Scientific Research Publishing

A Proposed Orthogonal Chirp Division Multiplexing (OCDM) Multicarrier Transceiver Based on the Discrete Fractional Cosine Transform

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Abstract

Doppler spread is a phenomenon caused by rapid changes in the channel response due to the movement of the transmitter/receiver through a multipath environment, which is also known as a doubly selective channel response. In doubly selective fading channel where the channel frequency response is regarded as rapidly time-varying, the inter carrier interference (ICI) occurs mainly because the traditional multicarrier transceivers are incapable to diagonalize the channel matrix. The proposed scenario is for an Orthogonal Chirp Division Multiplexing (OCDM) multicarrier system derived from Discrete Fractional Cosine Transform (DFrCT). The proposed DFrCT-OCDM multicarrier system is proofed to be less signal processing complexity and easier to implement, compared to the traditional Fast Fourier Transform (FFT). Simulation results reveal that the DFrCT provides superior performance reflected the significant improvement in the BitError Rate which reaches to 10 dB, better spectrum efficiency and better energy concentration properties than the conventional FFT. Finally, several equlizers are applied and they all show that the proposed DFrCT-OCDM over performs the other systems with all these equlizers.

1. Introduction

The origin of Orthogonal Frequency Division Multiplexing (OFDM) development was introduced as early as 1950's with the introduction of Frequency Divi-

sion Multiplexing (FDM) for data communications in early telephone lines. FDM divides the channel bandwidth into sub-channels and transmits multiple relatively low rate signals by carrying each signal on a separate carrier frequency. Guard-bands are left between the different sub-channels to ensure elimination of the Inter-Symbol-Interference (ISI) and separation between sub-channels, which led to bandwidth inefficiency [1] [2] [3] [4].

Doppler spread is a phenomenon caused by rapid changes in the channel response due to movement of the transmitter/receiver through a multipath environment which is also known as a doubly selective channel response. The amount of Doppler spread is proportional to the transmission frequency and the velocity of movement. The closer the sub-carriers are spaced together, the more OFDM signal is effected by the Doppler spread, thus a trade-off exists between the amount of multipath protection (length of the guard period) which is bandwidth inefficient and the Doppler spread tolerance, which is needed to reduce the Doppler spread effect, *i.e.* when increasing the mobile communication speed; the Doppler spread frequency increases resulting to the need for a longer guard period between the sub-carriers, which is regarded as bandwidth waste [4]-[10].

Moreover, the channels capacity and performance can be limited due to the multipath propagation problems, resulting to frequency selective fading response [8].

To fulfill the desires for high speed wireless communication at such high data rates transmission over delay-dispersive channels; the OFDM is widely implemented over delay-dispersive channels, which is a Multi-Carrier Modulation (MCM) scheme [10] [11] [12].

If only a single frequency band (single carrier) is used, then the symbol duration (T_S) has to be very small in order to achieve the required data rate, when symbol duration T_S is not larger than the channel delay spread (τ), ISI occurs and the communication systems will not be able to achieve good system performance [7]. Moreover, a very small symbol duration has a very long bandwidth, thus requires a complicated equaliser (mathematically expensive) with a large number of taps at the receiver and increases the computational effort [1] [2].

OFDM converts the high data rate stream into a number of low-data-rate streams (sub-channels) N and modulates each one of the low-data-rate streams over one of the sub-channels. These are then transmitted at the same time keeping the sum of the desired data rate constant, with the property that each of the sub-channels has $T_S \gg \tau$ and hence effectively ISI-free channels are obtained and the multipath fading channel can be seen as flat fading channel for each subcarrier (sub-channels experience relatively flat fading) as shown in Figure 1.

The sub-channels are orthogonal under ideal propagation conditions. Moreover ISI can be completely eliminated through the use of a cyclic prefix [13].

Traditional OFDM multicarrier technique is used to diagonalize the frequency domain for the frequency-selective channel matrix. Moreover, the single tap equalizer is implemented to compensate the distortion that affects each independent sub-channel through simple gain and phase adjustment [1] [2].

Digital Fourier Transform (DFT) is performed to enable implementing OFDM

and even makes it over performs the well-implemented single carrier systems. However, in doubly selective rapidly fading channels, such as the channels for the modern wireless systems, like Digital Video Broadcasting Handheld (DVB-H) and the high speed (beyond 100 km/h) mobile broadband services providers (microwave access WiMAX); it is regarded as less effective [3] [4] [5] [6] [7].

