Analysis of Conservative and Magnetically Induced Electric Fields in a Low-Frequency Birdcage Coil*

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ABSTRACT

Numerical methods are used to evaluate variations of the electromagnetic fields generated by a head-sized birdcage coil as a function of load (“loading effect”). The loading effect was analyzed for the cases of a coil loaded with a conductive cylindrical sample, a dielectric cylindrical sample, and an anatomically precise head model. Maxwell equations were solved by means of finite difference time domain (FDTD) method conducted at 12.8, 64, and 128 MHz. Simulation results indicate that at 12.8 MHz the conservative electric field \( E_c \) caused by the scalar electric potentials between the coil and the load or within the load was significantly higher than the magnetically-induced electric field \( E_i \) and was the major component of the total electric field \( E_{total} \). The amplitudes of \( E_c \) and \( E_{total} \) are seen to be lower within a sample than at a corresponding location in an empty coil, but approximately 65% higher in the space between coil and sample than at a corresponding location in an empty coil. This is due to polarization effects generating an additional scalar potential parallel to the original field. The increased electric field between coil and sample may cause increased power deposition at the surface of the sample and may affect the RF-induced currents in external leads used for physiological recording, i.e. ECG, during MRI scanning.

Keywords: MRI; FDTD; Loading Effect; Conservative Electric Field; Birdcage Coil

1. Introduction

In magnetic resonance imaging (MRI), the signal to noise ratio (SNR) and the specific energy absorption rate (SAR), the dosimetric parameter used to establish safety limits for human subjects by the International Electrotechnical Commission (IEC) [1] and the US Food and Drug Administration [2], depend upon the total electric field \( E_{total} \). The \( E_{total} \) can be decomposed into a conservative and magnetically-induced electric fields \( E\)-fields) [3] and a distinction is often needed between the two components. Conservative \( E\)-fields \( (E_c) \) caused by the scalar electrical potential on conductors give rise to a portion of sample loss also referred to as “dielectric loss” [4]. In some cases it is possible to reduce the losses due to \( E_c \) without changing the current distribution or magnetic field distribution using a so called “\( E_c\)-shield” [6], and thus maintaining the desired sensitivity and field of view (FOV) while reducing SAR in the sample and/or the noise received from the sample [6-9]. A previous study [6] showed that this method could be applied to a solenoid coil. This study evaluated whether the method of “\( E_c\)-shield” could be also extended to a birdcage coil, the most common type of coil used in human MRI. One of the motivations of this study to understand the mechanism of thermal injury to skin is currently the most common type of adverse event reported for MRI scans [10]. Another reason for this study is to find the effect of a conductive or a dielectric sample related to the safety assurance in a region of interest (ROI), particularly between the RF coil and the sample. Previous research [9,11] showed that the total electric field inside a coil would be decreased with addition of a loading sample.

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Conversely, the hypothesis driving the proposed study was that the electric field decreases only within the loading sample, but it is the same or higher in the space between coil and sample. Changes in electric field between coil and sample may cause increased power deposition in the subject skin, with possible related thermal injury [10]. Moreover, changes in electric field may also affect the RF-induced currents in external leads used for physiological recording during MRI (e.g., ECG or EEG leads). Additionally, there have been some suggestions that \( \mathbf{E}_c \) may play a significant role in the total sample loss, although it is generally believed that almost all of the sample loss is magnetically-induced [12].

The study was conducted by means of numerical simulations conducting a systematic analysis of the electromagnetic field, including \( \mathbf{E}_c \) and \( \mathbf{E}_r \) generated by the birdcage coil within and surrounding the load. The study took advantage of a recently developed method based on quasi-static approximation that allows separating numerically-calculated \( E \)-field distributions into conservative and magnetically-induced portions [3]. We performed numerical electromagnetic field simulations within and surrounding a high pass (HP) birdcage coil combined with a cylindrical conductive phantom and a human head model at different frequencies. Additionally, the volume charge density \( (\rho_v) \) distribution generating the scalar electric potential and \( \mathbf{E}_c \) was calculated to support the explanation of electromagnetic field variations. Results were analyzed to evaluate the contribution of \( \mathbf{E}_c \), \( \mathbf{E}_r \) and RF magnetic field \( (\mathbf{B}_r) \) to the total electromagnetic field distribution.

