Handling Observations with Low Interrater Agreement Values

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ABSTRACT

Considerable research has been conducted on how interrater agreement (IRA) should be established before data can be aggregated from the individual rater level to the organization level in survey research. However, little is known about how researchers should treat the observations with low IRA values fail to meet the suggested standard. We seek to answer this question by investigating the impact of two factors (the relationship strength and the overall IRA level of a sample) on the IRA decision. Using both real data from a service industry and simulated data, we find that both factors affect whether a researcher should include or exclude observations with low IRA values. Based on the results, practical guidelines on when to use the entire sample are offered.

Keywords: Interrater Agreement, Aggregation, Survey Research

1. Introduction

In survey research, using a single respondent to represent an organization is vulnerable to single-rater bias [1,2]; therefore, multiple raters are recommended. When multiple raters are used, a certain degree of agreement must be established before the responses can be aggregated to the organization level. Interrater agreement (IRA) refers to the absolute consensus in scores assigned by multiple raters to one or more targets. Estimates of IRA are used to address whether scores furnished by different raters are interchangeable or equivalent [3,4].

Previous research has suggested various measures of IRA. The most widely used one is the $r_{WG}$ index proposed by James et al. [4,5]. Values of 0.7 have been used as the traditional cutoff value denoting high versus low IRA using the $r_{WG}$ index [6,7]. Lebreton et al. [6] interpret $r_{WG}$ as the proportional reduction in error variance. A value of 0.7 suggests a 70% reduction in error variance and that just 30% of the observed variance among judges’ ratings should be credited to random responding.

Since $r_{WG}$ was first introduced, researchers have addressed various related issues [3,8-12], but little attention has been paid to dealing with the observations that have low $r_{WG}$ values, with only one noticeable exception. Lebreton and Senter [3] suggest that researchers examine the magnitude and pattern of $r_{WG}$ values and state that if only 5% of values are below the cutoff value, it is probably justified to use the entire sample of rated targets. Their statement, however, is rather vague, and no theoretical or empirical justification is provided for the suggested guideline of 5%. In addition, they fail to address what researchers should do if more than 5% of the $r_{WG}$ values are below the cutoff value. Hence, it is still largely an unanswered question in the literature.

Our aim is to address this issue with the current study. Specifically, we investigate when one can use the average value of $r_{WG}$, which is referred to as $R_{WG}$, to assess whether or not to include the entire sample, and when one has to examine $r_{WG}$ of each individual unit of analysis and determine if it should be included in the data analysis. In other words, we want to find out when we shall start looking at the trees instead of the forest regarding IRA.

2. Background and Research Questions

2.1. $r_{WG}$ as a Measure of IRA

When there is a single target or a single variable:

$$r_{WG} = 1 - \frac{S_X^2}{\sigma^2}$$

where $S_X^2$ is the observed variance across respondents, and $\sigma^2$ is the variance obtained from a theoretical null distribution representing a complete lack of agreement among raters (totally random). When a construct has more than one parallel measurement item, the index becomes
where \( J \) is the number of items in the construct, \( S_j^2 \) is the mean of the response variances across items.

The \( r_{WG} \) index is widely adopted in various fields such as services marketing [13], strategic management [14], organizational behavior [15], and operations management [16]. 0.7 has been used as the cutoff value denoting whether IRA is established while using the \( r_{WG} \) index [6,7]. \( r_{WG} \) is calculated for each object (observation, unit of analysis, usually organization) in a survey sample. Theoretically, for any organization with an \( r_{WG} \) below 0.7, aggregation of the data across raters is not justified. However, in checking IRA of their data, most researchers calculate the mean \( r_{WG} \) value of all observations in their data set, which is labeled as \( R_{WG} \).

\[
R_{WG} = \frac{\sum r_{WG}}{n}
\]

where \( n \) is the sample size, or the number of units (organizations) in the sample.

