Polarizations as States and Their Evolution in Geometric Algebra Terms with Variable Complex Plane

Alexander Soiguine

SOiGUINE Quantum Computing, Aliso Viejo, CA, USA
Email: alex@soiguine.com

Abstract

Recently suggested scheme [1] of quantum computing uses g-qubit states as circular polarizations from the solution of Maxwell equations in terms of geometric algebra, along with clear definition of a complex plane as bivector in three dimensions. Here all the details of receiving the solution, and its polarization transformations are analyzed. The results can particularly be applied to the problems of quantum computing and quantum cryptography. The suggested formalism replaces conventional quantum mechanics states as objects constructed in complex vector Hilbert space framework by geometrically feasible framework of multivectors.

Keywords

Quantum Mechanics, Quantum Computing, Geometric Algebra, Maxwell Equations

1. Introduction

The circular polarized electromagnetic waves are the only type of waves following from the solution of Maxwell equations in free space done in geometric algebra terms.

Let's take the electromagnetic field in the form:

$$F = F_0 \exp \left[ I_k \left( \omega t - k \cdot r \right) \right]$$

requiring that it satisfies the Maxwell system of equations in free space, which in geometrical algebra terms is one equation:

$$(\partial_e + \nabla) F = 0$$
where \( \nabla = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \) and multiplications are the geometrical algebra ones.

Element \( F_0 \) in (1) is a constant element of geometric algebra \( G_3 \) and \( I_S \) is unit value bivector of a plane \( S \) in three dimensions, that is a generalization of the imaginary unit [2], [3]. The exponent in (1) is unit value element of \( G^*_3 \) [3]:

\[
e^{i\omega \varphi} = \cos \varphi + I_S \sin \varphi, \quad \varphi = \omega t - k \cdot r
\]

Solution of (2) should be sum of a vector (electric field \( \mathbf{E} \)) and bivector (magnetic field \( I_S \mathbf{H} \)):

\[
F = \mathbf{E} + I_S \mathbf{H}
\]

with some initial conditions:

\[
E + I_S \mathbf{H} \big|_{t=0,r=0} = F_0 = E \big|_{t=0,r=0} + I_S \mathbf{H} \big|_{t=0,r=0} = E_0 + I_S \mathbf{H}_0
\]

In the magnetic field \( I_S \mathbf{H} \) the item \( I_S \) is unit pseudoscalar in three dimensions assumed to be the right-hand screw oriented volume, relative to an ordered triple of orthonormal vectors.

Substitution of (1) into the Maxwell’s (2) will exactly show us what the solution looks like.

2. Solution in the Geometric Algebra Terms

The derivative by time gives

\[
\frac{\partial}{\partial t} F = F_0 e^{i\omega \varphi} I_S \left( \omega t - k \cdot r \right) = F_0 e^{i\omega \varphi} I_S \omega = FI_S \omega
\]

The geometric algebra product \( \nabla F \) is:

\[
\nabla F = F_0 I_S e^{i\omega \varphi} \nabla \left( \omega t - k \cdot r \right) = -F_0 e^{i\omega \varphi} I_S k = -FI_S k
\]

or

\[
\nabla F = F_0 e^{i\omega \varphi} \nabla \left( \omega t - k \cdot r \right) I_S = -F_0 e^{i\omega \varphi} k I_S = -FKI_S,
\]

depending on do we write \( I_S \left( \omega t - k \cdot r \right) \) or \( \left( \omega t - k \cdot r \right) I_S \). The result should be the same since \( \omega t - k \cdot r \) is a scalar.

Commutativity \( I_S k = k I_S \) is true only if \( k \times I_S = 0 \). The following agreement takes place between orientation of \( I_S \), orientation of \( I_S \) and direction of vector \( k \) [3]. The vector \( I_S I_S = I_S I_S \) is orthogonal to the plane of \( I_S \) and its direction is defined by orientations of \( I_S \) and \( I_S \). Rotation of right/left hand screw defined by orientation of \( I_S \) gives movement of right/left hand screw. This is the direction of the vector \( I_S I_S = I_S I_S \). That means that the matching between \( \hat{k} \) and \( I_S \) should be \( \hat{k} = \pm I_S I_S \) or \( \hat{k} = \mp I_S \).

