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The Rational Solutions and the Interactions of the N-Soliton Solutions for Boiti-Leon-Manna-Pempinelli-Like Equation

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Abstract

These rational solutions which can be described a kind of algebraic solitary waves which have great potential in applied value in atmosphere and ocean. It has attracted more and more attention recently. In this paper, the generalized bilinear method instead of the Hirota bilinear method is used to obtain the rational solutions to the (2 + 1)-dimensional Boiti-Leon-Manna-Pempinelli-like equation (hereinafter referred to as BLMP equation). Meanwhile, the (2 + 1)-dimensional BLMP-like equation is derived on the basis of the generalized bilinear operators $D_{3,x}, D_{3,y}$ and $D_{3,t}$. And the rational solutions to the (2 + 1)-dimensional BLMP-like equation are obtained successively. Finally, with the help of the N-soliton solutions of the (2 + 1)-dimensional BLMP equation, the interactions of the N-soliton solutions can be derived. The results show that the two soliton still maintained the original waveform after happened collision.

1. Introduction

In recent years, with the emergence of high-powered computer and the development of mathematical research, various nonlinear problems increasingly cause attentions of scientists. Especially some key problems in engineering, science and modern physics are ultimately dependent on the specific solution of the nonlinear equations. So the solution of the nonlinear equations [1] [2] [3] [4] [5] occupies very important position no matter on theory research or practical application. But the diverse nonlinear equations of mathematical physics was descried with Hirota bilinear equations [6] [7] [8] and generalized bilinear equations [9] [11] [12] [13],

such as the KdV equations [10] [11] [14] [15], the BLMP equations [16] [17] [18], the NLS equation, the Boussinesq equation [19], the KP equations [9] [20] [21] and so on. There has been a growing attention on rational solutions [2] [3] [9] [10] [11] [22] to nonlinear differential equations in recent years. One kind of particular rational solutions are rogue wave solutions [24] [25] [26], which describe significant nonlinear wave phenomena in oceanography. So the rational solutions of the nonlinear equations become crucial in atmosphere and ocean.

As we know some rational solutions to integrable equations have been considered systematically on the basis of the Wronskian formulation, the Casoratian formulation and the Pfaffian formulation [9]. Rational solutions to the nonlinear differential equation are also considered by different approaches, such as the G'/G expansion methods [9] [10] [27] and the tanh-coth function method [28]. Moreover, the Hirota method is also used to construct rational solutions to the nonlinear differential equations. And it is very necessary and interesting for us to study rational solutions and generate the generalized bilinear form to nonlinear differential equations.

In this paper, the rational solutions to the (2 + 1)-dimensional BLMP equation can be derived by the Hirota bilinear method. And the (2 + 1)-dimensional BLMP-like nonlinear differential equation on the basic of existing BLMP bilinear differential equation can be derived via applying three generalized bilinear operators $D_{3,x}, D_{3,y}$ and $D_{3,t}$. Later, the rational solutions to the BLMP-like equation are obtained by Maple computation. Finally, the *N*-soliton solutions [4] [8] [29] to the (2 + 1)-dimensional BLMP equation are given under the Hirota method.

2. Hirota Bilinear D-Operators and the Rational Solutions to the (2 + 1)-Dimensional BLMP Equation

2.1. Hirota Bilinear D-Operators

The D-operators are defined in Refs. [6] [7] [8] as following:

$$D_x^m D_t^n F \cdot G = \left(\partial_x - \partial_{x'} \right)^m \left(\partial_t - \partial_{t'} \right)^n F(x,t) G(x',t') \Big|_{x=x',t=t'}, \tag{1}$$

where m, n are all non-negative integer.

Assume $m = 1, n = 0,$ Equation (1) can be reduced to

$$D_x F \cdot G = F_x G - F G_x. \tag{2}$$

Assume $m = 1, n = 1,$ Equation (1) can be reduced to

$$D_x D_t F \cdot G = F_{xt} G - F_t G_x - F_x G_t + F G_{xt}. \tag{3}$$

2.2. The Rational Solutions to the (2 + 1)-Dimensional BLMP Equation

Consider a (2 + 1)-dimensional BLMP equation

$$u_{yt} + u_{xxxy} - 3u_{xx}u_y - 3u_x u_{xy} = 0, \tag{4}$$

according to the formula Equation (1), the bilinear form as follows

$$2F_{yt}F - 2F_y F_t + 2F_{xxxy}F - 2F_{xxx}F_y - 6F_{xxy}F_x + 6F_{xx}F_{xy} = 0, \tag{5}$$

and the bilinear form with D-operators

$$\left(D_y D_t + D_x^3 D_y\right)F \cdot F = 0, \tag{6}$$

under the transformation $u = -2\left(\ln F\right)_x$.

Apply the direct Maple symbolic computation

$$F = \sum_{i=0}^{3}\sum_{j=0}^{2}\sum_{k=0}^{2} c_{ijk}\, x^i y^k t^t, \tag{7}$$

to obtain the polynomial solutions to Equation (5). where the c_{ijk} are constants.

The twelve classes of polynomial solutions to Equation (4) are obtained, as follows.

The first class of rational solutions to Equation (4):

$$u_1 = \frac{-m}{n} \tag{8}$$

there

$$m = 2\left(3t^2 x^2 c_{3,2,1} + 2t^2 x c_{2,2,1} + 3tx^2 c_{3,1,1} + t^2 c_{1,2,1} + 2txc_{2,1,1} + 3x^2 c_{3,0,1} + tc_{1,1,1} + 2xc_{2,0,1} + c_{1,0,1}\right),$$

$$n = t^2 x^3 c_{3,2,1} + t^2 x^2 c_{2,2,1} + tx^3 c_{3,1,1} + t^2 x c_{1,2,1} + tx^2 c_{2,1,1} + x^3 c_{3,0,1} + t^2 c_{0,2,1} + txc_{1,1,1} + x^2 c_{2,0,1} + tc_{0,1,1} + xc_{1,0,1} + c_{0,0,1}.$$

The second class:

$$u_2 = \frac{-m}{n} \tag{9}$$

there

$$\begin{aligned}
m = 6\big(&3x^2 y^2 c_{2,0,1}c_{3,0,2}^2 - 3x^2 y^2 c_{2,0,2}c_{3,0,1}c_{3,0,2} + 3x^2 y c_{2,0,1}c_{3,0,1}c_{3,0,2} \\
&- 3x^2 y c_{2,0,2}c_{3,0,1}^2 + 2xy^2 c_{2,0,1}c_{2,0,2}c_{3,0,2} - 2xy^2 c_{2,0,2}^2 c_{3,0,1} \\
&+ 3x^2 c_{2,0,1}c_{3,0,0}c_{3,0,2} - 3x^2 c_{2,0,2}c_{3,0,0}c_{3,0,1} + 2xyc_{2,0,1}^2 c_{3,0,2} \\
&- 2xyc_{2,0,1}c_{2,0,2}c_{3,0,1} + y^2 c_{1,0,2}c_{2,0,1}c_{3,0,2} - y^2 c_{1,0,2}c_{2,0,2}c_{3,0,1} \\
&+ 2xc_{2,0,0}c_{2,0,1}c_{3,0,2} - 2xc_{2,0,0}c_{2,0,2}c_{3,0,1} + yc_{1,0,1}c_{2,0,1}c_{3,0,2} \\
&- yc_{1,0,1}c_{2,0,2}c_{3,0,1} + c_{1,0,1}c_{2,0,0}c_{3,0,2} - c_{1,0,1}c_{2,0,2}c_{3,0,0} \\
&- c_{1,0,2}c_{2,0,0}c_{3,0,1} + c_{1,0,2}c_{2,0,1}c_{3,0,0}\big)c_{3,0,2}c_{3,1,2},
\end{aligned}$$