If \( R_{WG} \) is above 0.7, they will conclude that IRA is achieved on the entire data set. Therefore, they will aggregate the data on each observation, and subsequently use all observations to conduct data analysis [17,18]. The original papers introducing the \( r_{WG} \) indices [4,5] have been cited more than 700 times in different fields ranging from strategic management to nursing [3]. However, a review of the services marketing literature revealed that very few studies excluded observations with low \( r_{WG} \) values [19,20]. Rather, most studies used data on all observations once \( R_{WG} \) was found above 0.7. In some cases, \( R_{WG} \) values as low as 0.50 [21] were deemed acceptable.

Because \( R_{WG} \) is the mean of \( r_{WG} \) across all observations, even with some very low \( r_{WG} \) values, \( R_{WG} \) could still meet the suggested standard of 0.7. Therefore, a high \( R_{WG} \) value does not guarantee that IRA is achieved across all units of analysis, and the approach presented above is problematic. Simply having a mean value \( R_{WG} \) above the 0.7 threshold cannot justify data aggregation for those units that have lower \( r_{WG} \) values. Instead, the aggregation of data on each unit of analysis should be justified based on the individual \( r_{WG} \) of that specific unit.

Yet little research has been conducted on how to handle the observations that have low \( r_{WG} \) values. Hence, the question remains whether they should be excluded from the data analysis, using only those observations with high \( r_{WG} \), or whether all observations should be used as long as their average, \( R_{WG} \), is above 0.7, a practical approach adopted by most researchers. Although LeBreton and Senter [3] suggest that aggregation is justified if only 5% of \( r_{WG} \) values are below the 0.7 threshold, no theoretical or empirical support is provided for this assertion. Therefore, it remains an unanswered question in the literature, and one that we seek to answer in this paper. In doing so, we examined the potential impact of two factors on the IRA decision: relationship strength and overall IRA level of the sample.

### 2.2. Factors That Potentially Affect the IRA Decision

A number of different factors such as model complexity, number of items in a construct, and factor loading structure of a construct potentially affect whether \( R_{WG} \) is adequate for inclusion of all observations. We limit the scope of this study to two of these factors. The first is the strength of the investigated relationship. The stronger the relationship between two variables, the more robust it is to the working sample, and the less difference there will be between parameter estimates using the entire sample versus using only those observations with high IRA values. That is, when the relationship between two variables is very strong, using all data or only the valid data (those observations with \( r_{WG} \) above 0.7) likely makes little difference. When the relationship between two variables is fairly weak, better results may be realized if only the valid data are used.

The second factor that likely affects the decision is the overall magnitude of IRA, as represented by \( R_{WG} \) itself. As \( R_{WG} \) gets higher, the portion of the sample with low \( r_{WG} \) values decreases, so the difference between using the complete sample and using only those observations with high IRA values also decreases. When \( R_{WG} \) is very high (e.g., above 0.85), using only the valid data is probably as good as using all data. However, when \( R_{WG} \) is fairly low, the invalid data might cause too much noise in the analysis, suggesting that only the valid data should be used.

To the best of our knowledge, this is the first study to address the issue of whether \( R_{WG} \) is adequate for the inclusion of all observations. To date, none of the potential factors have been examined before, including the two factors we investigate in this study. Findings from this study have the potential to offer guidelines for researchers to determine when to aggregate and use the entire sample, and when only observations with high enough \( r_{WG} \) values should be used.

### 3. Research Design

To address these issues, we utilized two different approaches because triangulation enhances the validity of research results and helps develop a holistic understanding of the phenomena of interest [1]. The fist approach used actual
data which included two relationships among three variables that are well established in the services marketing literature; thus, the only purpose of the analyses was to answer questions about IRA. In addition, this data set employs multiple raters. Analysis of this real data should provide an initial sense of the phenomenon under investigation.

In the second approach, we used data generated through simulation. Simulation allowed us to generate multiple data sets with varying levels of relationship strength and varying levels of \( r_{WG} \), something not possible with real data. This approach enabled us to examine the impact of these two factors in the aggregation decision.