The applications mentioned above require to use OFDM in doubly dispersive channel (time-frequency fading channels) environments resulting to the lost in the orthogonallity of the sub-carrier, which causes the inter-carrier interference (ICI) [7] [8] [9] [10] [11].

To mitigate the change in the channel behaviour at high speed, *i.e.* the rapidly time-varying frequency response; we propose to make the sub-channel carrier frequencies to be time-varying as well, in order to cope with the distortion caused by the carriers channels. Applying Discrete Fractional Cosine Transform (DFrFT) instead of Fast Fourier Transformer (FFT) in multicarrier systems is well investigated in [11] [12] [14] and the references their-in, mainly to minimize the negative impact of Doppler frequency spreads. unlike the work performed in this paper, where Orthogonal Chirp Division Muliplex (OCDM) is implemented and fully investigated, hence the consequential DFrFT-OCDM system consists of orthogonal chirp signals of carriers, in the form of a block transfer system.

The chirp signal ideally is regarded as a frequency modulated signal which sweeps over a wide spectrum without any changing in its amplitude as shown in Figure 2.

The chirp signal uses a wideband spectrum for transmission and its main advantage that its resistant to the negative channel effects, such as the noise, multipath fading and Doppler effects [15].

In this paper we present a novel OCDM system that employs the Discrete Fractional Cosine Transform (DFrCT) basis as multicarrier system subcarriers. Unlike [12]-[17], where DFrCT based on the eigen-decomposition of DCT kernel was proposed. Moreover, the proposed scenario in this paper for the transformation is discrete and strictly invertible with less computation complexity, better spectral efficiency and energy concentration than the DFrFT, with maintaining the principle of using the same chirp like (time-frequency) signal properties. Furthermore, as will be shown in this paper, the complexity of the DFrCT-
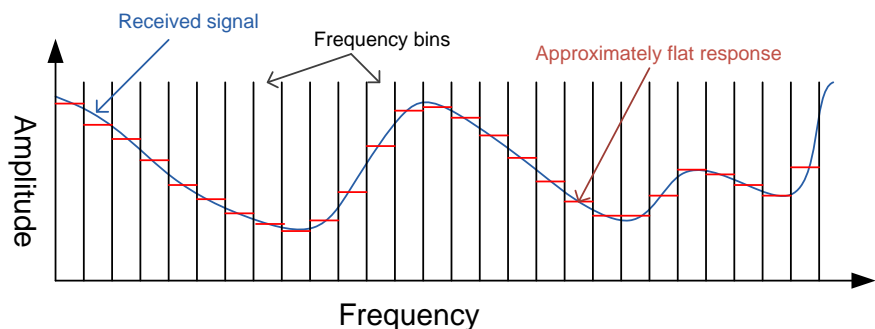

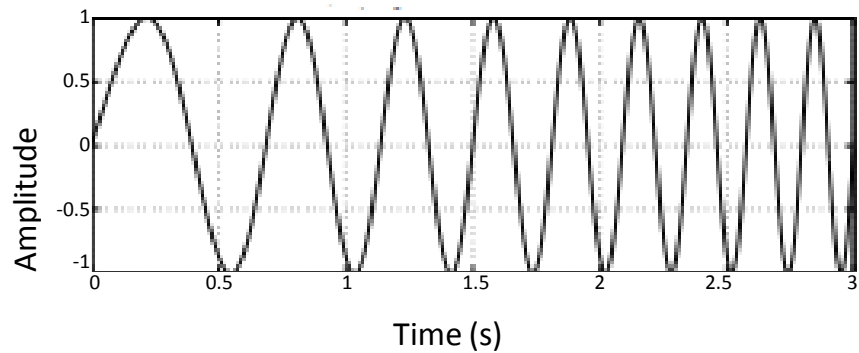**Figure 1.** Received OFDM signal in the frequency domain.

Figure 2. Illustration of chirps: A chirp signal from 1 to 3 Hz.

OCDM system is actually lower than that of conventional OFDM systems employing the DFT.

In [18], Peak-to-Average power ratio performance with BER analysis for Orthogonal Chirp Division Multiplexing Multicarrier Systems Based on Discrete Fractional Cosine Transform, and it showed how it is possible to improve the performance when using DFCT.

The rest of the paper is organized as follows: In Section 2, the Discrete Fractional Cosine Transform is introduced. In Section 3, the system model is presented and a description of the key realization of the transceiver is provided. In Section 4 the results of comparing the proposed system (DFrCT-OCDM) and the conventional OFDM and FrFT-OCDM systems are shown. Finally, Section 5 concludes the paper.