$$\begin{aligned}
n = &3x^3 y^2 c_{2,0,1}c_{3,0,2}^3 c_{3,1,2} - 3x^3 y^2 c_{2,0,2}c_{3,0,1}c_{3,0,2}^2 c_{3,1,2} + 3x^3 y c_{2,0,1}c_{3,0,1}c_{3,0,2}^2 c_{3,1,2} \\
&- 3x^3 y c_{2,0,2}c_{3,0,1}^2 c_{3,0,2}c_{3,1,2} + 3x^2 y^2 c_{2,0,1}c_{2,0,2}c_{3,0,2}^2 c_{3,1,2} \\
&- 3x^2 y^2 c_{2,0,2}^2 c_{3,0,1}c_{3,0,2}c_{3,1,2} + 36ty^2 c_{2,0,1}c_{3,0,2}^3 c_{3,1,2} \\
&- 36ty^2 c_{2,0,2}c_{3,0,1}c_{3,0,2}^2 c_{3,1,2} + 3x^3 c_{2,0,1}c_{3,0,0}c_{3,0,2}^2 c_{3,1,2} \\
&- 3x^3 c_{2,0,2}c_{3,0,0}c_{3,0,1}c_{3,0,2}c_{3,1,2} + 3x^2 y c_{2,0,1}^2 c_{3,0,2}^2 c_{3,1,2} \\
&- 3x^2 y c_{2,0,1}c_{2,0,2}c_{3,0,1}c_{3,0,2}c_{3,1,2} + 3xy^2 c_{1,0,2}c_{2,0,1}c_{3,0,2}^2 c_{3,1,2} \\
&- 3xy^2 c_{1,0,2}c_{2,0,2}c_{3,0,1}c_{3,0,2}c_{3,1,2} + 36tyc_{2,0,1}c_{3,0,1}c_{3,0,2}^2 c_{3,1,2} \\
&- 36tyc_{2,0,2}c_{3,0,1}^2 c_{3,0,2}c_{3,1,2} + 3x^2 c_{2,0,0}c_{2,0,1}c_{3,0,2}^2 c_{3,1,2} \\
&- 3x^2 c_{2,0,0}c_{2,0,2}c_{3,0,1}c_{3,0,2}c_{3,1,2} + 3xyc_{1,0,1}c_{2,0,1}c_{3,0,2}^2 c_{3,1,2} \\
&- 3xyc_{1,0,1}c_{2,0,2}c_{3,0,1}c_{3,0,2}c_{3,1,2} + 3y^2 c_{0,1,2}c_{2,0,1}c_{3,0,2}^3 \\
&- 3y^2 c_{0,1,2}c_{2,0,2}c_{3,0,1}c_{3,0,2}^2 - 36y^2 c_{2,0,1}c_{3,0,2}^4 + 36y^2 c_{2,0,2}c_{3,0,1}c_{3,0,2}^3 \\
&+ 36tc_{2,0,1}c_{3,0,0}c_{3,0,2}^2 c_{3,1,2} - 36tc_{2,0,2}c_{3,0,0}c_{3,0,1}c_{3,0,2}c_{3,1,2} + 3xc_{1,0,1}c_{2,0,0}c_{3,0,2}^2 \\
&- 3xc_{1,0,1}c_{2,0,2}c_{3,0,0}c_{3,0,2}c_{3,1,2} - 3xc_{1,0,2}c_{2,0,0}c_{3,0,1}c_{3,0,2}c_{3,1,2} \\
&+ 3xc_{1,0,2}c_{2,0,1}c_{3,0,0}c_{3,0,2}c_{3,1,2} + 3yc_{0,1,2}c_{2,0,1}c_{3,0,1}c_{3,0,2}^2 - 3yc_{0,1,2}c_{2,0,2}c_{3,0,1}^2 c_{3,0,2} \\
&+ yc_{1,0,1}c_{2,0,1}c_{2,0,2}c_{3,0,2}c_{3,1,2} - yc_{1,0,1}c_{2,0,2}^2 c_{3,0,1}c_{3,1,2} - yc_{1,0,2}c_{2,0,1}^2 c_{3,0,2}c_{3,1,2} \\
&+ yc_{1,0,2}c_{2,0,1}c_{2,0,2}c_{3,0,1}c_{3,1,2} - 36yc_{2,0,1}c_{3,0,1}c_{3,0,2}^3 + 36yc_{2,0,2}c_{3,0,1}^2 c_{3,0,2}^2 \\
&+ 3c_{0,1,2}c_{2,0,1}c_{3,0,0}c_{3,0,2}^2 - 3c_{0,1,2}c_{2,0,2}c_{3,0,0}c_{3,0,1}c_{3,0,2} + c_{1,0,1}c_{2,0,0}c_{2,0,2}c_{3,0,2}c_{3,1,2} \\
&- c_{1,0,1}c_{2,0,2}^2 c_{3,0,0}c_{3,1,2} - c_{1,0,2}c_{2,0,0}c_{2,0,1}c_{3,0,2}c_{3,1,2} + c_{1,0,2}c_{2,0,1}c_{2,0,2}c_{3,0,0}c_{3,1,2} \\
&- 36c_{2,0,1}c_{3,0,0}c_{3,0,2}^3 + 36c_{2,0,2}c_{3,0,0}c_{3,0,1}c_{3,0,2}^2.
\end{aligned}$$

The third class:

$$u_3 = \frac{-m}{n} \tag{10}$$

there

$$
\begin{aligned}
m = 2\big(& 2txy^2 c_{2,1,2}c_{3,0,2} + 3x^2 y^2 c_{3,0,2}^2 + 2txyc_{2,1,2}c_{3,0,1} + ty^2 c_{1,1,2}c_{3,0,2} + 3x^2 yc_{3,0,1}c_{3,0,2} \\
& + 2xy^2 c_{2,0,2}c_{3,0,2} + 2txc_{2,1,2}c_{3,0,0} + tyc_{1,1,2}c_{3,0,1} + 3x^2 c_{3,0,0}c_{3,0,2} + 2xyc_{2,0,2}c_{3,0,1} \\
& + y^2 c_{1,0,2}c_{3,0,2} + tc_{1,1,2}c_{3,0,0} + 2xc_{2,0,2}c_{3,0,0} + yc_{1,0,1}c_{3,0,2} + c_{1,0,0}c_{3,0,2} \big) c_{2,1,2}^3 c_{3,0,2},
\end{aligned}
$$

$$
\begin{aligned}
n = & \ tx^2 y^2 c_{2,1,2}^4 c_{3,0,2}^2 + x^3 y^2 c_{2,1,2}^3 c_{3,0,2}^3 + tx^2 yc_{2,1,2}^4 c_{3,0,1}c_{3,0,2} + txy^2 c_{1,1,2}c_{2,1,2}^3 c_{3,0,2}^2 \\
& + x^3 yc_{2,1,2}^3 c_{3,0,1}c_{3,0,2}^2 + x^2 y^2 c_{2,0,2}c_{2,1,2}^3 c_{3,0,2}^2 + tx^2 c_{2,1,2}^4 c_{3,0,0}c_{3,0,2} + txyc_{1,1,2}c_{2,1,2}^3 c_{3,0,1}c_{3,0,2} \\
& + ty^2 c_{1,0,2}c_{2,1,2}^4 c_{3,0,2} + ty^2 c_{1,1,2}^2 c_{2,1,2}^2 c_{3,0,2}^2 - ty^2 c_{1,1,2}c_{2,0,2}c_{2,1,2}^3 c_{3,0,2} + 12ty^2 c_{2,1,2}^3 c_{3,0,2}^3 \\
& + x^3 c_{2,1,2}^3 c_{3,0,0}c_{3,0,2}^2 + x^2 yc_{2,0,2}c_{2,1,2}^3 c_{3,0,1}c_{3,0,2} + x^2 c_{1,0,2}c_{2,1,2}^3 c_{3,0,2}^2 + txc_{1,1,2}c_{2,1,2}^3 c_{3,0,0}c_{3,0,2} \\
& + tyc_{1,0,1}c_{2,1,2}^4 c_{3,0,2} + tyc_{1,1,2}^2 c_{2,1,2}^2 c_{3,0,1}c_{3,0,2} - tyc_{1,1,2}c_{2,0,2}c_{2,1,2}^3 c_{3,0,1} + 12tyc_{2,1,2}^3 c_{3,0,1}c_{3,0,2}^2 \\
& + x^2 c_{2,0,2}c_{2,1,2}^3 c_{3,0,0}c_{3,0,2} + xyc_{1,0,1}c_{2,1,2}^3 c_{3,0,2}^2 - y^2 c_{1,0,2}c_{1,1,2}c_{2,1,2}^2 c_{3,0,2}^2 \\
& + y^2 c_{1,0,2}c_{2,0,2}c_{2,1,2}^3 c_{3,0,2} - y^2 c_{1,1,2}^3 c_{3,0,2}^3 + 2y^2 c_{1,1}^2 c_{2,0,2}c_{2,1,2}^2 c_{3,0,2}^2 - y^2 c_{1,1,2}c_{2,0,2}^2 c_{2,1,2}^2 c_{3,0,2} \\
& - 18y^2 c_{1,1,2}c_{2,1,2}c_{3,0,2}^4 + 12y^2 c_{2,0,2}c_{2,1,2}^2 c_{3,0,2}^3 + tc_{1,0,0}c_{2,1,2}^4 c_{3,0,2} + tc_{1,1,2}^2 c_{2,1,2}^2 c_{3,0,0}c_{3,0,2} \\
& - tc_{1,1,2}c_{2,0,2}c_{2,1,2}^3 c_{3,0,0} + 12tc_{2,1,2}^3 c_{3,0,0}c_{3,0,2}^2 + xc_{1,0,0}c_{2,1,2}^3 c_{3,0,2}^2 - yc_{1,0,1}c_{1,1,2}c_{2,1,2}^2 c_{3,0,2}^2 \\
& + yc_{1,0,1}c_{2,0,2}c_{2,1,2}^3 c_{3,0,2} - yc_{1,1,2}^3 c_{3,0,1}c_{3,0,2}^2 + 2yc_{1,1,2}^2 c_{2,0,2}c_{2,1,2}c_{3,0,1}c_{3,0,2} \\
& - yc_{1,1,2}c_{2,0,2}^2 c_{2,1,2}^2 c_{3,0,1} - 18yc_{1,1,2}c_{2,1,2}c_{3,0,1}c_{3,0,2}^3 + 12yc_{2,0,2}c_{2,1,2}^2 c_{3,0,1}c_{3,0,2}^2 \\
& - c_{1,0,0}c_{1,1,2}c_{2,1,2}^2 c_{3,0,2}^2 + c_{1,0,0}c_{2,0,2}c_{2,1,2}^3 c_{3,0,2} - c_{1,1,2}^3 c_{3,0,0}c_{3,0,2}^2 + 2c_{1,1,2}^2 c_{2,0,2}c_{2,1,2}c_{3,0,0}c_{3,0,2} \\
& - c_{1,1,2}c_{2,0,2}^2 c_{2,1,2}^2 c_{3,0,0} - 18c_{1,1,2}c_{2,1,2}c_{3,0,0}c_{3,0,2}^3 + 12c_{2,0,2}c_{2,1,2}^2 c_{3,0,0}c_{3,0,2}^2.
\end{aligned}
$$