For both data sets, we fitted each model twice. In the first model, we used the entire sample. In the second model, we used just a partial sample of those observations with \( r_{WG} \) above 0.7 on both constructs in the relationship. We then compared the differences between the results. The rationale is that the aggregated data are valid on observations with both \( r_{WG} \) values above 0.7, but not valid on observations with either \( r_{WG} \) value below 0.7. If no significant differences were found between these two analyses, aggregating the rater level data onto the organization level with all units of analysis was justified. However, if significant differences were found between these two analyses, under that specific condition, the researcher should not aggregate data on all units of analysis. Rather, they should only use those units with \( r_{WG} \) above 0.7.

### 3.1. Analysis with Real Data

As noted, the real data included well-established relationships in the services marketing literature, and we used two relationships among three variables to examine the research questions about IRA. In the services marketing literature, it is widely acknowledged that the server’s interaction quality with the customer greatly influences both the customer’s perceived value of the service and the customer’s satisfaction level. Therefore, we chose interaction quality, perceived value, and customer satisfaction as the variables of interest, and investigate the two relationships as shown in Figure 1.

Because these well-established relationships are very strong, the analysis was able to focus solely on the impact of IRA decisions on the relationships. Because we wanted to keep the relationship simple to analyze, focusing only on the impact of the IRA decisions, we investigated the two relationships separately rather than putting them together in one combined model. Each of the three variables was measured with a multi-item scale. Two types of Likert scales, 7-point and 10-point, were used to avoid common method bias. All measures were adapted from established scales in previous literature, and are included in Appendix A.

Data were collected via mail questionnaires from randomly selected customers of a national bank in New Zealand, who were given assurance of confidentiality and anonymity. Respondents were asked to rate their service experience with one specific bank branch. Of the 2500 questionnaires mailed, 872 were returned yielding a response rate of 34.9%. A total of 51 bank branches were rated by these customers, with each branch rated by an average of 11.2 customers. When we aggregated the data from individual customer level to the bank branch level, the \( r_{WG} \) values for the three scales were 0.853, 0.767, and 0.842, respectively.

We analyzed two structural equation models to test the two relationships. Each model was fitted twice first with the entire sample and then with a partial sample of those bank branches with \( r_{WG} \) above 0.7 on both constructs in the relationship. Results of these analyses are presented in Table 1.

The first relationship examined is the one between interaction quality and perceived value; 64.7% of the bank branches have a \( r_{WG} \) above 0.7 on both constructs. However, the model fit indices between the full sample and the partial sample are slightly different. The standardized path coefficient for this relationship is 0.613 on the full sample and 0.761 on the partial sample, and although both are significant at \( p < 0.001 \), the relationship appears stronger when observations with \( r_{WG} \) below 0.7 are not included in the analysis.

### Table 1. Real data analysis results.

<table>
<thead>
<tr>
<th></th>
<th>Interaction quality → Perceived value</th>
<th>Interaction quality → Customer satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full data</td>
<td>Partial data</td>
</tr>
<tr>
<td>Sample size</td>
<td>51</td>
<td>33 (64.7%)</td>
</tr>
<tr>
<td>Chi-square</td>
<td>47.63</td>
<td>38.65</td>
</tr>
<tr>
<td>Degree of freedom</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>Normed Chi-square</td>
<td>2.507</td>
<td>2.034</td>
</tr>
<tr>
<td>NFI</td>
<td>0.902</td>
<td>0.888</td>
</tr>
<tr>
<td>CFI</td>
<td>0.937</td>
<td>0.938</td>
</tr>
<tr>
<td>RMSEA</td>
<td>0.174</td>
<td>0.180</td>
</tr>
<tr>
<td>Std coefficient</td>
<td>0.613</td>
<td>0.761</td>
</tr>
<tr>
<td>( p )-value</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

Figure 1. Relationships in real data analysis.
The second relationship investigated is between interaction quality and customer satisfaction. Because both constructs have very high $R_{WG}$ (above 0.8), fewer bank branches actually have an $r_{WG}$ below 0.7 on either of the constructs, and 82.4% of the 51 bank branches (42) have an $r_{WG}$ above 0.7 on both constructs. As a result, the model fit statistics between the full sample and the partial sample are very close. In addition, the standardized path coefficient (Gamma) of the relationship is 0.819 for the full sample and 0.840 for the partial sample, both very high and very close to each other.