### Conservative and Magnetically-Induced Electric Field

The power loss \( (P) \) can be calculated as [8]:

\[
P = \frac{1}{2} \int_\text{vol} \sigma \mathbf{E}_{\text{total}}^2 \, dV
\]  

(1)

where \( \sigma \) is the conductivity (S/m) and \( \mathbf{E}_{\text{total}} = \mathbf{E}_{\text{total}} \) is the amplitude of the total electric field \( (\mathbf{E}_{\text{total}}) \) (V/m), which can be separated as two components:

\[
\mathbf{E}_{\text{total}} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi = \mathbf{E}_r + \mathbf{E}_c
\]  

(2)

where \( \mathbf{A} \) is the vector magnetic potential (Wb/m) and \( \phi \) is the scalar electric potential (V), respectively. In order to reduce the total power absorbed by the sample, the \( \mathbf{E}_{\text{total}} \) should be minimized, which means minimizing the components \( \mathbf{E}_c \) and/or \( \mathbf{E}_r \).

### 2. Materials and Methods

#### 2.1. Birdcage Head Coil

A high-pass (HP) birdcage head coil was modeled using 12 rods of 300 mm of length, disposed circularly with an inner diameter (ID) of 290 mm. To accurately simulate field distribution generated by an ideal high-pass birdcage coil, 12 sinusoidal voltage sources of 1 V amplitude in series with a 50 \( \Omega \) resistor were placed in both the top and bottom rings, in the middle of each of the segments between the rods of the coil (Figure 1). Each source was assigned a phase-shift equal to the azimuthal position of the segment (i.e., 30°C) between voltages in adjacent end ring segments, and with sources in opposite end rings having opposite orientation. The following frequencies were modeled: 12.8, 64, and 128 MHz, corresponding to 0.3, 1.5 and 3.0 T for water proton MRI.

#### 2.2. Load: Phantom and Head Model

The birdcage head coil was loaded with a cylindrical sample having ID of 200 mm and length of 300 mm with a 5 mm resolution. Three different electrical properties for the phantom were simulated, namely: a) conductive sample \( (\sigma = 0.2 \text{ S/m}, \varepsilon_r = 1) \), b) dielectric sample \( (\sigma = 0 \text{ S/m}, \varepsilon_r = 78) \), and weak saline \( (\sigma = 0.2 \text{ S/m}, \varepsilon_r = 78) \) [6].

Simulations were also performed with an anatomically-precise human head model (Figure 1). The human head model was created by segmenting the digital photographic data of the National Library of Medicine’s Visible Human Project [13, 14], and then transforming these segmented images into a 3D grid of Yee cell cubes [15]. The human head model had a 5 × 5 × 5 mm\(^3\) isotropic resolution and contained 20 tissue types [16,17] having different conductivity \( (\sigma) \) and relative permittivity \( (\varepsilon_r) \) values.

#### 2.3. Numerical Simulations and Data Processing

All simulations were performed using commercially available software (xFDTD, Remcom, Inc, State College, PA) and analysis of results was performed in Matlab (The MathWorks, Inc., Natick, MA). Simulation results of electromagnetic fields were normalized so that \( \left| \mathbf{B}_r \right| = 4 \mu \text{T} \) at the coil center corresponding to a 1.5 ms
90° pulse [14].

The parameters used for the FDTD simulations to ensure convergence of the simulations were: 500,000 number of time steps, −30 dB convergence threshold, and 61.5 periods. The $E_c$ and $E_i$ separation method was applied only at the 12.8 MHz because the methods assumes quasi-static approximation. Calculation procedures for the method were developed and explained in [3] and are reported in the appendix for the reader’s convenience.

3. Results

Figure 2 shows the normalized x-, y- and z-component of $|E_c|$, $|E_i|$, $|E_{total}|$ and $|B_z|$ within the empty coil in a single plane (YZ-plane) passing through the iso-center at 12.8 MHz. Values for $|E_i|$ were close to zero (i.e., less than 0.04 V/m) along the axis of the RF coil, increasing with distance from the center line following Faraday’s Law.

Table 1 reports the results of the simulations with the coil loaded with the conductive, dielectric, or weak saline phantom. There was approximately 25% reduction (i.e., 56 vs. 76 V/m) in the average $|E_c|$ and 70% increase (i.e., 231 vs. 134 V/m) in the maximum $|E_c|$ within the whole sample when the coil was loaded with the conductive, dielectric, or weak saline phantom compared to the empty coil.

Conversely, when comparing the results of the coil loaded with the head model vs. the empty coil, there was a 30% reduction (i.e., 53 vs. 76 V/m) in the average $|E_c|$ and a 430% increase (i.e., 716 vs. 134 V/m) for the maximum $|E_c|$, respectively (Table 1). Additionally, there was approximately a 20% reduction for average (i.e., 63 vs. 53 V/m) and maximum $|E_c|$ (i.e., 100 vs. 81 V/m) when comparing the empty coil vs. the coil loaded with the sample. Finally, when looking at the $E_{total}$, there was a destructive interference between $E_c$ and $E_i$ throughout the cylindrical sample or throughout the head, leading to an overall reduction of $E_{total}$ (i.e., 40 V/m in the empty coil vs. 20 V/m with the weak saline or 16 V/m in the Head).