2. The Fractional Cosine and Sine Transform

The FrFTis defined in [16] as an angle α rotation on the time frequency distribution, accordingly, for $\alpha = 0$, there will be no change after applying DFrFT, and for $\alpha = \pi/2$, DFrFT becomes a Fourier transform, which rotates the time frequency distribution with $\pi/2$. For other values of α, the DFrFT rotates the time frequency distribution according to α [19].

Figure 3 illustrates the relationships between the time, and frequency outputs from the DFrFT $\left(F_{\alpha} \right)$ when the fractional angle α changes as:

$$
\boldsymbol{F}_{\alpha} = \begin{cases} \boldsymbol{F}_0 \left[x(t) \right] = x(t) & \text{for} \quad \alpha = 0 \xrightarrow{\text{yields}} \text{Timedomain} \\ \boldsymbol{F}_{\pi/2} \left[x(t) \right] = X(f) & \text{for} \quad \alpha = \pi/2 \xrightarrow{\text{yields}} \text{Frequencydomain} \\ \boldsymbol{F}_{\pi} \left[x(t) \right] = x(-t) & \text{for} \quad \alpha = \pi \xrightarrow{\text{yields}} \text{Inversetimedomain} \\ \boldsymbol{F}_{3\pi/2} \left[x(t) \right] = X(-f) & \text{for} \quad \alpha = 3\pi/2 \xrightarrow{\text{yields}} \text{Inversefrequencydomain} \end{cases}
$$

It can also be seen from **Figure 3** as well that $\boldsymbol{F}_{2\pi} \left[x(t) \right] = \boldsymbol{F}_0 \left[x(t) \right] = x(t)$ [19].

Based on **Figure 3**; the FrFT is considered as a fractional power of the Fourier transform operator, where in the frequency-time plane; $f(t)$ is the original signal in the time domain and $F_{\alpha}(u)$ is equivalent to α^{th} order in the fractional domain.
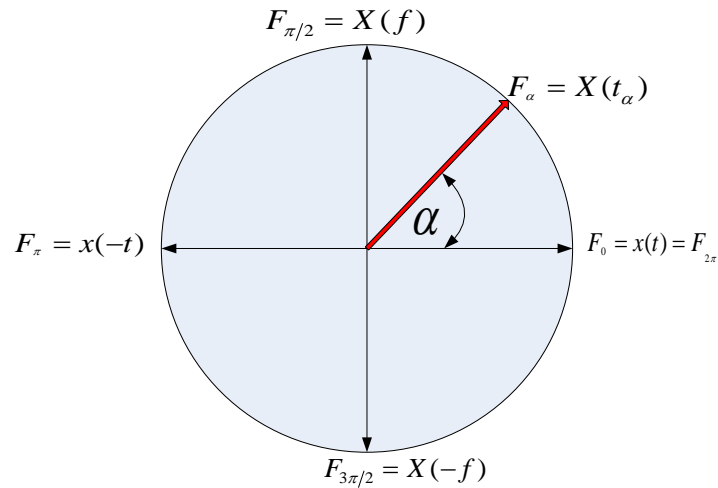
Figure 3. Signal $x(t)$ at different transformation angle αin the frequency-time plane.

Discrete Cosine Transformer (DCT) and Discrete Sine Transformer (DST) can be considered as a generalization of the cosine and sine transform respectively, where the αth order fractional cosine transform (FrCT) is $F_\alpha^c(u)$ and the αth order fractional sine transform (FrST) is $F_\alpha^c(u)$ shown in Equations (1) and (2) from [19] [20] [21]:

$$F_\alpha^c(u) = A_\alpha e^{j(u^2/2)\cot\cot\alpha} \int_{-\infty}^{\infty} \cos(\csc\csc\alpha.ut) e^{j(t^2/2)\cot\cot\alpha} f(t)\mathrm{d}t \qquad (1)$$