In comparing the two relationships, not only did the second relationship have a higher $R_{WG}$ on the constructs, but the relationship between the two variables was stronger than the first one (Gamma = about 0.8 compared to about 0.6). These results suggest that IRA decisions may be more critical for the first relationship because the results from the full sample as compared to the partial sample differ to a greater extent. However, because we cannot control the strength of the underlying relationship or the magnitude of $R_{WG}$ with the real data, we are unable to separate the effects of these two factors.

3.2. Analysis with Simulated Data

To separate the effects of the strength of the underlying relationship and the magnitude of $R_{WG}$, we used simulated data as a secondary approach to answer the research questions.

3.2.1. Data Generation

With simulated data, the analysis focused on the relationship between two hypothetical latent variables, $\xi$ and $\eta$. In this case, we assume each of the two constructs has four measurement items, $X1$, $X2$, $X3$, $X4$ for $\xi$, and $Y1$, $Y2$, $Y3$, $Y4$ for $\eta$. The model is shown in Figure 2.

We used simulation to generate multiple hypothetical data sets for these two variables with the following specifications: 1) The sample size was set to be 250, which is large enough for structural equation modeling analysis [26]. 2) The response scale was a seven-point Likert scale. 3) For each unit of analysis (organization), there were five raters. 4) The standardized item loading coefficients for $X1$, $X2$, $X3$ and $X4$ on the construct $\xi$ were set to be 0.72, 0.74, 0.76, and 0.80 respectively. The same coefficients apply to the loading of $Y1$, $Y2$, $Y3$ and $Y4$ onto the construct $\eta$. These coefficients were chosen to ensure that at least 50% (square of the standardized item loading coefficients) of the variance in the measurement items is explained by the underlying construct. In other words, the measurement items are all reliable indicators of their respective underlying constructs. This allows an exclusive focus on our real interest, the structural link between $\xi$ and $\eta$.

Because the magnitude of the difference may depend on the strength of the relationship and the overall magnitude of IRA, we controlled these two factors when generating the samples. To control the relationship strength between the latent variables, we chose five levels of Gamma: 0.1, 0.3, 0.5, 0.7, and 0.9. At each level of Gamma, we had 10 different target levels of $R_{WG}$ for both $\xi$ and $\eta$: 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, and 0.95. Empirical experience tells us that when $R_{WG}$ is below 0.50, a significant portion (usually above 80%) of the observations have $r_{WG}$ values below 0.7, which will almost inevitably affect the analysis. Therefore, we only chose levels of $R_{WG}$ above 0.50.

For each Gamma and target $R_{WG}$ combination, we generated five random samples. These five samples have exactly the same mean responses, but differ in the $r_{WG}$ value of each observation. Because we have five levels of Gamma and ten levels of target $R_{WG}$, the total number of samples generated is $5 \times 10 \times 5 = 250$. The detailed simulation process is described in Appendix B.

3.2.2. Data Analysis

Using Amos (Version 7.0), we tested the structural equation model shown in Figure 2 with each of the 250 samples. The analysis was conducted twice with each sample. The first analysis was conducted on the full sample of 250 observations, while the second analysis was conducted on a partial sample that had observations with $r_{WG}$ values above 0.7 on measures of both $\xi$ and $\eta$. A total of 500 analyses were conducted. For each analysis, we recorded the standardized path coefficient (Gamma) between $\xi$ and $\eta$ and the p-value corresponding to it.

Because we conducted the analysis twice with each sample, it was important to determine which of the two analyses provided a better estimate of the true population Gamma. Note that the data sets we generated are not raw responses, but aggregated responses across the raters of each target. For the aggregated data to be valid, IRA should be established first. For observations with low IRA values ($r_{WG}$ below 0.7), the aggregation itself is not justified. When comparing a full sample with its corresponding partial sample, we believe that the partial sample has a Gamma value that is closer to the true value because data aggregation is valid on all observations in the partial sample.
In comparison, the full sample has certain observations with low \( r_{WG} \) values on which the aggregation is not justified, introducing noise into the sample which leads to a biased estimate of Gamma. As a result, the partial sample provides a closer approximation of the true Gamma value than does the corresponding full sample.