The fourth class:

$$u_4 = \frac{-m}{n} \tag{11}$$

there

$$
\begin{aligned}
m = & \ 6\big(3x^2 yc_{3,0,1} + 2xyc_{2,0,1} + y^2 c_{1,0,2} + yc_{1,0,1} + c_{1,0,0}\big)c_{3,0,1}, \\
n = & \ 3x^3 yc_{3,0,1}^2 + 3x^2 yc_{2,0,1}c_{3,0,1} + 3xy^2 c_{1,0,2}c_{3,0,1} + 36tyc_{3,0,1}^2 + 3xyc_{1,0,1}c_{3,0,1} \\
& + y^2 c_{1,0,2}c_{2,0,1} + 3xc_{1,0,0}c_{3,0,1} + 3yc_{0,0,1}c_{3,0,1} + c_{1,0,0}c_{2,0,1}.
\end{aligned}
$$

The fifth class:

$$u_5 = \frac{-m}{n} \tag{12}$$

there

$$
\begin{aligned}
m = & \ 2\big(3x^2 c_{2,0,2}c_{3,0,1} + 2xyc_{2,0,2}^2 + 2xc_{2,0,1}c_{2,0,2} + yc_{1,0,2}c_{2,0,2} - 3c_{0,0,2}c_{3,0,1} + c_{1,0,2}c_{2,0,1}\big), \\
n = & \ x^3 c_{2,0,2}c_{3,0,1} + x^2 yc_{2,0,2}^2 + x^2 c_{2,0,1}c_{2,0,2} + xyc_{1,0,2}c_{2,0,2} + 12tc_{2,0,2}c_{3,0,1} \\
& - 3xc_{0,0,2}c_{3,0,1} + xc_{1,0,2}c_{2,0,1} + yc_{0,0,2}c_{2,0,2} + c_{0,0,1}c_{2,0,2}.
\end{aligned}
$$

The sixth class:

$$u_6 = \frac{-m}{n} \tag{13}$$

there

$$m = 2\left(c_{2,0,1}c_{3,1,1} - c_{2,1,1}c_{3,0,1}\right)c_{1,0,2}^2 \left(2txc_{1,0,2}c_{2,0,1}c_{3,0,1}c_{3,1,1} - 2txc_{1,0,2}c_{2,1,1}c_{3,0,1}^2\right.$$

$$- 3x^2c_{1,0,2}c_{3,0,1}^3 - tc_{0,0,2}c_{2,0,1}c_{3,0,1}c_{3,1,1} + tc_{0,0,2}c_{2,1,1}c_{3,0,1}^2 + tc_{1,0,2}c_{2,0,1}^2c_{3,1,1}$$

$$- tc_{1,0,2}c_{2,0,1}c_{2,1,1}c_{3,0,1} - 2xc_{1,0,2}c_{2,0,1}c_{3,0,1}^2 - yc_{1,0,2}^2c_{3,0,1}^2 - c_{1,0,1}c_{1,0,2}c_{3,0,1}^2\left.\right),$$

$$n = tx^2c_{1,0,2}^3c_{2,0,1}^2c_{3,0,1}c_{3,1,1}^2 - 2tx^2c_{1,0,2}^3c_{2,0,1}c_{2,1,1}c_{3,0,1}^2c_{3,1,1} + tx^2c_{1,0,2}^3c_{2,1,1}^2c_{3,0,1}^3$$

$$- x^3c_{1,0,2}^3c_{2,0,1}c_{3,0,1}^3c_{3,1,1} + x^3c_{1,0,2}^3c_{2,1,1}c_{3,0,1}^4 - txc_{0,0,2}c_{1,0,2}^2c_{2,0,1}^2c_{3,0,1}c_{3,1,1}$$

$$+ 2txc_{0,0,2}c_{1,0,2}^2c_{2,0,1}c_{2,1,1}c_{3,0,1}^2c_{3,1,1} - txc_{0,0,2}c_{1,0,2}^2c_{2,1,1}^2c_{3,0,1}^3 + txc_{1,0,2}^3c_{2,0,1}^3c_{3,1,1}^2$$

$$- 2txc_{1,0,2}^3c_{2,0,1}^2c_{2,1,1}c_{3,0,1}c_{3,1,1} + txc_{1,0,2}^3c_{2,0,1}c_{2,1,1}^2c_{3,0,1}^2 + tyc_{1,0,2}^4c_{2,0,1}^2c_{3,1,1}^2$$

$$- 2tyc_{1,0,2}^4c_{2,0,1}c_{2,1,1}c_{3,0,1}c_{3,1,1} + tyc_{1,0,2}^4c_{2,1,1}^2c_{3,0,1}^2 - x^2c_{1,0,2}^3c_{2,0,1}^2c_{3,0,1}^2c_{3,1,1}$$

$$+ x^2c_{1,0,2}^3c_{2,0,1}c_{2,1,1}c_{3,0,1}^3 - xyc_{1,0,2}^4c_{2,0,1}c_{3,0,1}^2c_{3,1,1} + xyc_{1,0,2}^4c_{2,1,1}c_{3,0,1}^3$$

$$+ tc_{0,0,2}^2c_{1,0,2}c_{2,0,1}^2c_{3,0,1}c_{3,1,1}^2 - 2tc_{0,0,2}^2c_{1,0,2}c_{2,0,1}c_{2,1,1}c_{3,0,1}^2c_{3,1,1} + tc_{0,0,2}^2c_{1,0,2}c_{2,1,1}^2c_{3,0,1}^3$$

$$- tc_{0,0,2}c_{1,0,2}^2c_{2,0,1}^3c_{3,1,1}^2 + 2tc_{0,0,2}c_{1,0,2}^2c_{2,0,1}^2c_{2,1,1}c_{3,0,1}c_{3,1,1} - tc_{0,0,2}c_{1,0,2}^2c_{2,0,1}c_{2,1,1}^2c_{3,0,1}^2$$

$$+ tc_{1,0,1}c_{1,0,2}^3c_{2,0,1}^2c_{3,1,1}^2 - 2tc_{1,0,1}c_{1,0,2}^3c_{2,0,1}c_{2,1,1}c_{3,0,1}c_{3,1,1} + tc_{1,0,1}c_{1,0,2}^3c_{2,1,1}^2c_{3,0,1}^2$$

$$- 12tc_{1,0,2}^3c_{2,0,1}c_{3,0,1}^3c_{3,1,1} + 12tc_{1,0,2}^3c_{2,1,1}c_{3,0,1}^4 - xc_{1,0,1}c_{1,0,2}^3c_{2,0,1}c_{3,0,1}^2c_{3,1,1}$$

$$+ xc_{1,0,1}c_{1,0,2}^3c_{2,1,1}c_{3,0,1}^3 - yc_{0,0,2}c_{1,0,2}^3c_{2,0,1}c_{3,0,1}^2c_{3,1,1} + yc_{0,0,2}c_{1,0,2}^3c_{2,1,1}c_{3,0,1}^3$$

$$- c_{0,0,2}^3c_{2,0,1}c_{3,0,1}^3c_{3,1,1} + c_{0,0,2}^3c_{2,1,1}c_{3,0,1}^4 + c_{0,0,2}^2c_{1,0,2}c_{2,0,1}^2c_{3,0,1}^2c_{3,1,1}$$

$$- c_{0,0,2}^2c_{1,0,2}c_{2,0,1}c_{2,1,1}c_{3,0,1}^3 - c_{0,0,2}c_{1,0,1}c_{1,0,2}^2c_{2,0,1}c_{3,0,1}^2c_{3,1,1} + c_{0,0,2}c_{1,0,1}c_{1,0,2}^2c_{2,1,1}c_{3,0,1}^3$$

$$+ 18c_{0,0,2}c_{1,0,2}^2c_{3,0,1}^5 - 6c_{1,0,2}^3c_{2,0,1}c_{3,0,1}^4.$$

The seventh class:

$$u_7 = \frac{m}{n} \tag{14}$$

there

$$m = 2\left(3x^2c_{3,0,1} + 2xc_{2,0,1} + c_{1,0,1}\right)y,$$

$$n = -x^3yc_{3,0,1} - x^2yc_{2,0,1} + 6tyc_{3,0,1} - xyc_{1,0,1} - y^2c_{0,0,2} - yc_{0,0,1} - c_{0,0,0}.$$