Assuming that orientation is \( I_S = \hat{k} I_S \), the Maxwell equation becomes:

\[
F \left( I_S \omega - I_S |k| \right) = F \left( \omega I_S - |k| \hat{k} I_S \right) = 0
\]

or

\[
\text{For any vector we write } \hat{a} = \hat{a} / |\hat{a}|.
\]
\[(E + I_s H) \omega = (E + I_s H) k\]

Left hand side is sum of vector and bivector, while right hand side is scalar \(E \cdot k\) plus bivector \(E \wedge k\), plus pseudoscalar \(I_s (H \cdot k)\), plus vector \(I_s (H \wedge k)\). It follows that both \(E\) and \(H\) lie on the plane of \(I_s\) and then:

\[
\omega E = I_s Hk, \omega l_s H = Ek \rightarrow \frac{\omega^2}{|k|} I_s Hk = \omega E
\]

Thus, \(\omega = |k|\) and we get equation \(I_s Hk = E\) from which particularly follows \(|E|^2 = |H|^2\) and \(E_\wedge H = I_s\).

The result for the case \(I_s = \hat{k}l_s\) is that the solution of (2) is

\[
F = (E_0 + I_s H_0) \exp[I_s (\omega t - k \cdot r)]
\]

where \(E_0\) and \(H_0\) are arbitrary mutually orthogonal vectors of equal length, lying on the plane \(S\). Vector \(k\) should be normal to that plane, \(\hat{k} = -I_s l_s\) and \(|k| = \omega\).

In the above result the sense of \(I_s\) orientation and the direction of \(\hat{k}\) were assumed to agree with \(I_s = \hat{k}l_s\). Opposite orientation, \(-I_s = \hat{k}l_s\), that’s \(k\) and \(I_s\) compose left hand screw and \(\hat{k} = I_s l_s\), will give solution

\[
F = F_0 \exp[I_s (\omega t - k \cdot r)] \text{ with } \hat{E}_\wedge H_k = I_s.
\]

**Summary:**

For a plane \(S\) in three dimensions Maxwell equation (2) has two solutions

- \(F_+ = (E_0 + I_s H_0) \exp[I_s (\omega t - k \cdot r)]\), with \(\hat{k} = l_s I_s\), \(\hat{E}_\wedge H_k = I_s\), and the triple \(\{\hat{E}, H, \hat{k}\}\) is right hand screw oriented, that’s rotation of \(\hat{E}\) to \(H\) by \(\pi/2\) gives movement of right hand screw in the direction of \(k = |k| l_s I_s\).
- \(F_- = (E_0 + I_s H_0) \exp[I_s (\omega t - k \cdot r)]\), with \(\hat{k} = -l_s I_s\), \(\hat{E}_\wedge H_k = -I_s\), and the triple \(\{\hat{E}, H, \hat{k}\}\) is left hand screw oriented, that’s rotation of \(\hat{E}\) to \(H\) by \(\pi/2\) gives movement of left hand screw in the direction of \(k = -|k| l_s I_s\) or, equivalently, movement of right hand screw in the opposite direction, \(-k\).

- \(E_0\) and \(H_0\) initial values of \(E\) and \(H\) are arbitrary mutually orthogonal vectors of equal length, lying on the plane \(S\). Vectors \(k_s = \pm |k_s| l_s I_s\) are normal to that plane. The length of the wave vectors \(|k_s|\) is equal to angular frequency \(\omega\).

Maxwell Equation (2) is a linear one. Then any linear combination of \(F_+\) and \(F_-\) saving the structure of (1) will also be a solution.