Figure 3 and Table 2 show the normalized y-component of the $|E_c|$, $|E_i|$, and $|E_{total}|$ at 12.8 MHz along the central sagittal plane (YZ-plane) with the coil empty.
Table 1. Normalized electromagnetic field properties within the whole sample when loaded with conductive (third row), dielectric (fourth row), weak saline (fifth row), and human head model (sixth row) using a high pass (HP) birdcage coil at 12.8 MHz. All values were normalized so that $B_1^+ = 4 \mu T$ at the coil center.

| 12.8 MHz   | $|B_1^+|$ | std [10$^{-7}$] | $|E_c|$ | $|E_i|$ | $|E_{total}|$ | $|E_{load}|$ |
|------------|----------|----------------|-------|-------|------------|------------|
| Air        | 3.4      | 9.2            | 76    | 134   | 63         | 100        | 40         | 90         |
| Conductive ($\sigma = 0.2$, $\varepsilon_r = 1$) | 3.5      | 7.5            | 56    | 231   | 52         | 81         | 20         | 210        |
| Dielectric ($\sigma = 0$, $\varepsilon_r = 78$) | 3.5      | 7.5            | 56    | 221   | 51         | 81         | 21         | 199        |
| Weak Saline ($\sigma = 0.2$, $\varepsilon_r = 78$) | 3.5      | 7.3            | 54    | 224   | 50         | 78         | 20         | 203        |
| Head Model | 2.5      | 15.2           | 53    | 716   | 49         | 109        | 16         | 693        |

Table 2. Normalized magnitude of 2D (YZ-plane) rotating RF magnetic field ($B_1^+$) and y-component of conservative $E$-field ($|E_y|$, first column), magnetically induced $E$-field ($|E_y,i|$), and total $E$-field ($|E_y|$) between the coil and the sample in Figure 3. All values were normalized so that $B_1^+ = 4 \mu T$ at the coil center.

| 12.8 MHz   | $|B_1^+|$ | std [10$^{-7}$] | $|E_y|_c$ | $|E_y,i|$ | $|E_y|$ |
|------------|----------|----------------|----------|---------|-------|
| Air        | 4.5      | 11.7           | 49       | 149     | 21    | 39    | 44 | 144 |
| Conductive ($\sigma = 0.2$, $\varepsilon_r = 1$) | 4.3      | 9.4            | 76       | 165     | 17    | 32    | 73 | 163 |
| Dielectric ($\sigma = 0$, $\varepsilon_r = 78$) | 4.2      | 9.4            | 73       | 161     | 17    | 32    | 71 | 159 |
| Weak Saline ($\sigma = 0.2$, $\varepsilon_r = 78$) | 4.2      | 9.1            | 73       | 158     | 17    | 31    | 70 | 155 |

Figure 3. Calculated magnitude y-component of total $E$-field ($E_y$, first column), magnetically induced $E$-field ($E_y,i$, second column) and conservative $E$-field ($E_y,c$, third column) at 12.8 MHz when loaded with air (first row), conductive ($\sigma = 0.2$ S/m, $\varepsilon_r = 1$, second row), dielectric sample ($\sigma = 0$ S/m, $\varepsilon_r = 78$, third row) and human head model (fourth row). The z-directional size of a head image (fourth row) is longer than others to include neck and shoulder region. The weak saline images, similar to conductive or dielectric ones, are not shown in this figure.
loaded with the conductive, the dielectric phantom, and the head model. The change of electric field near the end-ring with and without the head model can be observed (red arrows in the fourth row). The electric field distribution for the conductive sample, the dielectric sample, and the weak saline (not shown) was very similar (see also Table 3).