$$F_\alpha^s(u) = A_\alpha e^{j(\alpha-\pi/2)} e^{j(u^2/2)\cot\alpha} \int_{-\infty}^{\infty} \sin(\csc\alpha.ut) e^{j(t^2/2)\cot\alpha} f(t)\mathrm{d}t \qquad (2)$$
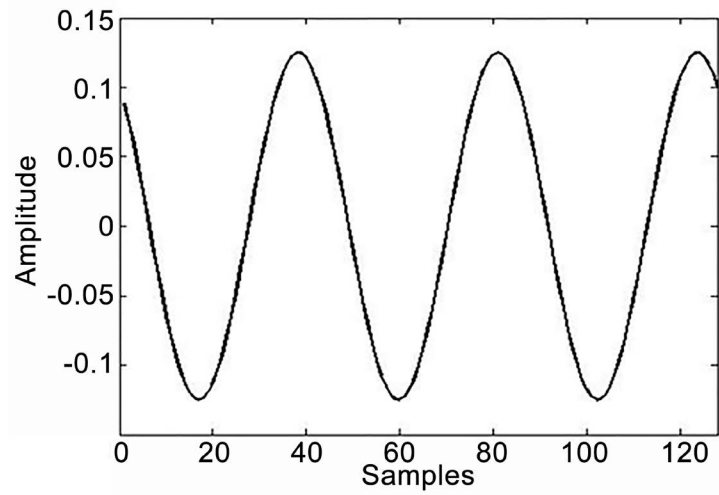
where $\alpha = -\dfrac{\pi}{2}$ to $\dfrac{\pi}{2}$, is the angle between the fractional order axis u and time axis t, and $A_\alpha = \sqrt{\dfrac{1-j\cot\alpha}{2\pi}}$.

Various definitions of the DFrCT differ in accuracy and complexity and may be derived using extensions from the FrFT by sampling the kernel real/ imaginary parts of FrFT or directly from the DCT itself [14]. The DFrCT definition in [19] is used in our work as it has minimal complexity and has a simple inverse transform. In the following we will denote the DFrCT as $F_\alpha = F_\alpha^c$ in [19] and the DFrCT uses the DFT Hermite eigenvector decomposition and the DCT transform kernel.
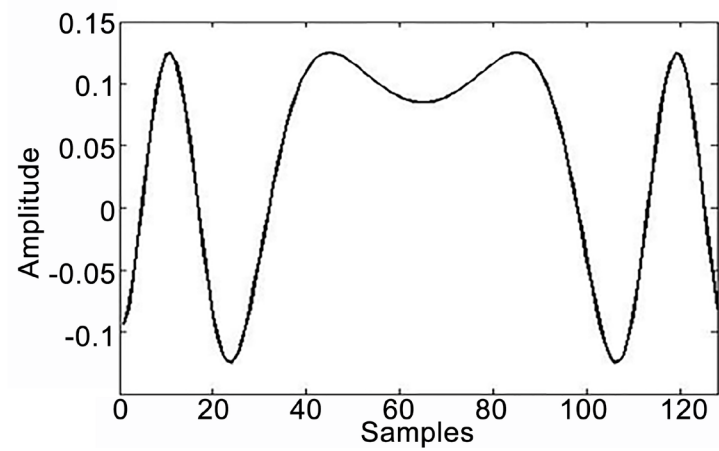
The vector notation for the DFrCT is given by Equation (3):

$$X = \begin{bmatrix} X_\alpha(0) \\ X_\alpha(1) \\ \vdots \\ X_\alpha(N-1) \end{bmatrix} = F_\alpha \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix} = F_\alpha \cdot x \qquad (3)$$
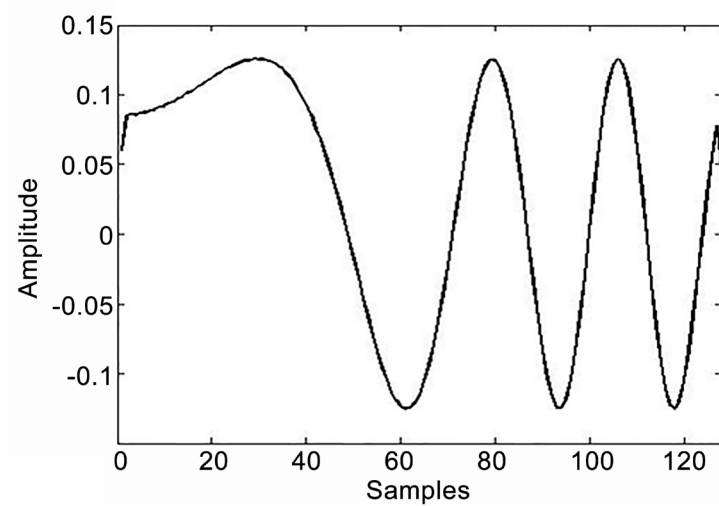
where F_α is the unitary $N \times N$ DFrCTmatrix, N is the number of samples and α is the transform rotational angle represented in the time-frequency plane shown in **Figure 4**. Note that when $\alpha = \pi/2$, the DFrCT will become the

(a) DFT-OFDM Subcarriers

(b) DFrFT-OCDM Subcarriers

(c) DFrCT-OCDM Subcarriers

Figure 4. DFrCT, DFrFT-OCDM with $\alpha = 0.95\frac{\pi}{2}$ and DFT-OFDM Subcarrier 2 (a) DFT-OFDM; (b) DFrFT-OCDM and (c) DFrCT-OCDM.

conventional DCT and when $\alpha = 0$, F_α is an identity matrix [19]. Similarly the inverse DFrCT (IDFrCT) is given in Equation (4):

$$x = F_{-\alpha} \cdot X \tag{4}$$

where $F_{-\alpha} = F_\alpha^H$ and $(.)^H$ denotes the complex conjugate transpose operation.

