

# Parameter Estimation of Maneuvering Target Using Maximum Likelihood Estimation for MIMO Radar with Colocated Antennas

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## Abstract

The estimation of target parameters in MIMO radar signal processing is one of the most important research topics. An efficient implementation of the Maximum Likelihood estimator is presented in this paper to estimate the DOA (Direction of Arrival), initial velocity and acceleration of a maneuvering target in colocated MIMO radar. The target's DOA is estimated in the first place, then a Maximum-Likelihood (ML) estimation based on peak search is applied to a two-dimensional grids providing estimation of initial velocity and acceleration. Simulations show that the MIMO radar has a better performance in DOA estimation than the phased array radar. By means of Monte Carlo simulations, the estimation error of initial velocity and acceleration on different SNRs are calculated. The results also suggest the effectiveness of this method.

## Keywords

MIMO Radar, Parameter Estimation, Maximum Likelihood Estimation

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## 1. Introduction

The Multiple-Input Multiple-Output (MIMO) Radio Transmission Technology, has the potential to enhance system capacity and improve spectral efficiency without requiring extra emission signal bandwidth or transmission power [1] [2]. The overwhelming popularity of MIMO systems draws radar researchers' attention to engage MIMO techniques in radar systems implementation.

MIMO radar employs multiple elements at both transmitting and receiving antennas as the phased array radar does, on flip side, MIMO radar transmits orthogonal or partially correlated waveforms. According to the antenna configuration, MIMO radar systems can be classified into two categories: colocated (co-

herent) MIMO radars are those with closely spaced antennas [3]. While distributed MIMO radars, also known as non-coherent MIMO radars, whose antennas are placed far from each other [4]. The latter configuration exploits the random fluctuation of the targets' Radar Cross Section (RCS). This fluctuation causes spatial decorrelation between target echoes, which prevents coherent processing. It is known that spatial resolution can be improved by taking advantage of coherent processing introduced by coherent MIMO radar. Several methods have been proposed for DOA estimation offering significant estimate performance without considering Doppler effects [5] [6]. As shown in [7] and [8], parameters of moving target can be measured by the maximum likelihood estimator for distributed MIMO radar. There are many efforts have been made on joint DOA and Doppler frequency estimation [9] [10], while few studies have yet been conducted on maneuvering targets.

This paper considers parameters estimation of maneuvering target in collocated MIMO radar by developing a maximum-likelihood estimator. The Multiple Signal Classification (MUSIC) is applied in the first place to estimate DOA, followed by a two-dimensional search applied to estimate the velocity and acceleration. Performance of estimation for phased array radar and MIMO radar are compared under the same condition. The root mean square errors (RMSE) of the ML estimation of target parameters are obtained by Monte Carlo simulations. Numerical results demonstrate that MIMO radar has better angle estimation performance compared with conventional phased array radar and verify the effectiveness of this method as well.

## 2. MIMO Radar Signal Model

Let's consider a MIMO radar system in which there are  $M$  transmitting antennas and  $N$  receiving antennas, with inter-element spacing of  $d_t$  and  $d_r$ , respectively. Radar working wavelength is  $\lambda$ . The signal emits from the  $m$ -th antenna, is defined as  $s_m(l) = [s_m(1), s_m(2), \dots, s_m(L)]^T, m = 1, 2, \dots, M$ , where  $L$  is the length of coded pulse signal, with  $(\cdot)^T$  denoting the transpose of matrices. Assuming the different transmitting signals are orthogonal, then transmit steering vector of the array towards direction  $\theta$  can be denoted as

$$\mathbf{a}_T(\theta) = \left[ 1, e^{j2\pi \frac{d_t \cos \theta}{\lambda}}, \dots, e^{j2\pi(M-1) \frac{d_t \cos \theta}{\lambda}} \right]^T \tag{1}$$

and the receive steering vector is

$$\mathbf{a}_R(\theta) = \left[ 1, e^{j2\pi \frac{d_r \cos \theta}{\lambda}}, \dots, e^{j2\pi(N-1) \frac{d_r \cos \theta}{\lambda}} \right]^T \tag{2}$$

Assuming that the target has a reflection coefficient  $\beta(\theta)$ , moves with an initial radial velocity  $v$  and an acceleration  $a$ . The received signal at antenna  $n$  and time index  $l$  can be described by

$$y_n(l) = \beta(\theta) e^{j2\pi(n-1) \frac{d_r \cos \theta}{\lambda}} \cdot e^{j2\pi f_d l} \cdot \sum_{m=1}^M s_m(l) e^{j2\pi(m-1) \frac{d_t \cos \theta}{\lambda}} + w_n(l) \tag{3}$$

where  $f_d$  represents Doppler shift caused by the maneuvering target, and  $w_n(l)$  is the noise components approximated by complex white Gaussian samples at the  $n$ -th receiver. From (3) the received space-time matrix can be expressed as

$$y(l) = \beta(\theta) \cdot a_r(\theta) a_t^T(\theta) \cdot s(l) \cdot e^{j2\pi f_d l} + w(l) \tag{4}$$