Figure 4 and Table 3 show the normalized z-compo-
Table 3. Normalized magnitude of 2D (YZ-plane) rotating RF magnetic field \( \mathbf{B}^c \) and \( z \)-component of conservative \( E \)-field \( E_Z^c \), magnetically induced \( E \)-field \( E_Z \) and total \( E \)-field \( E \) within the sample in Figure 4. Other parameters are same as Table 2.

| 12.8 MHz | \( |B^c| \) | \( |E_Z^c| \) | \( |E_Z| \) | \( |E| \) |
|----------|-----------|-----------|-----------|-----------|
|          | Mean [μT] | std [10⁻³] | Max [V/m] | Mean [V/m] | Max [V/m] | Mean [V/m] | Max [V/m] |
| Air      | 3.3       | 9.0       | 29        | 67         | 29        | 67         | 4         | 31        |
| Conductive (\( \sigma = 0.2, \varepsilon = 1 \)) | 3.4       | 7.3       | 32        | 105        | 29        | 67         | 10        | 83        |
| Dielectric (\( \sigma = 0, \varepsilon = 78 \)) | 3.4       | 7.3       | 31        | 100        | 29        | 67         | 10        | 77        |
| Weak Saline (\( \sigma = 0.2, \varepsilon = 78 \)) | 3.4       | 7.1       | 31        | 105        | 29        | 67         | 10        | 83        |

The value of \( E_y \) at the center was zero (Figures 2-5), as expected given the specific electrical configuration of the coil and can be explained by means of the magnetic vector potential \( \mathbf{A} \), proportional to the current density \( \mathbf{J} \) (Equation (3)). Because opposite sides of a birdcage coil in ideal mode 1 resonance have equal \( \mathbf{J} \) flowing in the opposite direction and generating an opposing \( \mathbf{A} \), the two \( \mathbf{A} \) having same amplitude and opposite direction cancel each other out at the center. In these results, the value of electric field in the isocenter of the coil was very close to zero but not exactly zero (i.e., 0.04 V/m, less than 0.2% of average total electric field within the whole sample). The \( \mathbf{E}_c \) was significantly different when the coil was loaded with a conductive, a dielectric, a weak-saline sample, or a human head model, with approximately a 25% - 30% change in the average \( \mathbf{E}_c \) and up to 430% change in maximum \( \mathbf{E}_c \) within the whole sample (Table 1). This was due to the additional scalar potential \( \phi \) within and surrounding the sample, as shown in Figure 3. When a conductive, dielectric, or weak-saline sample is located within the electric field generated by the RF coil, charged particles within the sample are moved to the boundaries of the sample, resulting in a polarization field which either has same or opposite direction of the original field depending on the specific region considered and on the components of the coil. Because of such polarization effects, the \( z \)-component of \( \mathbf{E}_c \) \( (|E_{Zc}|) \) within the sample increased (Figure 4 and Table 3); however, because the additional scalar electric potential had opposite direction of the original one, the \( y \)-component of \( \mathbf{E}_c \) \( (|E_{Yc}|) \) decreased within the sample. Moreover, because \( |E_{Yc}| \) was the dominant component of the \( \mathbf{E}_c \), this resulted in an overall reduction in the \( \mathbf{E}_c \) and an increase of magnetic field homogeneity within the sample (Figure 3, Tables 1 and 2). These results are in line with published literature [8,9,11]. However, the \( |E_{Yc}| \) between a coil and...
Figure 5. Total magnitude of conservative $E$-field ($E_c$, first row), magnetically-induced $E$-field ($E_i$, second row), total $E$-field ($E_{total}$, third row) and rotating RF magnetic field ($B_1^\ast$, fourth row) after normalization when loaded with air (first column), conductive sample (second column), dielectric sample (third column) and human head model (fourth column). The z-directional size of a head image (fourth column) is longer than others to include neck and shoulder region. The electric field distribution for the conductive sample, the dielectric sample, and the weak saline (not shown) was almost the same.

Table 4. Results of the 3D electromagnetic simulations within the sample at the different frequencies evaluated in this study. Mean and standard deviation (std) of circularly polarized RF magnetic field ($\mid B_1^\ast \mid$) and total electric field ($\mid E \mid$) when loaded with air, weak saline and human head model at three different frequencies of 12.8, 64, and 128 MHz.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\mid B_1^\ast \mid$ Mean $\ [\mu T]$</th>
<th>std $\ [10^{-7}]$</th>
<th>$\mid E \mid$ Sample Mean $\ [V/m]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (12.8 MHz)</td>
<td>3.4</td>
<td>9.2</td>
<td>40</td>
</tr>
<tr>
<td>Weak Saline (12.8 MHz)</td>
<td>3.5</td>
<td>7.3</td>
<td>20</td>
</tr>
<tr>
<td>Head (12.8 MHz)</td>
<td>2.5</td>
<td>15.2</td>
<td>16</td>
</tr>
<tr>
<td>Air (64 MHz)</td>
<td>3.6</td>
<td>6.9</td>
<td>159</td>
</tr>
<tr>
<td>Weak Saline (64 MHz)</td>
<td>2.9</td>
<td>7.1</td>
<td>85</td>
</tr>
<tr>
<td>Head (64 MHz)</td>
<td>2.3</td>
<td>14.4</td>
<td>75</td>
</tr>
<tr>
<td>Air (128 MHz)</td>
<td>3.6</td>
<td>6.4</td>
<td>315</td>
</tr>
<tr>
<td>Weak Saline (128 MHz)</td>
<td>1.5</td>
<td>8.4</td>
<td>98</td>
</tr>
<tr>
<td>Head (128 MHz)</td>
<td>1.9</td>
<td>12.7</td>
<td>119</td>
</tr>
</tbody>
</table>

