through a small area of the Gaussian surface, S_k is area of a small area of the surface of the disk-cylinder conductor, ε_0 is Permittivity constant.

Figure 7 shows the distribution of electric charge on the disk-cylinder conductor in the forward and backward electric field. The total quantity of the electric charge is −0.8854 nC. This is the same to the given electric charge. However
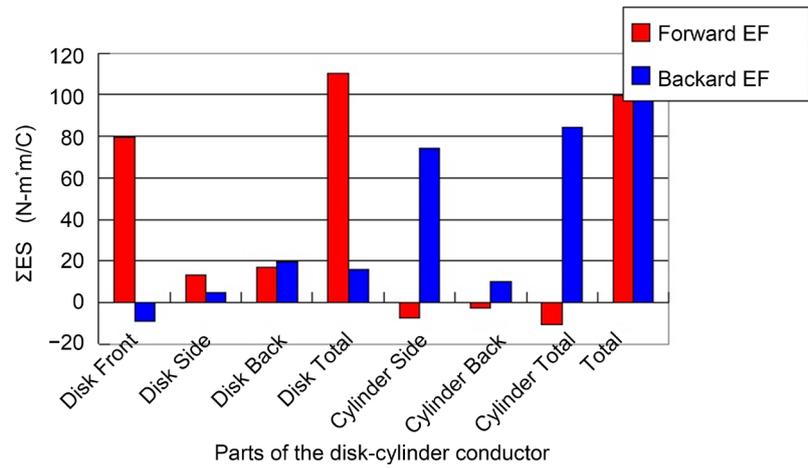
Figure 6. ΣES of each parts of the disk-cylinder shaped charged conductor in the forward and backward electric field.
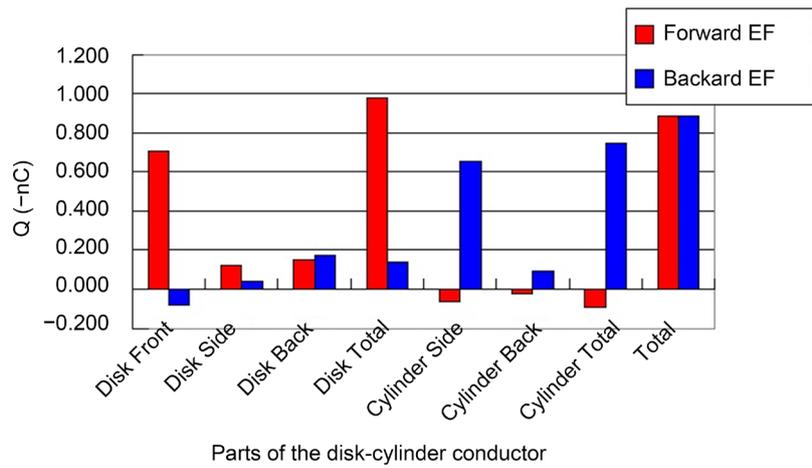

Figure 7. Quantity of charge of each parts of the disk-cylinder shaped charged conductor in the forward and backward electric field.

quantity of charge on the disk in the forward electric field is −0.977 nC. This is larger than quantity of the given electric charge. This difference is clearly explained by an electrostatic induction. Namely −0.092 nC was added to the disk and +0.092 nC was added to the cylinder by electrostatic induction in the forward electric field.

It is apparent from this figure that many charge (−0.71 nC) was gathered in the front surface of the disk parts in the forward electric field and many charge (−0.66 nC) was transferred into the circular side surface of the cylinder parts when the direction of the electric field was reversed.

3.2. Electrostatic Force Distribution Change

Next, electrostatic forces that acts on the different small area of the disk-cylinder charged conductor were calculated by the following equation [12].

$$F_k = \frac{q_k E_k}{2} = \frac{\varepsilon_0 S_k E_k^2}{2} \tag{9}$$

where q_k is charge on the small area of the disk-cylinder conductor, E_k is the intensity of the electric field that through a small area of the Gaussian surface, S_k is area of a small area of the surface of the disk-cylinder conductor, ε_0 is Permittivity constant.