Let’s write:

\[
\left\{ \begin{array}{l}
F_+ = (E_0 + I_s H_0) \exp[I_s \omega t] \exp[-I_s [(l_s I_s) \cdot r]] \\
F_- = (E_0 + I_s H_0) \exp[I_s \omega t] \exp[I_s [(l_s I_s) \cdot r]]
\end{array} \right. \tag{3}
\]

Then for arbitrary scalars \(\lambda\) and \(\mu\)

\[
\lambda F_+ + \mu F_- = (E_0 + I_s H_0) e^{i \omega t} \left( \lambda e^{-i_s [(l_s I_s) \cdot r]} + \mu e^{i_s [(l_s I_s) \cdot r]} \right) \tag{4}
\]

is solution of (2). The item in second parenthesis is weighted linear combination.
of two states with the same phase in the same plane but opposite sense of orientation. The states are strictly coupled, entangled if you prefer, because bivector plane should be the same for both, does not matter what happens with it.

One another option is:

\[
(\lambda_1 + i \mu_1)(E_0 + i_3 H_0) \exp \left[ I_3 \omega (t - (I_3) \cdot r) \right] \\
+ (\lambda_2 + i \mu_2)(E_0 + i_3 H_0) \exp \left[ I_3 \omega (t + (I_3) \cdot r) \right] \\
= \left[ \lambda_1 E_0 - \mu_1 H_0 + i_3 (\mu_1 E_0 + \lambda_1 H_0) \right] \exp \left[ I_3 \omega (t - (I_3) \cdot r) \right] \\
+ \left[ \lambda_2 E_0 - \mu_2 H_0 + i_3 (\mu_2 E_0 + \lambda_2 H_0) \right] \exp \left[ I_3 \omega (t + (I_3) \cdot r) \right]
\]

which is just rotation, along with possible change of length, of electric and magnetic initial vectors in their plane.

### 3. Transformations of Polarization States

Polarizations, in our approach, exponents in the solution of (3), have the form of states [3], that’s elements of \( G_3^+ \): \( G_3^+ \supset \alpha + i_3 \beta = \cos \varphi + i_3 \sin \varphi = e^{i\varphi} \), distributed in \((t, r)\) space. They are operators than can act on observables, also elements of \( G_3^- \), particularly other polarizations. Such states can be depicted in the current geometric algebra formalism using a triple of basis bivectors in three dimensions \( \{B_1, B_2, B_3\} \) (Figure 1):

The basis bivectors satisfy multiplication rules (in the right and screw orientation of \( I_3 \)):

\[
B_1B_2 = -B_3, \quad B_2B_3 = B_1, \quad B_3B_1 = -B_2
\]

One can identify basis bivectors with usual coordinate planes: \( B_1 = j \hat{z}, \) \( B_2 = k \hat{x}, \) \( B_3 = \hat{x} y \). Any one of these three bivectors can be taken as explicitly identifying imaginary unit, though any unit value bivector in three dimensions can take the role [2], [4].

Thus:

\[
\alpha + i_3 \beta = \alpha + \beta (b_1B_1 + b_2B_2 + b_3B_3) = \alpha + \beta_1B_1 + \beta_2B_2 + \beta_3B_3
\]

The difference between units of information in classical computational scheme, quantum mechanical conventional computations (qubits) and geometric algebra

---

**Figure 1.** Basis of bivectors and unit value pseudoscalar.
scheme (g-qubits) with variable explicitly defined complex plane is seen from Figure 2.

Circular polarizations received as solutions of Maxwell Equation (2) is an excellent choice to have such g-qubits in a lab.

Commonly accepted idea to use systems of qubits to tremendously increase speed of computations is based on assumption of entanglement – roughly speaking when touching one qubit all the other in the system react instantly, in no time. A bit strange, though you should not care about that because our paradigm is very different.

Assume we have some general state:

\[ \begin{align*}
\alpha + I_3 \beta &= \alpha + \beta \left( b_1 B_1 + b_2 B_2 + b_3 B_3 \right) = \alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3 \\
\end{align*} \]

The state can be identified as a point \( (\alpha, \beta_1, \beta_2, \beta_3) \) on unit sphere \( S^3 \). It can be subjected to a Clifford translation

\[ \alpha + I_3 \beta \Rightarrow e^{\text{cI} \Delta \psi} \left( \alpha + I_3 \beta \right) \]

executing displacement \( \Delta \psi \) at point \( (\alpha, \beta_1, \beta_2, \beta_3) \) along intersection of \( S^3 \) with the unit bivector plane \( I_{cI} \).