A sample was increased because the additional scalar electric potential—due to the presence of the sample—had the same direction of the original one. This result extends previous published literature showing that the total electric field would decrease within the sample when the coil is loaded with a conductive or a dielectric sample (as shown in (9,11)) (Table 1) and additionally demonstrating that the electric field increases between coil and the sample (Table 2). This result may have consequences on subject safety. For example, the accumulation of charge...
on the boundary region of the sample may increase local and 10 g-average SAR. Moreover, when external conductive leads for physiological monitoring (e.g., ECG, EEG) are present, the increase in electric field between coil and sample may result in increasing induced currents along the monitoring leads, with possible increase of local SAR at the interface between leads and patient skin. Because of difficulties of SAR and temperature calculation in free space, direct comparisons of losses and heating in the objects were not studied.

The electric field variation of the x-component with addition of the samples was not shown in the figures because of the small absolute amplitude (less than 15% compared to the y- and z-component for the sagittal view) and because no difference was noticed among all the loading samples considered. Additionally, the results with a weak saline sample were similar to the results obtained with a conductive or a dielectric sample (Tables 2 and 3).

The change for the magnetically induced E-field \( E_i \) with addition of the sample was much less when compared to \( E_c \) (i.e., about 13% - 18% change in the average \( \| E_c \| \) and up to 22% change in the maximum \( \| E_i \| \)) (Table 1 and Figures 3-5). The reason that the value of \( E_i \) within and surrounding the sample appears to be relatively independent of sample properties at 12.8 MHz can be explained using Faraday’s law. \( E_i \) is induced by a time varying vector magnetic potential (Equation (2)) which is mainly caused by the conduction current flowing in the RF coil. Because the RF coil used in this case is very small (300 mm in length and 290 mm ID) compared to the electrical wavelength (free space wavelength at 12.8 MHz equal to 23.4 m), the presence of the sample does not significantly affect the distribution of coil currents (no wavelength effect) and specifically \( E_i \) (Figure 5). However, as the frequency increased from 12.8 MHz to 128 MHz (free space wavelength equal to 2.34m), \( E_c \) and \( E_i \) both proportional to the frequency (Equation (2)) also increased and the \( E_{total} \) was much higher, (i.e., about 644% increase in average \( \| E_{total} \| \).
within the head model compared to 12.8 MHz) (Figure 6 and Table 4).

The calculated volume charge density in Figure 7 was highly concentrated on the boundary region of the sample and the head model, matching well with the increased conservative electric field between the coil and the loading.

A previous study [6] showed that in a solenoid coil the $E_c$-shield can be used effectively to reduce $E_{\text{total}}$ without changing $B_i$ because the direction of $E_c$-shield in the solenoid coil is orthogonal to the direction of current in the coil wires [6,8,9]. However, the results of this study show that the difference in current directions along the structure between a birdcage and a solenoid coil is significant enough to do not allow using the same $E_c$-shield approach with a birdcage coil.

### 5. Conclusion

This study presents the variations of electromagnetic field inside a birdcage coil when loaded with a conductive cylindrical sample, a dielectric cylindrical sample, a weak-saline cylindrical sample, or a head model. The results were presented using a designed $E_c$ and $E_i$ separation method at the frequency of 12.8 MHz. The additional scalar potential caused by the polarization effects within the load caused an increase of the $y$-component of $E_c$ between coil and sample, and a decrease within the sample at 12.8 MHz resulting in higher possibility of increased power deposition in the subject skin and induced RF currents in external leads used for physiological recording, i.e., ECG. The proposed $E_c$ and $E_i$ separation method can be applied as long as the current density in the RF coil is much greater than that in the sample and no significant wavelength effects are present for the accurate calculation of magnetic vector potential ($A$). As the frequency increased from 12.8 MHz to 128 MHz, the total $E$-field within and surrounding the sample increased significantly. Results indicate that the $E_c$-shield approach, previously proposed for a solenoid coil to reduce sample heating, cannot be used with a birdcage coil.

### REFERENCES


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