We used the absolute difference between the Gamma estimates from the full sample and the corresponding partial sample as the dependent variable; hence, we refer to this variable as “Gamma difference”. The first independent variable is the relationship strength, and the second independent variable is the IRA level, which is reflected by the magnitude of \( R_{WG} \). While generating the samples, the target \( R_{WG} \) values for measures of \( \xi \) and \( \eta \) are the same for each sample. As a result, the calculated \( R_{WG} \) values for measures of \( \xi \) and \( \eta \) are very close to each other and we used the mean of these two \( R_{WG} \) values as a measure of the IRA level.

We conducted a regression analysis with the one dependent variable and the two independent variables, as well as the interaction between the two IVs. To avoid multicollinearity between the first order variables and the interaction term, we standardized the two independent variables prior to forming the interaction term through multiplication [27]. To assess the difference in variance explained when the interaction terms was included, we conducted a stepwise regression analysis, where the two independent variables were included in the first step and the interaction term entered into the model in the second step. The results are summarized in Table 2.

The base model (without the interaction term) is statistically significant. The two independent variables explain a significant portion of variance (38.5%) in the dependent variable. In addition, both independent variables have a significant negative relationship with the dependent variable \((p < 0.01 \text{ for both})\). This indicates that each of the independent variables contributes to the dependent variable in a unique way. Basically, when the relationship gets stronger, the full sample becomes more valid. In addition, when IRA level of the sample increases, the full sample becomes more valid.

Table 2. Results of regression analyses.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Without interaction term</th>
<th>With interaction term</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R^2 )</td>
<td>0.390</td>
<td>0.399</td>
</tr>
<tr>
<td>Adjusted ( R^2 )</td>
<td>0.385</td>
<td>0.392</td>
</tr>
<tr>
<td>( F )-value</td>
<td>78.797 ( *** )</td>
<td>54.532 ( *** )</td>
</tr>
<tr>
<td>Standardized coefficient</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IRA level</td>
<td>-0.621 ( *** )</td>
<td>-0.617 ( *** )</td>
</tr>
<tr>
<td>Relationship strength</td>
<td>-0.145 ( *** )</td>
<td>-0.138 ( *** )</td>
</tr>
<tr>
<td>Interaction term</td>
<td>-</td>
<td>0.100 ( ** )</td>
</tr>
</tbody>
</table>

\( *** p < 0.01; ** p < 0.05; * p < 0.1 \).

The model with the interaction term is also significant \((p < 0.01)\), and the interaction term itself is statistically significant at \( p < 0.05 \). While the effects of the two independent variables are both negative, the interaction effect is positive. When an interaction effect is statistically significant, it should be further analyzed and interpreted as a conditional effect on the main effects [28]. Following the approach suggested by Aiken and West [27], we created an interaction plot using the following equation:

\[
\text{Gamma difference} = -0.617 \text{ IRA} - 0.138 \text{ Rel. Str.} + 0.100 \text{ IRA} \times \text{Rel. Str.}
\]

When overall IRA level is low (one standard deviation below the mean), an increase of relationship strength by one unit was estimated to decrease Gamma difference by 0.238 units (calculated as \(-0.138 - 0.100\)). When overall IRA level is at its mean level, an increase of relationship strength by one unit was estimated to decrease Gamma difference by 0.138 units (calculated as \(-0.138 + 0\)). However, when overall IRA level is high (one standard deviation above the mean), an increase of relationship strength by one unit was estimated to decrease Gamma difference by only 0.038 units (calculated as \(-0.138 + 0.100\)). The plot is shown in Figure 3. As the plot illustrates, relationship strength has a fairly strong negative effect on Gamma difference when overall IRA level is relatively low. As overall IRA level becomes higher, the relationship between relationship strength and Gamma difference becomes weaker and weaker. When overall IRA level is very high, relationship strength has minimal impact on Gamma difference. These results indicate that between the two factors—relationship strength and overall IRA level—as one gets lower, the influence of the other on the IRA decision becomes stronger.