Let’s make notations more like conventional quantum mechanical ones. I will write:

\[ \begin{align*}
\alpha + I_3 \beta &= \langle g \rangle_{(\alpha, \beta, I_3)}; & \overline{\alpha + I_3 \beta} &= \alpha - I_3 \beta = \langle g \rangle_{(\alpha, \beta, I_3)} \\
\end{align*} \]

and use Hamiltonian like form of the Clifford translation bivector.

Conventional Hamiltonian

\[ \text{Figv} \text{ure 2. Differences between bits, qubits and g-qubits.} \]
\[
\begin{pmatrix}
\gamma + \gamma_1 & \gamma_2 - i\gamma_3 \\
\gamma_2 + i\gamma_3 & \gamma - \gamma_1
\end{pmatrix},
\]
with removed not important scalar \(\gamma\), has the lift in \(G^*\) \[3\]:

\[\mathcal{H} = I_s (\gamma_1 B_1 + \gamma_2 B_2 + \gamma_3 B_3)\]

Then the associated Clifford translation plane bivector is \(-I_s \mathcal{H}(t)\). By normalizing the bivector to unit value we get generalization of imaginary unit

\[i \Rightarrow I_s \frac{\mathcal{H}(t)}{|\mathcal{H}(t)|},\]

that is critical for the whole approach. Therefore, for some \(\Delta t\), Clifford translation for a given Hamiltonian is:

\[
\left| g(t + \Delta t) \right|_{(0, I_s B_1, \beta_1 I_s B_2, \beta_2 I_s B_3)} = e^{\frac{\mathcal{H}(t)}{\|\mathcal{H}(t)\|}} \left| g(t) \right|_{(0, I_s B_1, \beta_1 I_s B_2, \beta_2 I_s B_3)}
\]

(5)

For an arbitrary sequence of infinitesimal Clifford translations, the final state is integral\(^2\)

\[
\int e^{-\frac{\mathcal{H}(l)}{|\mathcal{H}(l)|} \|dI\|} \left| g(I) \right|_{(0, I_s B_1, \beta_1 I_s B_2, \beta_2 I_s B_3)}
\]

along the curve on unit sphere \(S^3\) composed of infinitesimal displacements by

\[-\left( I_s \frac{\mathcal{H}(t)}{|\mathcal{H}(t)|} \right)|\mathcal{H}(t)|dI\]

Let’s calculate the result of the right-hand side of (5) in general case when the plane of \(I_s \frac{\mathcal{H}(t)}{|\mathcal{H}(t)|}\) differs from \(S(t)\).

To calculate the geometric algebra product of the two exponents in Clifford translation with not coinciding exponent planes, \(e^{I_s \mathcal{H}(t)} e^{I_s \mathcal{H}(t)}\), \(S_1 \neq S_2\), let’s first expand \(I_s\) in original basis to get formulas for generators of Clifford translation. If \(I_{S_1} = \gamma_1 B_1 + \gamma_2 B_2 + \gamma_3 B_3\) then a part of geometrical product \(e^{I_s \mathcal{H}(t)} e^{I_s \mathcal{H}(t)}\) is:

\[
I_{S_1} I_{S_2} = (\gamma_1 B_1 + \gamma_2 B_2 + \gamma_3 B_3) (\beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3)
\]

\[
= - (\gamma_1 \beta_1 + \gamma_2 \beta_2 + \gamma_3 \beta_3) + (\gamma_2 \beta_1 - \gamma_1 \beta_2) B_3 + (\gamma_3 \beta_1 - \gamma_1 \beta_3) B_2 + (\gamma_3 \beta_2 - \gamma_2 \beta_3) B_1
\]

\[
= - (\gamma \cdot \beta) - I_3 (\gamma \times \beta) = I_{S_1} \cdot I_{S_2} + I_{S_1} \wedge I_{S_2}
\]

(see Figure 3)

where \(\gamma\) and \(\beta\) are vectors dual to bivectors \(I_{S_1}\) and \(I_{S_2}\).