The total  $L$  samples can be collected into a single matrix

$$Y = \beta(\theta) \cdot A(\theta) \cdot S \cdot D(f_d) + W \tag{5}$$

where  $A(\theta) = a_r(\theta) a_t^T(\theta)$ ,  $D(f_d) = \text{diag}(1, e^{j\omega_1}, \dots, e^{j(L-1)\omega_1})$  is an  $L \times L$  diagonal matrix,  $\omega_1 = 2\pi f_d / f_r = 2\pi \cdot 2v \cdot T_p / \lambda + \pi \cdot 2a \cdot T_p^2 / \lambda$ , where  $T_p$  is the pulse width, and  $W$  is an  $N \times L$  noise matrix with a variance of  $\sigma_w^2$ .

### 3. Maximum Likelihood Estimation

Let  $\psi = [\theta \ v \ a]^T$  be the vector denoting the unknown parameters that need to be estimated. We use data  $Y$  as stated in (5) and be vectorized, then the received data vector  $y$  can be obtained, which follows a Gaussian probability density function whose mean is  $\gamma(\psi) = \text{vec}(\beta(\theta)A(\theta) \cdot S \cdot D(f_d))$ . Since a maneuvering target is discussed, the Doppler frequency  $f_d$  involves two variables: initial velocity  $v$  and acceleration  $a$ .

The probability density function of  $y$  is

$$P(y|\theta, f_d) = \frac{1}{\pi^{NL} \sigma^{2NL}} e^{-\frac{1}{\sigma_w^2} (y - \gamma(\psi))^H (y - \gamma(\psi))} \tag{6}$$

where  $(\cdot)^H$  denotes the conjugate transpose. The corresponding log-Likelihood function for radar data can then be described as

$$P'(\psi) = C - \frac{1}{\sigma_w^2} (y^H y - y^H \gamma(\theta, f_d) - \gamma(\theta, f_d)^H y + \gamma(\theta, f_d)^H \gamma(\theta, f_d)) \tag{7}$$

where  $C$  is a constant which does not depend on  $\psi$ . To estimate the value of  $\theta$  and  $f_d$ , the cost-function to be maximized is given by

$$\begin{aligned} \hat{\psi} &= \arg \max_{\psi} \{P'(\psi)\} \\ &= \arg \min_{\psi} \{y^H y - y^H \gamma(\theta, f_d) - \gamma(\theta, f_d)^H y + \gamma(\theta, f_d)^H \gamma(\theta, f_d)\} \end{aligned} \tag{8}$$

The first term  $y^H y$  in (8) does not depend on  $\psi$ . The final term  $\gamma(\theta, f_d)^H \gamma(\theta, f_d)$  is constant with respect to  $f_d$  and  $\theta$ . By exploiting the relation  $\text{vec}(ASD) = (D^T \otimes A) \text{vec}(S)$ , where  $\otimes$  denotes Kronecker product, we have

$$\begin{aligned} \gamma(\theta, f_d)^H \gamma(\theta, f_d) &= \text{vec}(\beta(\theta)A(\theta)SD(f_d))^H \text{vec}(\beta(\theta)A(\theta)SD(f_d)) \\ &= \left[ (D^T(f_d) \otimes A(\theta)) \text{vec}(S) \right]^H \left[ (D^T(f_d) \otimes A(\theta)) \text{vec}(S) \right] \\ &= \text{vec}(S)^H (D^*(f_d) \otimes A^H(\theta)) (D^T(f_d) \otimes A(\theta)) \text{vec}(S) \\ &= \text{vec}(S)^H \left[ (D^*(f_d) D^T(f_d)) \otimes (A^H(\theta) A(\theta)) \right] \text{vec}(S) \\ &= \text{vec}(S)^H \left[ I_{L \times L} \otimes (A^H(\theta) A(\theta)) \right] \text{vec}(S) \\ &= (N^2 \times L) \cdot I_{L \times L} \end{aligned} \tag{9}$$

It can be easily shown that the product in (9) yields a diagonal matrix with diagonal elements maintain a constant which equals to  $N^2L$ . Therefore the cost-function in (8) can be simplified as

$$\hat{\psi} = \arg \max_{\psi} \left\{ \mathbf{y}^H \boldsymbol{\gamma}(\theta, f_d) + \boldsymbol{\gamma}(\theta, f_d)^H \mathbf{y} \right\} \quad (10)$$

In order to find the maximum likelihood estimations of the unknown parameters, it is necessary to perform a three-dimensional (3-D) search in the parameter space. From a practical point of view, such a 3-D search, for all possible values of DOA, velocity and acceleration of a target, can cause tremendous computational burden. Therefore, we propose a two-stage sequential estimation process.