DFrCT Complexity

Implementing the FrFT for a given signal requires one DFT and two chirp multiplications, since an efficient DFT requires approximately $(P/3) \cdot \log_2 P$ complex multiplications (using the split-radix algorithm) where P is the total number of sampling points. Therefore a total of approximately $2P + (P/3) \cdot \log_2 P$ Complex Multiplications (CMult) are required to implement the FrFT [14].

The complex number multiplication CMult requires a minimum of three folders more than the com required in the real number multiplications, resulting to Equation (5) which gives the amount of real number multiplications required for the FrFT:

$$FrFT_{C_{\text{Mult}}} = 6P + P \cdot \log_2 P \tag{5}$$

where $FrFT_{C_{\text{Mult}}}$ is the FrFT complex multiplication number.

Using similar arguments, when computing the DFrCT from the first-type DCT kernel [14], the required complex multiplication number for DFrCTis given in Equation (6):

$$DFrCT_{C_{\text{Mult}}} = 2P + (P/2) \cdot \log_2 (P/2) \tag{6}$$

where $DFrCT_{C_{\text{Mult}}}$ is the DFrCT complex multiplication number.

Based on above and by comparing Equation (5) with Equation (6); it is obvious that when implementing FrCT; the system complexity is approximately decreased to the half when compared with the complexity of the FrFT.

3. The DFrCT Based OCDM System Model

3.1. The DFrCT-OCDM Bases

A comparison between the DFrCT, DFrFT-OCDM and OFDM real part of Subcarrier 2 are shown in Figure 4 where $\alpha = 0.95\frac{\pi}{2}$ has been used. It is well known that the OFDM subcarriers are constant frequency signals. the DFrCT-OCDM subcarrier are chirp like signal as shown in Figure 4 with twice the frequency variations in the DFrFT-OCDM chirp signal which give more frequency diversity and thus potentially more robustness against doubly dispersive channels.

3.2. The DFrCT-OCDM System

A conventional OFDM system applies an inverse discrete Fourier transform (IDFT) matrix to a data vector d_n and introduces a cyclic prefix (CP) prior to multiplexing to eliminate the ISI. The transmitted OFDM symbol is passed through a dispersive channel $h[m,v]$ corrupted by additive white Gaussian noise $v[n]$. At the receiver, after demultiplexing the received signal and re-

moving the CP, a DFT matrix reconstructs the transmitted data vector $\hat{d}_n$.

In a DFrCT based OCDM system, the DFT matrix is replaced by a DFrCT matrix F_α as shown in **Figure 5** below:

The subcarriers for the OCDM system are modulated by the IDFrCT where the transmitted data vector $d_n = \left[d_0 d_1 \cdots d_{N_a-1} \right]^{\mathrm{T}}$ and we additionally introduce a binary matrix $P \in \mathbb{Z}^{N \times N}$ which assigns a data vector $d_n \in C^{N_a}$ to N subcarriers, of which only N_a are active according to Equation (7)

$$P = \left[0_{N_a \times (N-N_a)/2} \, I_{N_a} \, 0_{N_a \times (N-N_a)/2} \right] \tag{7}$$

where $0_{L \times M}$ is an $L \times M$ matrix with zero entries, and I_{N_a} is an $N_a \times N_a$ identity matrix. The vector $s = \left[s_\alpha(0), s_\alpha(1) \cdots s_\alpha(N-1) \right]^{\mathrm{T}}$ is calculated from Equation (4):

$$s_n = F_{-\alpha} \cdot P \cdot d_n \tag{8}$$

At the receiver after PC removal in **Figure 5**, the multiplexed signal $r_n = H_n s_n + v_n$ is characterised by a system matrix with elements

$$[H_n]_{i,j} = \begin{cases} h[n-L+i, i-j] & i \geq j \\ h[n-L+i, L+i-j-1] & i < j \end{cases} \tag{9}$$

In stationary channel conditions, H_n is circulant, and can be decoupled by the DFT matrix. Received subcarriers are demodulated using the DFrCT

$$y_\alpha = F_\alpha r = F_\alpha H s \tag{10}$$

and the equalizer matrix $W_n \in C^{N_a \times N_a}$ operates on the input:

$$\hat{r}_n = P^H F_\alpha H_n F_{-\alpha} \cdot P \cdot d_n + P^H F_\alpha v \tag{11}$$

with a system matrix $C_{n,\alpha} \in C^{N_a \times N_a}$. The binary matrix P is purposed to operate as frequency guard bands, and hence to eliminate the components that could appear in the upper right and lower left corners $C_{n,\alpha}$ [22].