The eighth class:

$$u_8 = \frac{-m}{n} \tag{15}$$

there

$$m = 2\left(2t^2xc_{2,2,2} + 3tx^2c_{3,1,2} + t^2c_{1,2,2} + 2txc_{2,1,2} + tc_{1,1,2} + c_{1,0,2}\right),$$

$$n = t^2x^2c_{2,2,2} + tx^3c_{3,1,2} + t^2xc_{1,2,2} + tx^2c_{2,1,2}$$

$$+ t^2c_{0,2,2} + txc_{1,1,2} + tc_{0,1,2} + xc_{1,0,2} + c_{0,0,2}.$$

The ninth class:

$$u_9 = \frac{-m}{n} \tag{16}$$

there

$$m = 2\left(2t^2xc_{2,2,2} + 2txc_{2,1,2} + tc_{1,1,2} + 2xc_{2,0,2} + c_{1,0,2}\right),$$

$$n = t^2x^2c_{2,2,2} + tx^2c_{2,1,2} + t^2c_{0,2,2} + txc_{1,1,2} + x^2c_{2,0,2} + tc_{0,1,2} + xc_{1,0,2} + c_{0,0,2}.$$

The tenth class:

$$u_{10} = \frac{-m}{n} \tag{17}$$

there

$$
\begin{aligned}
m = 2\big(&3x^2 yc_{2,0,0}c_{3,0,1} + 2xy^2 c_{2,0,0}c_{2,0,2} + 2xyc_{2,0,0}c_{2,0,1} + y^2 c_{1,0,0}c_{2,0,2} \\
&+2xc_{2,0,0}^2 - 3yc_{0,0,0}c_{3,0,1} + yc_{1,0,0}c_{2,0,1} + c_{1,0,0}c_{2,0,0}\big),
\end{aligned}
$$

$$
\begin{aligned}
n = &x^3 yc_{2,0,0}c_{3,0,1} + x^2 y^2 c_{2,0,0}c_{2,0,2} + x^2 yc_{2,0,0}c_{2,0,1} + xy^2 c_{1,0,0}c_{2,0,2} \\
&+12tyc_{2,0,0}c_{3,0,1} + x^2 c_{2,0,0}^2 - 3xyc_{0,0,0}c_{3,0,1} + xyc_{1,0,0}c_{2,0,1} \\
&+ y^2 c_{0,0,0}c_{2,0,2} + xc_{1,0,0}c_{2,0,0} + yc_{0,0,1}c_{2,0,0} + c_{0,0,0}c_{2,0,0}.
\end{aligned}
$$

The eleventh class:

$$u_{11} = \frac{-m}{n} \tag{18}$$

there

$$
\begin{aligned}
m = 2\big(&6c_{1,2,1}t^2 yc_{3,0,1}c_{2,0,0}^2 + 8txyc_{0,0,0}c_{1,2,1}c_{2,0,0}c_{3,0,1} - 2txyc_{1,0,0}^2 c_{1,2,1}c_{3,0,1} \\
&+ 4ty^2 c_{0,0,0}c_{1,2,1}c_{2,0,0}c_{2,0,2} - ty^2 c_{1,0,0}^2 c_{1,2,1}c_{2,0,2} + 18c_{3,0,1}^2 x^2 yc_{2,0,0}^2 \\
&+ 12c_{2,0,2}xy^2 c_{3,0,1}c_{2,0,0}^2 + tyc_{0,0,0}c_{1,0,0}c_{1,2,1}c_{3,0,1} + 3tyc_{0,0,0}c_{1,2,1}c_{2,0,0}c_{2,0,1} \\
&- tyc_{1,0,0}^2 c_{1,2,1}c_{2,0,1} + 12c_{2,0,1}xyc_{3,0,1}c_{2,0,0}^2 + 6c_{1,0,0}c_{2,0,2}y^2 c_{3,0,1}c_{2,0,0} \\
&+ 4tc_{0,0,0}c_{1,2,1}c_{2,0,0}^2 - tc_{1,0,0}^2 c_{1,2,1}c_{2,0,0} + 12c_{2,0,0}^3 xc_{3,0,1} - 18yc_{0,0,0}c_{2,0,0}c_{3,0,1}^2 \\
&+ 6yc_{1,0,0}c_{2,0,0}c_{2,0,1}c_{3,0,1} + 6c_{1,0,0}c_{3,0,1}c_{2,0,0}^2\big)c_{3,0,1}c_{2,0,0},
\end{aligned}
$$

$$
\begin{aligned}
n = &-tc_{0,0,0}c_{1,2,1}c_{2,0,0}^3 c_{2,0,1} - tc_{1,0,0}^3 c_{1,2,1}c_{2,0,0}c_{3,0,1} - txc_{1,0,0}^2 c_{1,2,1}c_{2,0,0}^2 c_{3,0,1} \\
&- tyc_{0,0,0}c_{1,2,1}c_{2,0,0}^2 c_{2,0,1}^2 - tyc_{1,0,0}^3 c_{1,2,1}c_{2,0,1}c_{3,0,1} - ty^2 c_{1,0,0}^3 c_{1,2,1}c_{2,0,2}c_{3,0,1} \\
&+ 6c_{1,2,1}xt^2 yc_{3,0,1}^2 c_{2,0,0}^3 + 5tc_{0,0,0}c_{1,0,0}c_{1,2,1}c_{2,0,0}^2 c_{3,0,1} + 4txc_{0,0,0}c_{1,2,1}c_{2,0,0}^3 c_{3,0,1} \\
&+ 6xyc_{1,0,0}c_{2,0,0}^2 c_{2,0,1}c_{3,0,1}^2 - 16tyc_{0,0,0}^2 c_{1,2,1}c_{2,0,0}c_{3,0,1}^2 + 3tyc_{0,0,0}c_{1,0,0}^2 c_{1,2,1}c_{3,0,1}^2 \\
&- 6t^2 yc_{1,0,0}c_{1,2,1}c_{2,0,0}^2 c_{3,0,1}^2 + 6t^2 yc_{1,2,1}c_{2,0,0}^3 c_{2,0,1}c_{3,0,1} + 6c_{1,0,0}c_{2,0,2}xy^2 c_{3,0,1}^2 c_{2,0,0}^2 \\
&+ 6t^2 y^2 c_{1,2,1}c_{2,0,2}c_{3,0,1}c_{2,0,0}^3 - tx^2 yc_{1,0,0}^2 c_{1,2,1}c_{2,0,0}c_{3,0,1}^2 - ty^2 c_{0,0,0}c_{1,2,1}c_{2,0,0}^2 c_{2,0,1}c_{2,0,2} \\
&+ txyc_{0,0,0}c_{1,0,0}c_{1,2,1}c_{2,0,0}c_{3,0,1}^2 - txyc_{1,0,0}^2 c_{1,2,1}c_{2,0,0}c_{2,0,1}c_{3,0,1} - txy^2 c_{1,0,0}^2 c_{1,2,1}c_{2,0,0}c_{2,0,2}c_{3,0,1} \\
&+ 6tyc_{0,0,0}c_{1,0,0}c_{1,2,1}c_{2,0,0}c_{2,0,1}c_{3,0,1} + 3txyc_{0,0,0}c_{1,2,1}c_{2,0,0}^2 c_{2,0,1}c_{3,0,1} \\
&+ 5ty^2 c_{0,0,0}c_{1,0,0}c_{1,2,1}c_{2,0,0}c_{2,0,2}c_{3,0,1} + 4txy^2 c_{0,0,0}c_{1,2,1}c_{2,0,0}^2 c_{2,0,2}c_{3,0,1} \\
&+ 4tx^2 yc_{0,0,0}c_{1,2,1}c_{2,0,0}^2 c_{3,0,1}^2 - 18xyc_{0,0,0}c_{2,0,0}^2 c_{3,0,1}^3 + 6c_{0,0,0}c_{2,0,2}y^2 c_{3,0,1}^2 c_{2,0,0}^2 \\
&+ 6c_{2,0,1}x^2 yc_{3,0,1}^2 c_{2,0,0}^3 + 6c_{2,0,2}x^2 y^2 c_{3,0,1}^2 c_{2,0,0}^3 + 6c_{2,0,0}^4 x^2 c_{3,0,1}^2 + 6c_{0,0,0}c_{3,0,1}^2 c_{2,0,0}^3 \\
&+ 6c_{1,0,0}xc_{3,0,1}^2 c_{2,0,0}^3 + 6c_{3,0,1}^3 x^3 yc_{2,0,0}^3 + 72tyc_{2,0,0}^3 c_{3,0,1}^3 + 6t^2 c_{1,2,1}c_{2,0,0}^4 c_{3,0,1}^4,
\end{aligned}
$$