These results spawn some practical guidelines that can assist researchers in determining when to use the full sample rather than a partial sample.

![Figure 3. Plot of the interaction between relationship strength and IRA level.](image-url)
3.2.3. Proposed Guidelines

To provide some guidelines for researchers on determining the use of a full or partial sample, we used the unstandardized coefficients from the regression results to calculate the appropriate IRA levels for different levels of relationship strength. We report three sets of guidelines, each of which assumes it is acceptable if the estimated Gamma is within 10%, 20%, and 30% of the actual value, respectively. The guidelines are summarized in Table 3.

At a given Gamma level and a chosen estimation accuracy level (such as 10%), when the group IRA level is above the suggested threshold, the full sample is regarded as valid and therefore, can be used to conduct the analysis. When the group IRA level is below the suggested threshold, the full sample would result in a much distorted estimate of the relationship; in this case, the best solution is to use a partial sample of those cases with $r_{WG}$ above 0.7 on both constructs in the investigated relationship.

These threshold values suggest that the appropriateness of the full sample depends on both the relationship strength and the overall IRA level of the sample. When the relationship under estimation is so strong that Gamma is 0.8—even if the $R_{WG}$ for the sample is as low as 0.60—the full sample still provides a very valid estimate of the relationship, which is within 10% of the actual value. As the relation strength gets weaker, a higher level of $R_{WG}$ is needed. When the true Gamma is 0.4, the full sample $R_{WG}$ must be above 0.80 to get an estimate that is within 10% of the actual value. However, when the true Gamma is only 0.2, our results suggest that a $R_{WG}$ of 0.87 is needed to justify the use of the full sample.

Table 3. Appropriate IRA levels for different levels of Gamma.

<table>
<thead>
<tr>
<th>Gamma</th>
<th>Estimate within 10% of actual value</th>
<th>Estimate within 20% of actual value</th>
<th>Estimate within 30% of actual value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.90</td>
<td>0.87</td>
<td>0.84</td>
</tr>
<tr>
<td>0.2</td>
<td>0.87</td>
<td>0.81</td>
<td>0.75</td>
</tr>
<tr>
<td>0.3</td>
<td>0.84</td>
<td>0.74</td>
<td>0.64</td>
</tr>
<tr>
<td>0.4</td>
<td>0.80</td>
<td>0.66</td>
<td>0.53</td>
</tr>
<tr>
<td>0.5</td>
<td>0.76</td>
<td>0.58</td>
<td>0.40</td>
</tr>
<tr>
<td>0.6</td>
<td>0.71</td>
<td>0.49</td>
<td>0.26</td>
</tr>
<tr>
<td>0.7</td>
<td>0.66</td>
<td>0.38</td>
<td>0.11</td>
</tr>
<tr>
<td>0.8</td>
<td>0.60</td>
<td>0.27</td>
<td>-</td>
</tr>
<tr>
<td>0.9</td>
<td>0.54</td>
<td>0.14</td>
<td>-</td>
</tr>
</tbody>
</table>

4. Discussion

4.1. Contributions

Researchers have addressed numerous issues around the $r_{WG}$ measure of IRA [3]. However, little has been posited about what should be done with observations displaying low $r_{WG}$ values. Rather, most researchers generally use all observations as long as $R_{WG}$ is above 0.7. In this study, we show how problematic this convenient approach could be. In addition, we make a first attempt to provide guidance about how to handle observations with low $r_{WG}$ values.

We used two approaches to investigate the research question, one with real data and the other with hypothetical data generated via simulation. Our results show that whether or not the full sample could be used in the analysis depends on: 1) the strength of the relationship under investigation, and 2) the overall IRA level of a sample, as reflected by $R_{WG}$. As the underlying relationship gets stronger, a sample becomes more robust to IRA and the impact of low IRA values decreases. Moreover, as $R_{WG}$ gets higher, the proportion of observations with low $r_{WG}$ values gets lower, as does their impact on the estimation of the relationship. The results from both approaches are consistent, improving the validity of our findings [1].