The equivalent $N_a \times N_a$ channel matrix in the fractional Cosine domain is given by $\tilde{H}_\alpha = F_\alpha H F_{-\alpha}$ and $\tilde{v}_\alpha = F_\alpha v_\alpha$ is the noise vector in the fractional domain. $\tilde{H}_\alpha$ is a nondiagonal subcarrier channel matrix that introduces ICI, which is the case when the dispersive channel comprises a multipath doubly selective channel that destroys the orthogonality between the subcarriers and introduce power leakage between them. This will make the symbol estimation task particularly complicated requiring a complex equalizer.

3.3. Equalization in the Fractional Cosine Domain

Linear MMSE and zero-forcing (ZF) estimates [22] may be expressed as follows in Equation (12):

$$\hat{d}_{\mathrm{MMSE}} = \tilde{H}_\alpha^H \left(\tilde{H}_\alpha \tilde{H}_\alpha^H + \gamma^{-1} I_{N_a} \right)^{-1} y_\alpha \tag{12}$$

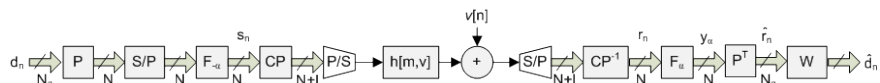$$\hat{d}_{\mathrm{MMSE}} = \tilde{H}_\alpha^+ y_\alpha \tag{13}$$

Figure 5. The DFrCT-OCDM system.

where $\hat{d}_{\text{MMSE}}$ is estimated data after MMSE equalization, I_{N_A} is identity matrix with $N_a \times N_a$ elements, γ is the signal-to-noise ratio (SNR) and $\tilde{H}_\alpha^+$ is the Moore-Penrose pseudo-inverse of the channel matrix in the fractional cosine domain [22]. In Equations (12) and (13) perfect knowledge of the channel matrix H is assumed and the equalizer does not use guard subcarriers. Furthermore it is assumed that $E\{d^{(i)}\} = E\{n^{(i)}\} = 0$,

$$E\{d^{(i)}d^{(i)H}\} = I, \quad E\{d^{(i)}n^{(i)H}\} = 0, \quad E\{n^{(i)}n^{(i)H}\} = \sigma^2 I \qquad (14)$$

ZF equalizer performance is poor due to noise enhancement while the MMSE equalizer is particularly complex due to channel matrix inversion which requires $\mathcal{O}(N_v^3)$ flops. This is not practical for high values of N_A. Low complexity equalizers were considered in [23] [24] [25] [26] [27], however, this is beyond this paper work though it is among future work plan.

3.4. The Selection of Optimal Order α

We will now investigate the effect of the fractional order α on the DFrCT-OCDM multicarrier system performance. To improve the multicarrier system performance, α should be chosen to ensure the best match between the sub-channel carrier frequencies variation and the fast time-frequency distortion of the channel.

Selecting α depends on the number of subcarriers N, time sample interval Ts, the Doppler shift f_D, the number of resolvable channel paths and the channel power delay profile. Offline optimization of α for DFrFT-OCDM was proposed in [11] using calculations of the channel statistical expectations. The same method can be used with the DFrCT-OCDM by extracting the channel properties at the receiver and define the optimum α which give the lowest ICI then feedback the α value to the transmitter.

In this paper, we believe that it is extremely important to determine the optimum fractional angle α, however, there is no need to repeat the work in [11]. It is important to mention that the used angle fraction is selected to be optimum with the parameters used in this paper based on [11], which means, we believe that there is no need to investigate this point any further.