Thus, the full product is:

\(^2\)In the case of constant plane of Hamiltonian, it easily follows the Schrodinger equation of conventional quantum mechanics with clearly defined imaginary unit.
Figure 3. Two bivector geometrical product.

\[ e^{I_S \Delta \phi} e^{I_S \phi_2} = \cos \Delta \phi_1 \cos \phi_2 + \sin \Delta \phi_1 \cos \phi_2 I_{S_1} + \cos \Delta \phi_1 \sin \phi_2 I_{S_2} + \sin \Delta \phi_1 \sin \phi_2 I_{S_2} \]

\[ = \cos \Delta \phi_1 \cos \phi_2 - \sin \Delta \phi_1 \cos \phi_2 I_{S_1} \]

\[ - I_{S_1} \left( \cos \Delta \phi_1 \sin \phi_2 I_{S_1} I_{S_2} \right) + \sin \Delta \phi_1 \sin \phi_2 I_{S_2} \]

\[ = \cos \Delta \phi_1 \cos \phi_2 + \sin \Delta \phi_1 \sin \phi_2 \left( I_{S_1} \cdot I_{S_2} \right) \]

\[ + \sin \Delta \phi_1 \cos \phi_2 I_{S_1} + \sin \Delta \phi_1 \sin \phi_2 I_{S_2} + \sin \Delta \phi_1 \sin \phi_2 I_{S_1} \wedge I_{S_2} \]

4. Transformations of Circular Polarized Electromagnetic Fields

Now we have everything to retrieve action of Clifford translation generated by a Hamiltonian on general solution (4):

\[ e^{-\frac{H(t)}{\hbar} \frac{\Delta \phi}{\alpha}} \left( E_0 + iI_S H_0, \lambda e^{i e^{I_S \phi}} + \mu e^{i e^{I_S \phi}} \right) \]

To make expressions simpler I will use notations \( I_S \frac{\Delta \phi}{\alpha} \equiv I_{S_1} \), \( \omega(t - (I, I_S) \cdot r) \equiv \phi_\alpha \), and \( \omega(t + (I, I_S) \cdot r) \equiv \phi_- \). Then we get (see Sections 1.3 and 1.6 in [3] for multiplication details):

\[ e^{-\frac{H(t)}{\hbar} \frac{\Delta \phi}{\alpha}} \left( E_0 + iI_S H_0, \lambda e^{i e^{I_S \phi}} + \mu e^{i e^{I_S \phi}} \right) \]

\[ = - (E_0 + iI_S H_0) \lambda \left( \cos \left( \frac{H(t)}{\hbar} \Delta t \right) \cos \phi_- - \sin \left( \frac{H(t)}{\hbar} \Delta t \right) \sin \phi_\alpha \right) \left( I_{S_1} \cdot I_{S_2} \right) \]

\[ - (E_0 + iI_S H_0) \lambda \left( \sin \left( \frac{H(t)}{\hbar} \Delta t \right) \cos \phi_\alpha I_{S_1} + \cos \left( \frac{H(t)}{\hbar} \Delta t \right) \sin \phi_\alpha I_{S_1} \right) \]

\[ + \sin \left( \frac{H(t)}{\hbar} \Delta t \right) \sin \phi_\alpha \left( I_{S_1} \wedge I_{S_2} \right) - (E_0 + iI_S H_0) \mu \left( \sin \left( \frac{H(t)}{\hbar} \Delta t \right) \cos \phi_\alpha I_{S_1} \right) \]

\[ + \cos \left( \frac{H(t)}{\hbar} \Delta t \right) \sin \phi_\alpha I_{S_1} + \sin \left( \frac{H(t)}{\hbar} \Delta t \right) \sin \phi_\alpha \left( I_{S_1} \wedge I_{S_2} \right) \]

\[ 3 \text{In the case } I_{S_1} = I_{S_2}, \text{ we trivially have rotation of } e^{i e^{I_S \phi}} \text{ by angle } \Delta \phi. \]
Let’s take popular case of \( I_3 = B_3 = \hat{x}\hat{y} \) (plane orthogonal to \( \hat{z} \) axis) and \( I_H = B_1 = \hat{y}\hat{z} \) (or \( I_H = B_2 = \hat{z}\hat{x} \), does not matter.) The above formula becomes:

\[
-(E_0 + I, H_0)[\hat{z} \cdot \cos(|H(t)|\Delta t) \cos \varphi_0 + \sin(|H(t)|\Delta t) \cos \varphi_0 B_1 \\
+ \sin(|H(t)|\Delta t) \sin \varphi_0 B_2 + \cos(|H(t)|\Delta t) \sin \varphi_0 B_3 \\
+ \mu \cos(|H(t)|\Delta t) \cos \varphi_0 + \sin(|H(t)|\Delta t) \cos \varphi_0 B_1 \\
+ \sin(|H(t)|\Delta t) \sin \varphi_0 B_2 + \cos(|H(t)|\Delta t) \sin \varphi_0 B_3 ]
\]

It makes simpler if \( F_+ \) and \( F_- \) are equally weighted, say both \( \lambda \) and \( \mu \) are equal to one:

\[
-(E_0 + I, H_0)[\hat{z} \cdot \cos(|H(t)|\Delta t) \cos \varphi_0 + \sin(|H(t)|\Delta t) \cos \varphi_0 B_1 \\
+ \sin(|H(t)|\Delta t) \sin \varphi_0 B_2 + \cos(|H(t)|\Delta t) \sin \varphi_0 B_3 ]
\]

\[
= -2(E_0 + I, H_0)[\cos(\hat{z} \cdot r) \cos(|H(t)|\Delta t) \cos \omega t + \sin(|H(t)|\Delta t) \cos \omega t B_1 \\
+ \sin(|H(t)|\Delta t) \sin \omega t B_2 + \cos(|H(t)|\Delta t) \sin \omega t B_3 ]
\]

\[
(6)
\]

5. Action of Polarization States on Observables

Since a state in the described formalism is operator that gives the result of measurement when acting on observable, which can be any element of geometric algebra \( G_3 \), the following is detailed description of the case when the element in parenthesis of the (6) expression acts on some bivector. Such operation is generalization of the Hopf fibration and rotates the bivector in three dimensions.

Denoting:\n
\[
\cos(|H(t)|\Delta t) \cos \omega t + \sin(|H(t)|\Delta t) \cos \omega t B_1 \\
+ \sin(|H(t)|\Delta t) \sin \omega t B_2 + \cos(|H(t)|\Delta t) \sin \omega t B_3 \\
= \alpha + \beta_1 B_1 + \beta_2 B_2 + \beta_3 B_3 \equiv \mathcal{E}^{\parallel, \omega t}
\]

where

\[
I_{H,\omega} = (\gamma_1 B_1 + \gamma_2 B_2 + \gamma_3 B_3 )
\]

\[
\gamma_1 = \frac{\sin(|H(t)|\Delta t) \cos \omega t}{\sqrt{\sin^2(|H(t)|\Delta t) + \cos^2(|H(t)|\Delta t) \sin^2 \omega t}}
\]

\[
\gamma_2 = \frac{\sin(|H(t)|\Delta t) \sin \omega t}{\sqrt{\sin^2(|H(t)|\Delta t) + \cos^2(|H(t)|\Delta t) \sin^2 \omega t}}
\]

\[
\gamma_3 = \frac{\cos(|H(t)|\Delta t) \sin \omega t}{\sqrt{\sin^2(|H(t)|\Delta t) + \cos^2(|H(t)|\Delta t) \sin^2 \omega t}}
\]

\[\text{Easy to see that the left-hand side is unit value element of } G^+_3.\]
\[ \psi = \cos^{-1}(\cos(\mathcal{H}(t)\Delta t)\cos\omega t) \]

and taking a bivector operand (observable) \( c_1B_1 + c_2B_2 + c_3B_3 \), we get the result of measurement, action of the state on observable (see [3], [4] for details):