The sub-group analyses with the simulated data enabled us to suggest some guidelines for empirical researchers on determining when to use the full sample for data analysis. These guidelines show the combined effect of relationship strength and $R_{WG}$ level on the validity of the aggregated full sample. Most previous researchers regarded the entire aggregated sample as valid as long as $R_{WG}$ of the sample is above 0.7. Our results presented in Table 3 indicate that only when the investigated relationship has a Gamma that is above 0.6, the estimated Gamma using the full sample is within 10% of the actual value. When the true population Gamma is below 0.6, a higher $R_{WG}$ is needed to ensure that the full sample provides an estimate that is close enough to the valid partial sample, which contains only those observations with $r_{WG}$ above 0.7.

In real world applications, researchers need some prior information on the relationship strength before making the IRA decision. If the relationship has been investigated in previous studies, researchers could use the previous Gamma as an estimate of the relationship strength. However, if the relationship under investigation is a new one and there is no existing research to provide this estimate, we suggest that researchers take a conservative approach and assume that the true Gamma is relatively weak. In this case, a $R_{WG}$ above 0.80 is needed to justify the use of the entire sample.

4.2. Limitations and Future Research Directions

As noted, multiple factors such as model complexity,
number of items in a construct, and factor loading structure of a construct could potentially affect the IRA decision. In this first attempt to address the IRA issue, we focused specifically on two of these factors: relationship strength and overall IRA level. While generating data with simulation, we controlled the other factors to eliminate their effects on the study. Specifically, we selected a very simple model between two variables, set the number of measurement items in each variable to be four, and decided the factor loading structures to ensure construct reliability. In reality, the model could be much more complicated, the number of measurement items could be variable, and the factor loadings may not be strong. It was unrealistic to examine all these factors in this initial study. To investigate the impact of these factors on the IRA decision, future studies could relax some of our restrictions. For example, as the model gets more complicated, more stringent guidelines might be required with regard to the validity of the full sample. Future research could investigate situations with more complicated research models.

Further, when generating samples with simulation, we assumed that both constructs in the relationship have comparable $R_{WG}$ values. In reality, this is not always the case. One variable could have much higher $R_{WG}$ than another variable in a relationship. If we consider multiple relationships in a model, the situation may become even more complicated. Future studies could investigate situations where variables in a relationship have different $R_{WG}$ levels and suggest corresponding guidelines for those more complicated situations.

Even with these limitations, the current study is a first attempt to address the issue of dealing with observations with low $r_{WG}$ values in survey research. We believe this study provides an enriched understanding of IRA and encourages additional research on this important, but understudied issue.

5. Acknowledgements

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REFERENCES


Appendix A. Measures of Variables

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<thead>
<tr>
<th>Interaction quality [22]</th>
<th>RWG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employees of XYZ Bank are (strongly disagree = 1 to strongly agree = 7)</td>
<td>0.853</td>
</tr>
<tr>
<td>• IQ1. Dependable.</td>
<td></td>
</tr>
<tr>
<td>• IQ2. Competent.</td>
<td></td>
</tr>
<tr>
<td>• IQ3. Knowledgeable.</td>
<td></td>
</tr>
<tr>
<td>• IQ4. Reliable.</td>
<td></td>
</tr>
<tr>
<td>• IQ5. Willing to provide service in a timely manner.</td>
<td></td>
</tr>
</tbody>
</table>

Perceived value [29]
Please rate XYZ Bank relative to other banks (much worse = 1 to much better = 10) | 0.767 |
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>• VAL1. The quality of service given the fees I pay.</td>
</tr>
<tr>
<td>• VAL2. The fees I pay for the quality of service I receive.</td>
</tr>
<tr>
<td>• VAL3. Overall value from the service I receive.</td>
</tr>
</tbody>
</table>

Customer satisfaction [30]
Please rate your feelings about your interactions with XYZ Bank (10-point scale) | 0.842 |
<table>
<thead>
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</thead>
<tbody>
<tr>
<td>• SAT1. Unhappy (1) - Happy (10).</td>
</tr>
<tr>
<td>• SAT2. Displeased (1) - Pleased (10).</td>
</tr>
<tr>
<td>• SAT3. Terrible (1) - Delighted (10).</td>
</tr>
<tr>
<td>• SAT4. Dissatisfied (1) - Satisfied (10).</td>
</tr>
</tbody>
</table>

Appendix B. Simulation Process
1) To control the strength of the relationship (Gamma) between the two latent constructs $\xi$ and $\eta$, we first defined a covariance matrix among all eight measurement items (X1 to Y4) based on the chosen Gamma and the specified item loading coefficients. Using it as the covariance matrix of a multivariate standard distribution, each time we generated one random sample of size 250 from the population.