4. Performance Analysis

The uncoded bit error rate (BER) performance of the OFDM, DFrFT-OCDM and the proposed DFrCT-OCDM with the same block MMSE equalizer are investigated by means of simulation. The following system parameters are used: N = 128, NA = 96, L = 8, $\alpha = 0.5\pi/2$ (from offline optimization search) and QPSK modulation is assumed. Rayleigh fading channels with exponential power delay profile, and Jakes' Doppler spectrum. The root-mean-square delay spread of the channel, normalized to the sampling period T_S, is $\sigma = 3$ with maximum Doppler frequency $f_D = 0.15\Delta f$. The carrier frequency is $f_C = 10$ GHz and the subcarrier spacing is $\Delta f = 20$ kHz. This Doppler frequency corresponds to a high mobile speed $V = 324$ Km/h.

Figure 6 shows the performance of an uncoded BER averaged over 10,000 multicarrier blocks, from which it can be seen that there is an improvement in the performance of the DFrCT-OCDM system compared to both the OFDM and FrFT-OCDM systems with complexity less than the FrFT-OCDM system. From Figure 6, it is observed that the proposed DFrCT at 10-3 Bit Error Rate provides approximately 10 dB better performance than the OFDM and about 1 dB improvements over the FrFT-OCDM system and at 10-4 Bit Error Rate provides approximately 3 dB improvements over the FrFT-OCDM system.

Same results are obtained when using the low comlexity LDL^{H} factorization equalizer.

DFrCT-OCDM System Performance Using the Low complexity LDL^{H} factorization equaliser:

The low complexity LDL^{H} factorization equaliser is used to shown the good performance it provides with the DFrFT-OCDM system, The LDL^{H} factorization equaliser can be used with the DFrCT-OCDM system using the same proposed technique. The uncoded BER performance of the DFrCT-OCDM compared to the OFDM system is investigated by means of simulation over 100000 multicarrier blocks. An FrCT-OCDM system with N = 128, NA = 96, L = 8, and QPSK modulation is assumed. The channel simulation parameters are the same as the Rayleigh fading channel used in the clock MMSE equaliser in Figure 6.

Figure 7 shows that the DFrCT-OCDM system performance is superior to the conventional OFDM system with the same low complexity equaliser. In [28] Cancellation Schemes for BER Performance Improvement of OFDM System is shown, and the itsresults can be compared with these results to show the improvement achieved by DFrCT-OCDM.

DFrCT-OCDM System Performance Using the Low complexity LSMR Equaliser:

The low complexity LSMR equalizer shows almost the same performance as the block MMSE equalizerequaliser in Figure 6 and the LDL^{H} factorization
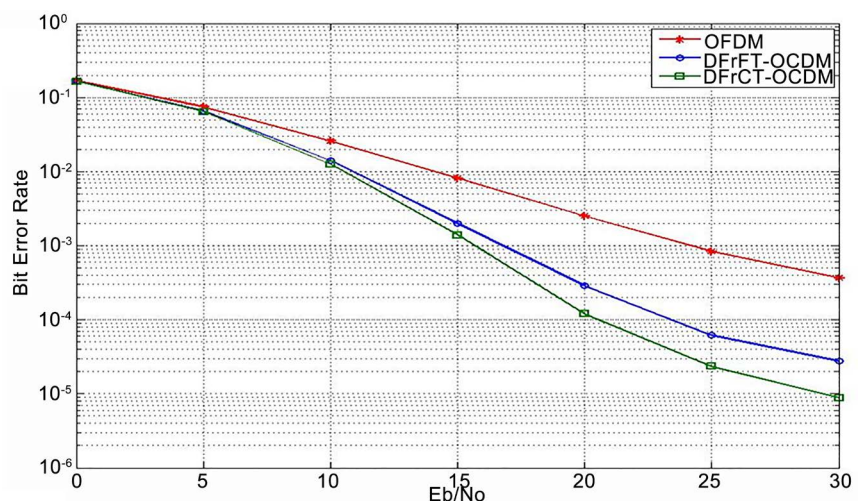

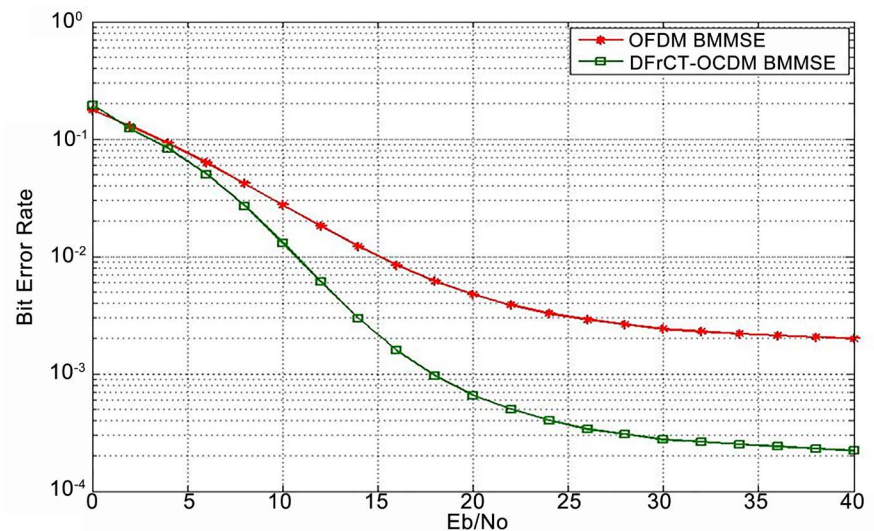Figure 6. The BER Comparison between DFrCT, DFrFT-OCDM and OFDM using block MMSE equalizer.