where

$$
\begin{aligned}
&16c_{0,0,0}^3 c_{3,0,1}^2 c_{2,0,0} - 3c_{0,0,0}^2 c_{1,0,0}^2 c_{3,0,1}^2 - 6c_{0,0,0}^2 c_{1,0,0}c_{2,0,1}c_{3,0,1}c_{2,0,0} + c_{0,0,0}^2 c_{2,0,0}^2 c_{2,0,1}^2 \\
&+ c_{0,0,0}c_{1,0,0}^3 c_{2,0,1}c_{3,0,1} + \big(6c_{0,0,0}c_{1,0,0}c_{2,0,0}c_{3,0,1} - 2c_{0,0,0}c_{2,0,0}^2 c_{2,0,1} - c_{1,0,0}^3 c_{3,0,1}\big)\_Z \\
&+ c_{2,0,0}^2 \_Z^2 = 0.
\end{aligned}
$$

The twelfth class:

$$u_{12} = \frac{-m}{n} \tag{19}$$

there

$$
\begin{aligned}
m = 2\big(& 6c_{1,2,1}t^2 c_{3,0,1}c_{2,0,2}^2 + 8txc_{0,0,2}c_{1,2,1}c_{2,0,2}c_{3,0,1} - 2txc_{1,0,2}^2 c_{1,2,1}c_{3,0,1} \\
& + 4tyc_{0,0,2}c_{1,2,1}c_{2,0,2}^2 - tyc_{1,0,2}^2 c_{1,2,1}c_{2,0,2} + 18c_{3,0,1}^2 x^2 c_{2,0,2}^2 + 12c_{2,0,2}^3 xyc_{3,0,1} \\
& + tc_{0,0,2}c_{1,0,2}c_{1,2,1}c_{3,0,1} + 3tc_{0,0,2}c_{1,2,1}c_{2,0,1}c_{2,0,2} - tc_{1,0,2}^2 c_{1,2,1}c_{2,0,1} \\
& + 12c_{2,0,1}xc_{3,0,1}c_{2,0,2}^2 + 6c_{1,0,2}yc_{3,0,1}c_{2,0,2}^2 - 18c_{0,0,2}c_{3,0,1}^2 c_{2,0,2} \\
& + 6c_{1,0,2}c_{2,0,1}c_{2,0,2}c_{3,0,1} \big)c_{3,0,1}c_{2,0,2},
\end{aligned}
$$

$$
\begin{aligned}
n = & \; 6c_{2,0,1}x^2 c_{3,0,1}^2 c_{2,0,2}^3 + 6c_{0,0,2}yc_{3,0,1}^2 c_{2,0,2}^3 + 6c_{2,0,2}^4 x^2 yc_{3,0,1}^2 - 18xc_{0,0,2}c_{2,0,2}^2 c_{3,0,1}^3 \\
& + 6c_{3,0,1}^3 x^3 c_{2,0,2}^3 + 72tc_{2,0,2}^3 c_{3,0,1}^3 + 6tc_{0,0,2}c_{1,0,2}c_{1,2,1}c_{2,0,1}c_{2,0,2}c_{3,0,1} \\
& + 3txc_{0,0,2}c_{1,2,1}c_{2,0,1}c_{2,0,2}^2 c_{3,0,1} + 5tyc_{0,0,2}c_{1,0,2}c_{1,2,1}c_{2,0,2}^2 c_{3,0,1} \\
& + 4txyc_{0,0,2}c_{1,2,1}c_{2,0,2}^3 c_{3,0,1} + txc_{0,0,2}c_{1,0,2}c_{1,2,1}c_{2,0,2}c_{3,0,1}^2 \\
& - txyc_{1,0,2}^2 c_{1,2,1}c_{2,0,2}^2 c_{3,0,1} - txc_{1,0,2}^2 c_{1,2,1}c_{2,0,1}c_{2,0,2}c_{3,0,1} + 6c_{1,2,1}c_{2,0,2}^4 t^2 yc_{3,0,1} \\
& + 6c_{1,2,1}xt^2 c_{3,0,1}^2 c_{2,0,2}^3 + 6c_{1,0,2}xyc_{3,0,1}^2 c_{2,0,2}^3 + 6xc_{1,0,2}c_{2,0,1}c_{2,0,2}^2 c_{3,0,1}^2 \\
& - 16tc_{0,0,2}^2 c_{1,2,1}c_{2,0,2}c_{3,0,1}^2 + 3tc_{0,0,2}c_{1,0,2}^2 c_{1,2,1}c_{3,0,1}^2 - 6t^2 c_{1,0,2}c_{1,2,1}c_{2,0,2}^2 c_{3,0,1}^2 \\
& + 6t^2 c_{1,2,1}c_{2,0,1}c_{2,0,2}^3 c_{3,0,1} - tc_{0,0,2}c_{1,2,1}c_{2,0,1}^2 c_{2,0,2}^2 - tc_{1,0,2}^3 c_{1,2,1}c_{2,0,1}c_{3,0,1} \\
& - tx^2 c_{1,0,2}^2 c_{1,2,1}c_{2,0,2}c_{3,0,1}^2 - tyc_{0,0,2}c_{1,2,1}c_{2,0,1}c_{2,0,2}^3 \\
& - tyc_{1,0,2}^3 c_{1,2,1}c_{2,0,2}c_{3,0,1} + 4tx^2 c_{0,0,2}c_{1,2,1}c_{2,0,2}^2 c_{3,0,1}^2,
\end{aligned}
$$

where

$$
\begin{aligned}
& 16c_{0,0,2}^3 c_{3,0,1}^2 c_{2,0,2} - 3c_{0,0,2}^2 c_{1,0,2}^2 c_{3,0,1}^2 - 6c_{0,0,2}^2 c_{1,0,2}c_{2,0,1}c_{3,0,1}c_{2,0,2} + c_{0,0,2}^2 c_{2,0,1}^2 c_{2,0,2}^2 \\
& + c_{0,0,2}c_{1,0,2}^3 c_{2,0,1}c_{3,0,1} + \big(6c_{0,0,2}c_{1,0,2}c_{2,0,2}c_{3,0,1} - 2c_{0,0,2}c_{2,0,1}c_{2,0,2}^2 - c_{1,0,2}^3 c_{3,0,1}\big)\_Z \\
& + c_{2,0,2}^2 \_Z^2 = 0.
\end{aligned}
$$

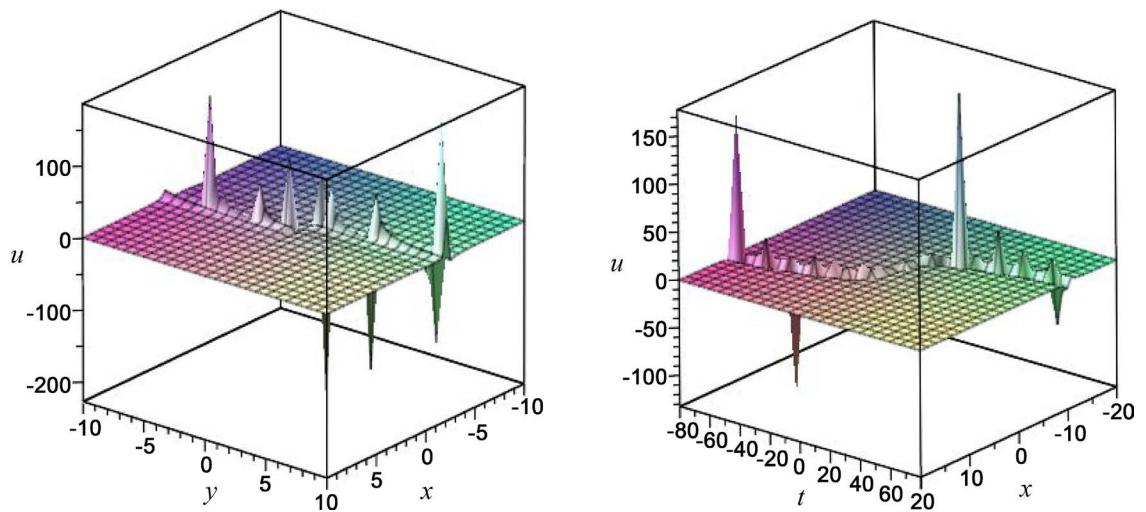The rational solutions in Equations (8)-(11) are depicted in Figure 1 & Figure 2, respectively.