2) The original data sets generated were standardized data. We then unstandardized them to make them fit on a 1 - 7 scale.

3) These unstandardized data sets were treated as the aggregated data sets, i.e., each value in a data set represented the mean of five individual responses on a specific measurement item. Each data set generated was treated as an aggregated data set. Instead of generating the raw responses from this aggregated data set, we just allocated a possible $r_{WG}$ value for $\xi$ and $\eta$ respectively on each of the 250 observations.

4) We assumed that the raw responses are not limited to integers between 1 and 7, but they could assume any value between 1 and 7. This gave us more flexibility in treating the response means and response variances. Without this assumption, certain response means that we generated might not be legitimate. For example, when all five responses to an item are 4, the response mean is 4.0. When four responses are 4 and the fifth response is 5, the response mean is 4.2. Therefore, any response mean value between 4.0 and 4.2, such as 4.13, is not legitimate if we use responses strictly from a 1 - 7 Likert scale. Given our assumption of continuous responses, any response mean value between 1 and 7 is possible and legitimate.

5) In order to find a possible $r_{WG}$ value for any response mean, we used the following approach. For each mean response $\bar{X}_i$ through $\bar{Y}_i$, we calculated the theoretical maximum response variance. Taking item $X_1$ as an example, the theoretical maximum response variance of five ratings given the response mean of $\bar{X}_1$ is (see the equation below)

$$V_{\bar{X}_i} = \frac{\text{Int} \left( \frac{5 \times (\bar{X}_i - 1)}{6} \right) \times (7 - \bar{X}_i)^2 + \left( 4 - \text{Int} \left( \frac{5 \times (\bar{X}_i - 1)}{6} \right) \right) \times (1 - \bar{X}_i)^2 + \left( 5 \times (\bar{X}_i - 1) - 6 \times \text{Int} \left( \frac{5 \times (\bar{X}_i - 1)}{6} \right) + 1 - \bar{X}_i \right)^2}{5}$$

6) Under the continuous response assumption, for any response mean, the minimum response variance is always 0. We controlled the target $R_{WG}$ value of a sample by controlling the range of response variance from which to choose a specific response variance. Specifically, we randomly chose 5% of observations in a sample. For
these observations, the maximum response variance limit is set to the theoretical maximum response variance. For the rest 95% of the observations, we set a maximum response variance limit, which was a percentage of the theoretical maximum response variance. For example, when the target $R_{WG}$ is 0.50, the maximum response variance limit was set to be 88% of the theoretical maximum response variance. This way we were able to control the approximate level of the $R_{WG}$ value of each sample. In addition, this approach ensured that the chosen response variance was practical for the given response mean.

7) For each measurement item in any observation in a data set, we randomly chose a response variance from 0 to its maximum response limit. Then we used the four response variances for $X_i$ to calculate $r_{WG}$ for construct $\xi$, and used the four response variances for $Y_i$ to calculate $r_{WG}$ for construct $\eta$. Here are the formulas for $r_{WG,\xi}$ [4,5]:

Mean response variance for $\xi$:  $S_{\xi}^2 = \frac{v_{x_1} + v_{x_2} + v_{x_3} + v_{x_4}}{4}$

If $S_{\xi}^2 < 4$, $r_{WG,\xi} = \left( \frac{S_{\xi}^2}{4} \right) \left( \frac{S_{\xi}^2}{S_{\xi}^2} \right) + \frac{S_{\xi}^2}{4}$

Otherwise, $r_{WG,\xi} = 0$