Figure 8 shows the calculation results. It is apparent from this figure that total electrostatic force that acts on the charged disk-cylinder conductor in the forward electric fields is about same to total electrostaitc force in the backward electric field. Namely, the former was 0.57 mN and the latter was 0.52 mN. However the electrostatic forces on the Z direction were different each other. Namely intensity of the electrostatic force in the forward electric field was 0.32 mN and intensity of the electrostatic force in the backward electric field was 0.17 mN. As mentioned before this big difference between the Z direction electrostatic forces is called as Asymmetric Electrostatic Force.

The reason of this phenomenon depends on the charge distribution pattern. Many charge (−0.71 nC) gathered into the front surface of the disk parts in the forward electric field and many charge (−0.66 nC) transferred into the circular side surface of the cylinder parts in the backward electric field. As a result, big electrostatic force (0.38 mN) acts on the front surface of the disk parts in the forward electric field and big electrostatic force (0.31 mN) acts on the circular side surface of the cylinder parts in the backward electric field. However the latter force become zero, Because the circular side surface is parallel to the direction of the electric field and the direction of the electric force that acts on this surface is perpendicular to this surface and they cancel each other at an interval of 180 degrees and ultimately become zero.

On the contrary, the front surface of the disk parts is perpendicular to the direction of the electric field and the direction of the electric force that acts on this surface is perpendicular to this surface. As a result this force contributes to the Z direction electrostatic force 100%.
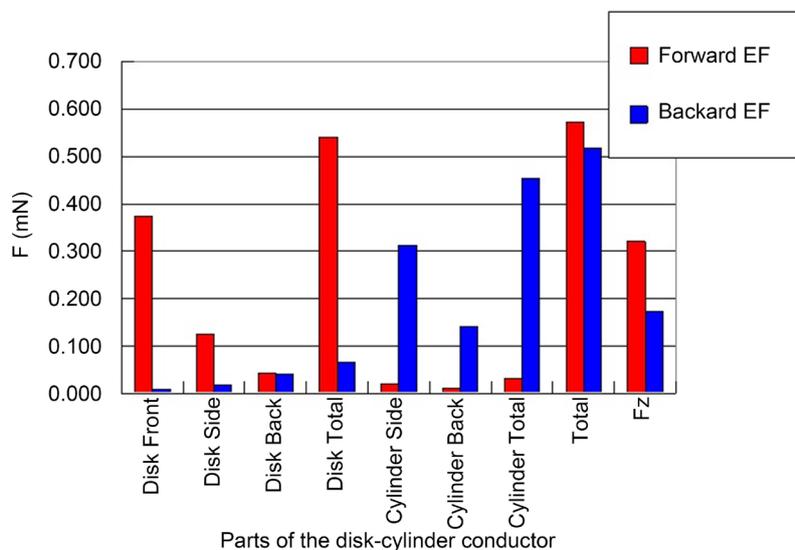

Figure 8. Intensity of the electrostatic force that acts on each parts of the disk-cylinder shaped charged conductor in the forward and backward electric field.

4. Conclusion

Total ΣES on Gaussian surface that encloses the asymmetric shaped charged conductor is the same even if the direction (Z) of the electric field is reversed. And total electrostatic force that acts on this asymmetric shaped conductor is about same even if the direction of the electric field is reversed. However the distribution of ΣES changes when the direction of the electric field is reversed. This result means that the distribution of charge changes. When many charge transferred from the surface that is perpendicular to the direction of the electric field to the other surface that is parallel to the direction of the electric field, the electrostatic force that acts on the former surface contributes 100% to the Z direction electrostatic force; on the contrary the electrostatic force that acts on the latter surface does not contributes to the Z direction electrostatic force. This is the reason of Asymmetric Electrostatic Force.

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