Figure 1. Pictures of (8) with $y = 0$:3d plot (left) and Pictures of (9) with $y = 0$:3d plot (right).

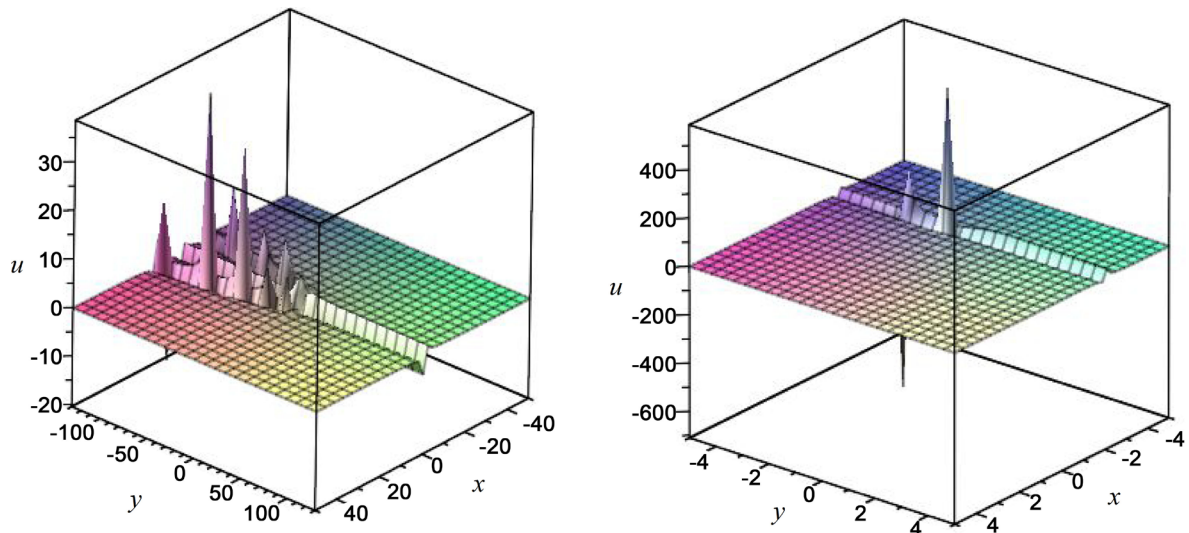**Figure 2.** Pictures of (10) with $t = 0$:3d plot (left) and Pictures of (11) with $t = 1$:3d plot (right).

3. Generalized Bilinear D_p-Operators and the (2 + 1)-Dimensions BLMP-Like Equation

3.1. The (2 + 1)-Dimensions BLMP-Like Equation

Consider a generalized bilinear differential equation of (2 + 1)-dimensions BLMP type:

$$\left(D_{3,y}D_{3,t} + D_{3,x}^3 D_{3,y} \right) F \cdot F = 0. \tag{20}$$

Which has the same bilinear form as the BLMP equation. The above differential operators are a kind of generalized bilinear differential operators that are put forward by professor Ma in [11].

The formula reads.

$$
\begin{aligned}
D_{p,x}^m D_{p,t}^n F \cdot F &= \left(\frac{\partial}{\partial x} + \alpha_p \frac{\partial}{\partial x'} \right)^m \left(\frac{\partial}{\partial t} + \alpha_p \frac{\partial}{\partial t'} \right)^n F(x,t) F(x',t') \big|_{x'=x,t'=t} \\
&= \sum_{i=0}^{m} \sum_{j=0}^{n} \binom{m}{i}\binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m-i}}{\partial x^{m-i}} \frac{\partial^i}{\partial x'^i} \frac{\partial^{n-j}}{\partial t^{n-j}} \frac{\partial^j}{\partial t'^j} F(x,t) F(x',t') \big|_{x'=x,t'=t} \quad (21) \\
&= \sum_{i=0}^{m} \sum_{j=0}^{n} \binom{m}{i}\binom{n}{j} \alpha_p^i \alpha_p^j \frac{\partial^{m+n-i-j} F(x,t)}{\partial x^{m-i} \partial t^{n-j}} \frac{\partial^{i+j} F(x,t)}{\partial x^i \partial t^j}, \quad m,n \geq 0,
\end{aligned}
$$

where α_p^s meet the following formula,

$$\alpha_p^s = (-1)^{r_p(s)}, \quad s = r_p(s) \bmod p. \tag{22}$$

The prime number associated with this kind of generalized bilinear differential operators Equation (20) is $p = 3$.

When $p = 3$, Equation (22) gives rise to:

$$\alpha_3 = -1, \ \alpha_3^2 = 1, \ \alpha_3^3 = 1, \ \alpha_3^4 = -1, \ \alpha_3^5 = 1, \ \alpha_3^6 = 1, \ \alpha_3^7 = 1, \ \cdots \tag{23}$$

Substituting Equation (23) into Equation (21) yields

$$
\begin{aligned}
D_{3,y}D_{3,t} F \cdot F &= 2F_{y,t}F - 2F_y F_t, \\
D_{3,x}^3 D_{3,y} F \cdot F &= -6F_{xxy}F_x + 6F_{xx}F_{xy}.
\end{aligned} \tag{24}
$$

Then the generalized bilinear type to the (2 + 1)-dimensional BLMP-like equation as follows,

$$\left(D_{3,y}D_{3,t}+D_{3,x}^3 D_{3,y}\right)F\cdot F = 2F_{y,t}F - 2F_y F_t - 6F_{xxy}F_x + 6F_{xx}F_{xy}. \quad (25)$$

Through the transformation $u = 2(\ln F)_x$, Equation (25) can be reduced to the (2 + 1)-dimensional BLMP-like nonlinear differential equation

$$\frac{\left(D_{3,y}D_{3,t}+D_{3,x}^3 D_{3,y}\right)F\cdot F}{F^2} = u_{yt}+\frac{3}{2}u_{xx}u_y-\frac{3}{2}uu_{xxy}-\frac{3}{4}u^2u_{xy}-\frac{3}{2}uu_xu_y \quad (26)$$
$$= 0.$$

3.2. The Rational Solutions to the (2 + 1)-Dimensional BLMP-Like Equation

Apply the maple symbolic computing Equation (7), twelve classes of rational solutions to the (2 + 1)-dimensional BLMP-like equation Equation (26) as follows:

The first class of rational solutions to Equation (26):

$$u_1 = \frac{m}{n} \quad (27)$$

there

$$m = 2\left(3x^2 c_{3,0,2}+2xc_{2,0,2}+c_{1,0,2}\right),$$
$$n = y^2 x^3 y^2 c_{3,0,2}+x^2 y^2 c_{2,0,2}+xy^2 c_{1,0,2}+y^2 c_{0,0,2}+yc_{0,0,1}+c_{0,0,0}.$$

The second class:

$$u_2 = \frac{m}{n} \quad (28)$$

there

$$m = 2\left(3xy^2 c_{3,0,2}+2y^2 c_{2,0,2}+2yc_{2,0,1}+2c_{2,0,0}\right)x,$$
$$n = x^3 y^2 c_{3,0,2}+x^2 y^2 c_{2,0,2}+18ty^2 c_{3,0,2}+x^2 yc_{2,0,1}+x^2 c_{2,0,0}+y^2 c_{0,0,2}.$$

The third class:

$$u_3 = \frac{m}{n} \quad (29)$$

there

$$m = 2\big(3xy^2 c_{3,0,0}c_{3,0,2}+3xyc_{3,0,0}c_{3,0,1}+2y^2 c_{2,0,0}c_{3,0,2}$$
$$+3xc_{3,0,0}^2+2yc_{2,0,0}c_{3,0,1}+2c_{2,0,0}c_{3,0,0}\big)x,$$
$$n = x^3 y^2 c_{3,0,0}c_{3,0,2}+x^3 yc_{3,0,0}c_{3,0,1}+x^2 y^2 c_{2,0,0}c_{3,0,2}+x^3 c_{3,0,0}^2$$
$$+x^2 yc_{2,0,0}c_{3,0,1}+x^2 c_{2,0,0}c_{3,0,0}+y^2 c_{0,0,2}c_{3,0,0}.$$