As we simplify the target moving scenario by assuming the target is moving along a certain direction, a classical sub-space method, multiple signal classification, is used to estimate the DOA of the target which can be treated as static or relatively static. Then the estimation of initial velocity and acceleration can be obtained when the cost function in (10) achieves its maximum.

#### 4. Simulations

We consider a MIMO radar with 6 antennas both on transmitting and receiving ends. Antennas are uniformly spaced with an distance of  $\lambda/2$ , where  $\lambda = 0.23$  m. Each transmitter transmits different Hadamard codes with the same carrier frequency. The code length is set to  $L=64$  and code duration is  $T_p = 5 \times 10^{-4}$  s. The azimuth of the target is  $\theta = 45^\circ$ , the initial velocity of the maneuvering target is  $v = 24.01$  m/s and acceleration is  $a = 99.9$  m/s<sup>2</sup>. Initial velocity searching range is from 0m/s to 40m/s, acceleration searching range is from 70m/s<sup>2</sup> to 130m/s<sup>2</sup>, the step for searching of initial velocity and acceleration are set to 0.25 m/s and 0.05m/s<sup>2</sup>, respectively.

Monte Carlo simulations with 200 trials per signal-to-noise ratio (SNR) were conducted to acquire the RMSE of the ML estimation. The RMSE of DOA is shown in **Figure 1**, which indicates that MIMO radar has better angular estimation performance than phased array radar. The RMSE of initial velocity is shown in **Figure 2**, and that of acceleration is shown in **Figure 3**. Under various SNRs, MIMO radar and phased array radar both can obtain excellent estimation performance in estimating initial velocity and acceleration of maneuvering targets.

#### 5. Conclusions

This paper studies the performance of parameters estimation of maneuvering target in coherent MIMO radars with the application of maximum likelihood estimation. Numerical simulation results show that MIMO radar has a better angular resolution than conventional phased array radar. The effectiveness of maximum likelihood estimator is also verified through the simulations.

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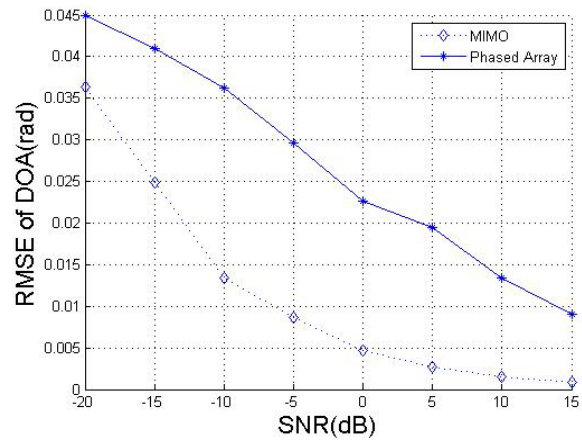


Figure 1. RMSE of DOA.

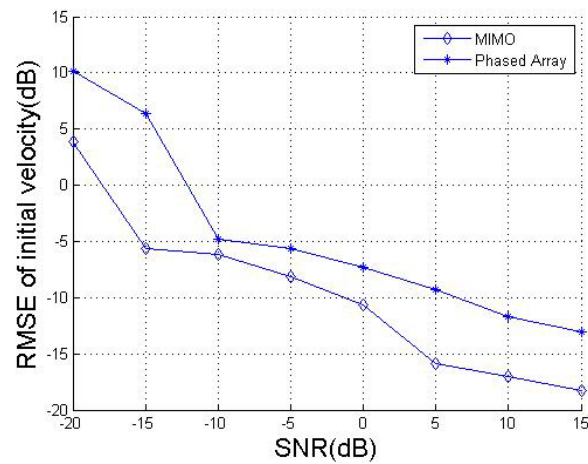


Figure 2. RMSE of initial velocity.

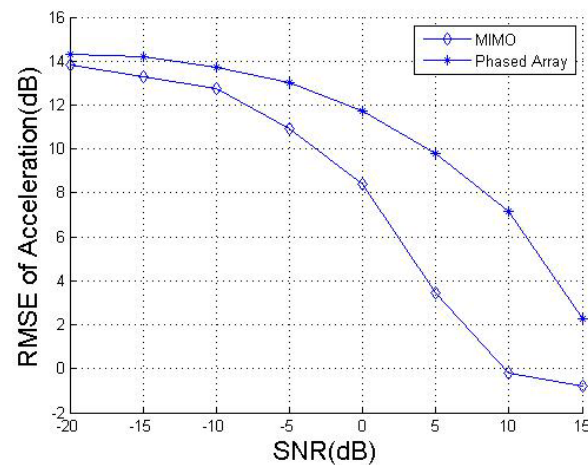


Figure 3. RMSE of acceleration.

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