\[
e^{-i\omega t} (c_1B_1 + c_2B_2 + c_3B_3) e^{i\omega t} \\
= (c_1\left(\alpha^2 + \beta_1^2\right) - \left(\beta_2^2 + \beta_3^2\right) + 2c_2(\beta_2\beta_3 - \alpha\beta_1) + 2c_3(\beta_1\beta_3 + \alpha\beta_2))B_1 \\
+ (2c_1(\alpha\beta_3 + \beta_1\beta_2) + c_2\left(\alpha^2 + \beta_2^2\right) - (\beta_1^2 + \beta_3^2)) + 2c_3(\beta_2\beta_3 - \alpha\beta_1))B_2 \\
+ (2c_1(\beta_2\beta_3 + \alpha\beta_2) + 2c_2(\beta_1\beta_3 + \alpha\beta_1) + c_3\left(\alpha^2 + \beta_3^2\right) - (\beta_1^2 + \beta_2^2))B_3 \\
= (c_1\cos 2\omega t - \sin 2\omega t\left(c_2\cos(2\mathcal{H}(t)\Delta t)) - c_3\sin(2\mathcal{H}(t)\Delta t))\right)B_1 \\
+ (c_1\sin 2\omega t + \cos 2\omega t\left(c_2\cos(2\mathcal{H}(t)\Delta t)) - c_3\sin(2\mathcal{H}(t)\Delta t))\right)B_2 \\
+ (c_2\sin(2\mathcal{H}(t)\Delta t)) + c_3\cos(2\mathcal{H}(t)\Delta t))B_3 \\
\]

One interesting remark. If the observable belongs only to the \( B_1 \) plane, that’s \( c_2 = c_3 = 0 \), the result of measurement has only components in \( B_1 \) and \( B_2 \), projections of the value \( c_1 \) due to rotation with angular velocity \( 2\omega \) around the \( \hat{z} \) axis.

6. Polarization States Acting on Multiple Observables

The core of quantum computing should not be in entanglement as it understood in conventional quantum mechanics, which only formally follows from representation of the many particle states as tensor products of individual particle states and not supported by really operating physical devices. The core of quantum computing scheme should be in manipulation and transferring of sets of states as operators decomposed in geometrical algebra variant of qubits (g-qubits), or four-dimensional unit sphere points, if you prefer. Such operators can act on observables, particularly through measurements. From the recent calculation we realize that the action of state, which depends on \((t,r)\), on an observable can be done only if observable is defined at the same point \((t,r)\) where the state is defined [5]. In this way quantum computer is an analog computer keeping information in sets of objects with infinite number of degrees of freedom, contrary to the two value bits or two-dimensional Hilbert space elements, qubits.

Thus, if we have a state

\[ \alpha + I_\beta = |g\rangle_{(a,\beta,\beta_3)} = |g\rangle_{(a(r),\beta(t,r),\beta_3(t,r))} \]

as in the case of polarization defined states, it becomes a state acting on a set of observables if the latter are defined at some given points:

\[ |c_n\rangle = |c\rangle_{(\xi(t_n,r_n),\xi(t_n,r_n))} = c_0(t_n,r_n) + I_{C(t_n,r_n)} \sqrt{1 - c_0^2(t_n,r_n)}, n = 1, \ldots, N \]

Then the state transforms into multi-observable one:
This formula for $|g_1 \cdots g_N\rangle$ bears clear physical and geometrical sense, contrary to conventional quantum mechanics definition following formally from tensor product which does not have good physical interpretation but is the root of entanglement-based quantum computing.

The formula also prompts how quantum encryption decoding can be effectively implemented with the bivector value security key (see Figure 4).

The formula can also be applied to challenging area of anyons in three dimensions.

7. Conclusion

Two seminal ideas—variable and explicitly defined complex plane in three dimensions, and the $G^\s_1$ states as operators acting on observables—allow to put forth comprehensive and much more detailed formalism appropirate for quantum mechanics in general and particularly for quantum computing schemes. The approach may be thought about, for example, as a far going geometric algebra generalization of some proposals for quantum computing formulated in terms of light beam time bins, see [6], [7], but giving much more strength and flexibility in practical implementation.

References

https://doi.org/10.1103/PhysRevLett.111.150501

https://doi.org/10.1103/RevModPhys.79.135