The forth class:

$$u_4 = \frac{m}{n} \quad (30)$$

there

$$m = 2\left(3t^2x^2c_{3,2,2} + 2t^2xc_{2,2,2} + 3tx^2c_{3,1,2} + t^2c_{1,2,2}\right.$$
$$\left. + 2txc_{2,1,2} + 3x^2c_{3,0,2} + 2xc_{2,0,2} + c_{1,0,2}\right),$$
$$n = t^2x^3c_{3,2,2} + t^2x^2c_{2,2,2} + tx^3c_{3,1,2} + t^2xc_{1,2,2} + tx^2c_{2,1,2}$$
$$ + x^3c_{3,0,2} + t^2c_{0,2,2} + x^2c_{2,0,2} + xc_{1,0,2} + c_{0,0,2}.$$

The fifth class:

$$u_5 = \frac{m}{n} \tag{31}$$

there

$$m = 2\left(3x^2yc_{1,2,1}c_{3,2,0} + 3x^2c_{1,2,0}c_{3,2,0} + 2xyc_{1,2,1}c_{2,2,0}\right.$$
$$\left. + 2xc_{1,2,0}c_{2,2,0} + yc_{1,2,0}c_{1,2,1} + c_{1,2,0}^2\right),$$
$$n = x^3yc_{1,2,1}c_{3,2,0} + x^3c_{1,2,0}c_{3,2,0} + x^2yc_{1,2,1}c_{2,2,0} + x^2c_{1,2,0}c_{2,2,0}$$
$$ + xyc_{1,2,0}c_{1,2,1} + y^2c_{0,2,2}c_{1,2,0} + xc_{1,2,0}{}^2 + yc_{0,2,1}c_{1,2,0} + c_{0,2,0}c_{1,2,0}.$$

The sixth class:

$$u_6 = \frac{m}{n} \tag{32}$$

there

$$m = 2\left(3t^2xc_{3,2,0} + 2t^2c_{2,2,0} + 3txc_{3,1,0} + 2tc_{2,1,0} + 3xc_{3,0,0}\right)x,$$
$$n = t^2x^3c_{3,2,0} + t^2x^2c_{2,2,0} + tx^3c_{3,1,0} + tx^2c_{2,1,0} + x^3c_{3,0,0} + t^2c_{0,2,0}.$$

The seventh class:

$$u_7 = \frac{m}{n} \tag{33}$$

there

$$m = 2x\left(3xy^2c_{3,1,2} + 3xyc_{3,1,1} + 2y^2c_{2,1,2} + 3xc_{3,1,0} + 2yc_{2,1,1} + 2c_{2,1,0}\right),$$
$$n = x^3y^2c_{3,1,2} + x^3yc_{3,1,1} + x^2y^2c_{2,1,2} + 18ty^2c_{3,1,2} + x^3c_{3,1,0}$$
$$ + x^2yc_{2,1,1} + 18tyc_{3,1,1} + x^2c_{2,1,0} + 18tc_{3,1,0}.$$

The eighth class:

$$u_8 = \frac{m}{n} \tag{34}$$

there

$$m = 2y\left(3x^2c_{3,2,1} + 2xc_{2,2,1} + c_{1,2,1}\right),$$
$$n = x^3yc_{3,2,1} + x^2yc_{2,2,1} + xyc_{1,2,1} + y^2c_{0,2,2} + yc_{0,2,1} + c_{0,2,0}.$$

The ninth class:

$$u_9 = \frac{m}{n} \tag{35}$$

there

$$m = 2x\left(3xc_{3,2,0} + 2c_{2,2,0}\right),$$
$$n = x^3c_{3,2,0} + x^2c_{2,2,0} + y^2c_{0,2,2} + yc_{0,2,1} + c_{0,2,0}.$$

The tenth class:

$$u_{10} = \frac{m}{n} \tag{36}$$

there

$$m = 2\left(3t^2 x c_{0,2,1} c_{3,0,1} + 2t^2 c_{0,0,1} c_{2,2,1} + 2tc_{0,0,1} c_{2,1,1} + 3xc_{0,0,1} c_{3,0,1} + 2c_{0,0,1} c_{2,0,1}\right)x,$$
$$n = t^2 x^3 c_{0,2,1} c_{3,0,1} + t^2 x^2 c_{0,0,1} c_{2,2,1} + tx^2 c_{0,0,1} c_{2,1,1} + x^3 c_{0,0,1} c_{3,0,1}$$
$$+ t^2 c_{0,0,1} c_{0,2,1} + x^2 c_{0,0,1} c_{2,0,1} + c_{0,0,1}^2.$$

The eleventh class:

$$u_{11} = \frac{m}{n} \tag{37}$$

there

$$m = 4\left(yc_{2,0,2} + c_{2,0,1}\right)x,$$
$$n = x^2 yc_{2,0,2} + x^2 c_{2,0,1} + yc_{0,0,2} + c_{0,0,1}.$$

The twentieth class:

$$u_{12} = \frac{m}{n} \tag{38}$$

there

$$m = 2\left(3xyc_{3,0,1} + 2y^2 c_{2,0,2} + 2yc_{2,0,1} + 2c_{2,0,0}\right)x,$$
$$n = x^3 yc_{3,0,1} + x^2 y^2 c_{2,0,2} + x^2 yc_{2,0,1} + 18tyc_{3,0,1} + x^2 c_{2,0,0} + yc_{0,0,1}.$$

The solutions in Equations ((27), (30), 31 and (34)) are depicted in **Figure 3** & **Figure 4**, respectively.

4. The N-Soliton Solutions of the (2 + 1)-Dimensional BLMP Equation

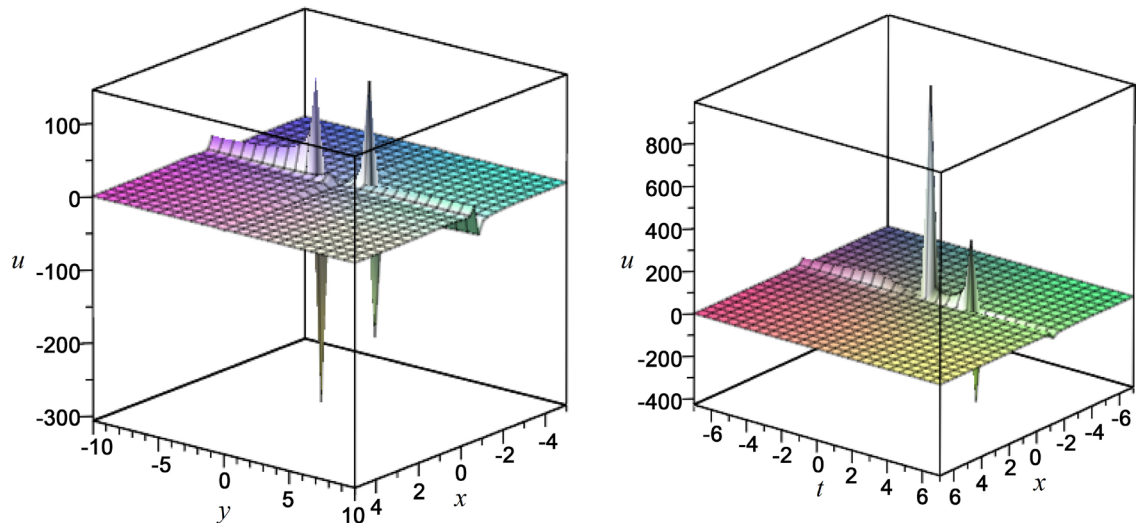Consider the N-soliton solutions of Equation (4) by using the perturbation ap-

Figure 3. Pictures of (27) with $t = 1$:3d plot (left) and Pictures of (30) with $y = 1$:3d plot (right).
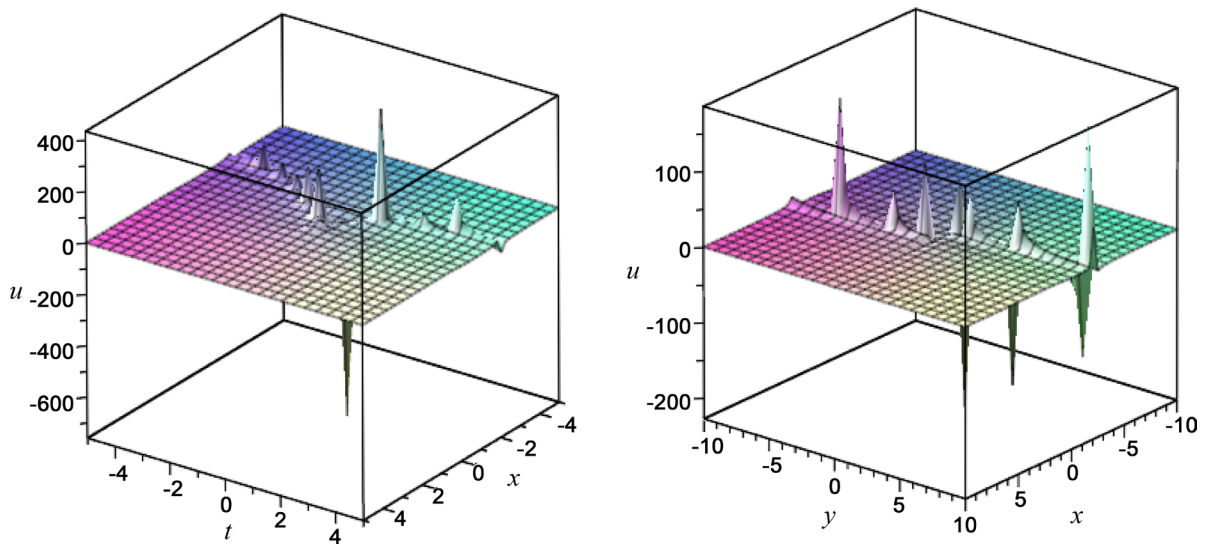
Figure 4. Pictures of (31) with $t = 1$:3d plot (left) and Pictures of (34) with $y = 1$:3d plot (right).