Figure 7. The BER Comparison between DFrCT-OCDM and OFDM using the LDL^H factorization equalizer.
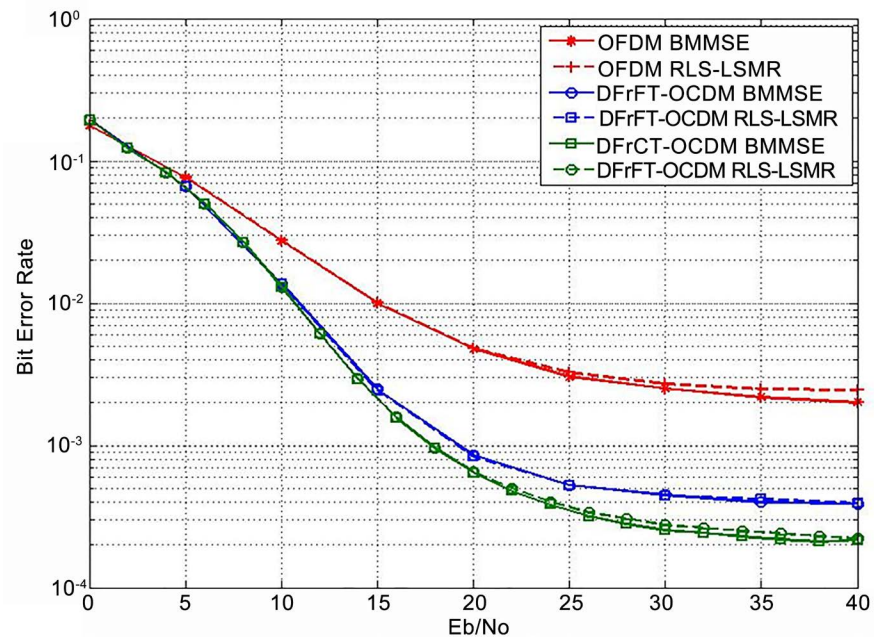

Figure 8. The BER Comparison between DFrCT-OCDM and OFDM using the RLS-LSMR equalizer.

equaliserin **Figure 7**, but with lower complexity. LSMR equaliser can be used with the DFrCT-OCDM following the same technique that was followed with the DFrFT-OCDM.

Figure 8 indicates that the DFrCT-OCDM system performance is better than both the conventional OFDM system and the DFrFT-OCDM system with the same low complexity equaliser.

The different equaliser techniques that are based on the LSMR method can be used all with the DFrCT-OCDM system and it is predicted that the DFrCT-OCDM system will be better than the DFrFT-OCDM system and the OFDM

systems.

5. Conclusions

A novel multicarrier transceiver based on the Discrete Fractional Cosine Transform (DFrCT) was proposed. The DFrCT is an optimal modulation/demodulation basis when the multicarrier system faces doubly selective fading channels. It was shown that the DFrCT-OCDM multicarrier system BER performance is superior to the OFDM and the FrFT-OCDM systems with less computational complexity which make the DFrCT a viable multicarrier transceiver in some practical applications that need low power consumption that can be effective in high speed requirement conditions such as DVB-H and WiMAX.

Our ongoing research includes using of low complexity equalizers, testing the peak to average power ratio, time-frequency synchronization and the anti-jamming properties of the new system.

The future work will be directed to investigate the most proper equalizer for the proposed system; moreover, it is important to investigate in more details the effect changing the guard bank with the operation frequency as the basic parameter to mitigate the effect of the Doppler frequency.

Moreover, as total new direction; it is important to go for the downlink communication to investigate this system more with high performance encoders and the ability to apply the new techniques over this system such as Network Coding as it is a band width efficient technique and could help to solve the inefficiency of the bandwidth when increasing the guard band to decrease ICI as a result of the Doppler frequency.

Acknowledgements

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