proach and the Hirota direct method. Equation (6) is the bilinear form of Equation (4) by using transformation $u = -2(\ln F)_x$. Expand the F into small parameter exponential as follows

$$F = 1 + \varepsilon f_1 + \varepsilon^2 f_2 + \varepsilon^3 f_3 + \cdots, \tag{39}$$

and $\lambda = 0$.

Substituting Equation (39) into Equation (6) and comparing the coefficients of the same powers of ε give rise to

$$F = \sum_{\mu_i, \mu_j = 0,1} \exp\left\{ \sum_{i>j}^{N} A_{ij} \mu_i \mu_j + \sum_{j=1}^{N} \mu_j \mu_j \right\}, \tag{40}$$

where $\eta_j = k_j x + l_j y + w_j t$, $e^{A_{ij}} = \dfrac{2(l_j - l_i)\left[w_j - w_i + (k_i - kj)_3 \right]}{2(l_i + l_j)\left[w_i + w_j + (k_i + k_j)_3 \right]}$.

When $\varepsilon = 1$, the one-soliton solution and two-soliton solution as follows.

1) One-soliton solution

$F = 1 + f_1, f_1 = e^{\eta}, \eta = kx + ly + wt$, where the coefficient k, l, w satisfy with $lw + k^3 l = 0$, the one-soliton solution is

$$u = 2(\ln F)_{xx} = \frac{k^2}{2} \operatorname{sech}^2 \frac{\eta}{2}. \tag{41}$$

2) Two-soliton solution

$F = 1 + f_1 + f_2, f_1 = e^{\eta_1} + e^{\eta_2}, f_2 = e^{\eta_1 + \eta_2 + A_{12}}$, where

$$\eta_1 = k_1 x + l_1 y + w_1 t, \quad \eta_2 = k_2 x + l_2 y + w_2 t, \quad e^{A_{12}} = \frac{2(l_2 - l_1)\left[w_1 - w_2 + (k_1 - k_2)^3 \right]}{2(l_1 + l_2)\left[w_1 + w_2 + (k_1 + k_2)^3 \right]}.$$

There, coefficients $k_i, l_i, w_i \ (i = 1, 2)$ satisfy with $l_i w_i + k_i^3 l_i = 0$, the two-soliton solution is

$$u = 2(\ln F_{xx}) = 2\left[\ln(1 + f_1 + f_2) \right]_{xx}. \tag{42}$$
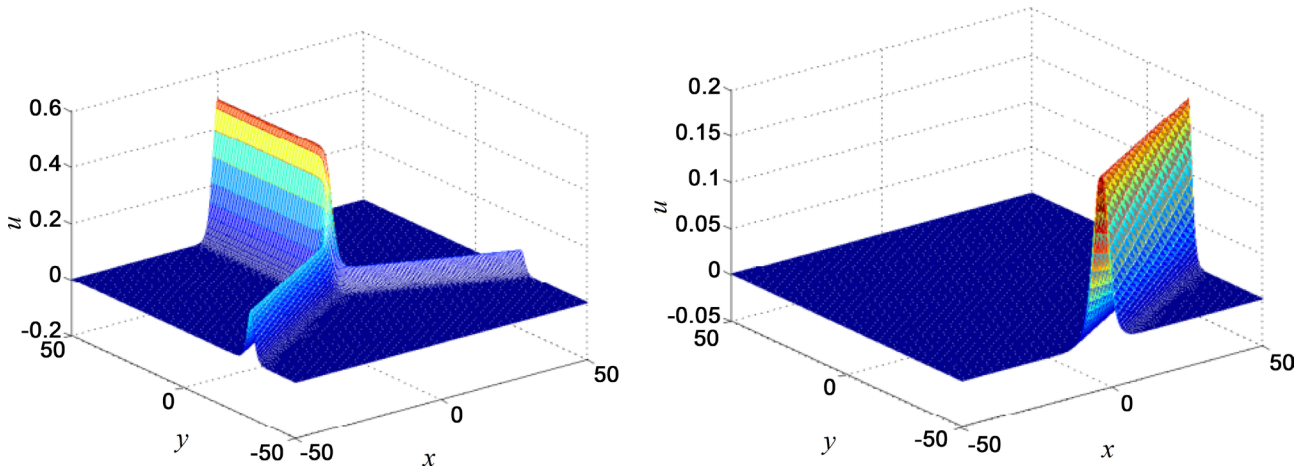
Figure 5. The resonance solution to Equation (4) with the parameters $k_1 = 0.4, k_2 = -0.6, l_1 = l_2 = 1$, (left) indicates $t = 0$, and (right) indicates $t = 220$.
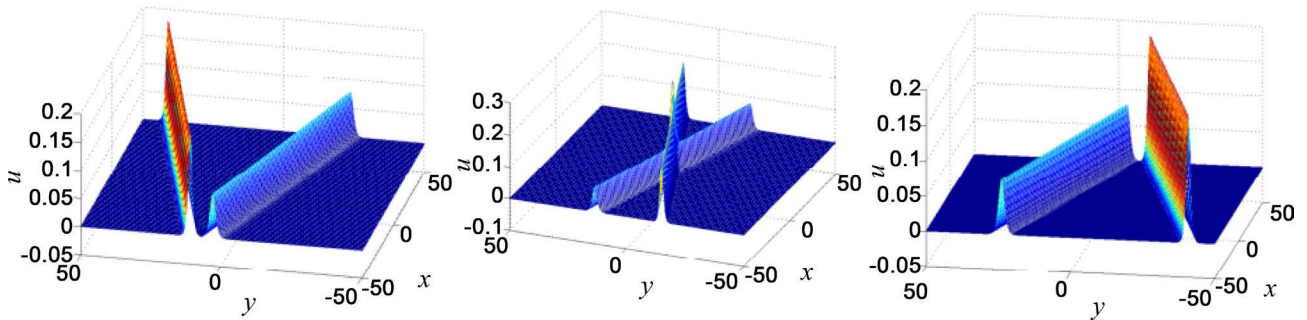

Figure 6. The resonance solution to Equation (42) with the parameters $k_1 = 0.4$, $k_2 = -0.6$, $l_1 = 1.5$, $l_2 = 2$, (left) indicates $t = -240$, (middle) indicates $t = 0$ and (right) indicates $t = 240$.

When the coefficient $e^{A_{ij}} = 0$, the soliton solution is changed into the resonance solution. Its propagation will be showed in **Figure 5**.

Let the coefficient $e^{A_{ij}} \neq 0$, the two-soliton happen in the collision because of the different soliton speed. The pursue collision between the two soliton will be showed in **Figure 6**.

5. Conclusion

With the help of the generalized bilinear methods [9] [11] [12] [15], we present the bilinear form to the (2 + 1)-dimensional BLMP equation and the (2 + 1)-dimensional BLMP-like equation. Afterwards, the twelve classes of rational solutions are obtained respectively. The 3d graphics to the rational solutions shows some properties about these rational solutions, such as symmetry. These rational solutions which can be described as a kind of algebraic solitary waves have great potential in applied value in atmosphere and ocean. Every solitary wave velocity is different (the faster speed wave will overtake the slower speed wave), so it comes to a conclusion that these solutions are changed into the resonance solution when the coefficient $e^{A_{ij}} = 0$, and the soliton collision will happen when the coefficient $e^{A_{ij}} \neq 0$. After the collision, the two solitons will continue to spread as the pre-

vious speed and the direction. The generalized bilinear method could be applied to more high dimensional equation to obtain its rational solutions and deduce new equation. This method also could be applied to the half a discrete nonlinear differential equation and discrete nonlinear differential equation. Continuing to study this generalized bilinear method in depth is meaningful and interesting.

Acknowledgements

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