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Scalar Particles' Tunneling and Effect of Quantum Gravity

Guoping Li, Xiaotao Zu

School of Physical Electronics, University of Electronic Science and Technology of China, Chengdu, China
Email: li_gp2009@163.com

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Abstract

According to the generalized uncertainty principle (GUP), the Klein-Gordon equation is corrected by the quantum gravity exactly. Hence, the corrected Klein-Gordon equation will be more precise on the expression of the tunneling behavior. Then, the corrected Hawking temperature of the Gibbons-Maeda-Dilaton black hole is obtained near the horizon by quantum gravity. Analyzing the results carefully, it is obvious for us that the tunneling result is not only related to the mass of black hole, but also related to the mass and energy of outgoing fermions. Finally, we also infer that the tunneling radiation would be stopped at some particular temperature.

Keywords

The Quantum Gravity, The Gibbons-Maeda-Dilaton Black Hole, The Corrected Klein-Gordon Equation, The Generalized Uncertainty Principle

1. Introduction

Hawking radiation can be regard as the tunneling process which the vacuum fluctuation caused at the event horizon of black holes [1] [2]. Then, more and more attentions of theoretical physicists are focused on the Hawking radiation. Since 1976, T Dmour and Sannan developed Hawking radiation greatly with the relativistic quantum mechanics and the quantum field theory. In this way, a lot of work had been done by Zhao *et al.* [3]. Later, a semi-classical quantum tunneling method was proposed to study the Hawking radiation of black holes by Parikh *et al.* [4] [5]. After that, the process of calculation on the Hawking temperature has been greatly simplified, and then many effects take into account. The centre of this method can be regard as two parts; one is the expression of the wave function $\psi = \exp\left[\frac{i}{\hbar} I\left(t, r, \theta, \varphi\right)\right]$, and the other is the WKB approximation. Later, the dynamical black hole, the de Sitter black hole and the higher dimensional black hole, have been studied by using this method, and some important similar results were obtained. In 2007, Kerner and Mann developed this method once more [6]. Then, the tunneling method can be applied to study the dynamics behavior of spin 1/2 particles, and the Hawking temperature of 1/2-spin particles was also obtained. After that, many effects work on it and important results are obtained [7]-[12]. Now, the quantum tunneling theory has been the one of most popular theories on the calculation of Hawking temperature in black holes.

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Recently, the quantum gravity theory came into a period of rapid development. For example, the most important symbol is the appearance of the supergravity theory and loop quantum gravity theory. Obviously, the best application model of the quantum gravity is black hole model. More and more evidences imply that the generalized uncertainty principle (GUP) can be modified by the modified fundamental commutation relation; therefore the momentum operator will be corrected with it. Finally, the dynamics equation of particles in black holes can be modified by the quantum gravity, and the Hawking radiation is corrected. Also, many other various modifications can be found in [13]. Through the quantum tunneling method and the GUP, the tunneling behavior of the scalar particle of Schwarzschild black hole has been studied by K Nozari [14]. And many other studies of the tunneling behavior have been discussed in [15]-[18]. Lots of evidences indicate that the quantum gravity research has the important correction on the Hawking radiation.

The aim of this paper is to study the tunneling radiation of scalar particles in the Gibbons-Maeda-Dilaton black hole with the Klein-Gordon equation near the horizon. With the careful calculation, the corrected tunneling rate and corrected Hawking temperature are obtained. The rest paper proceeds as follows: Section 2 introduces the modified Klein-Gordon equation; Section 3 studies the Hawking radiation of scalar particles in the Gibbons-Maeda-Dilaton black hole with the Klein-Gordon equation; Section 4 calculates the residuum of black hole; Section 5 is only a conclusion.

2. The Corrected Klein-Gordon Equation

In this section, we will discuss the modified Klein-Gordon equation by the quantum gravity. More and more evidences indicate that the generalized uncertainty principle (GUP) can describe the minimum measurable length [19]-[23]. Based on the modified fundamental commutation relation [13]

$$\left[x_i, p_j\right] = i\hbar\delta_{ij}\left[1+\beta p^2\right] \tag{1}$$

The expression of GUP can be express as [17]

$$\Delta x\Delta p \geq \frac{\hbar}{2}\left[1+\beta\left(\Delta p\right)^2\right] \tag{2}$$

where, M_p is the Planck mass, $\beta = \beta_0 / M_P^2$, β_0 is a dimensionless parameter and $\beta_0 \leq 10^{34}$, x_i and p_i can be found in the reference [17],

$$x_i = x_{0i}, \quad p_i = p_{0i}\left(1+\beta p^2\right) \tag{3}$$

$$p^2 = p_i p^i \simeq -\hbar^2\left[\partial_i\partial^i - 2\beta\hbar^2\left(\partial_j\partial^j\right)\left(\partial^i\partial_i\right)\right] \tag{4}$$

The canonical commutation relations express as $\left[x_{0\mu}, p_{0\nu}\right] = i\hbar\delta_{\mu\nu}$ should be satisfied. The Klein-Gordon equation without the electromagnetic field is given by the following form [24],

$$-P_\mu P^\mu = m^2 \tag{5}$$

To studied the effect which the quantum gravity have on the Klein-Gordon equation, we expand the Klein-Gordon equation as two parts, so we rewrite this equation as,

$$-\left(i\hbar\right)^2\partial^t\partial_t = \left(i\hbar\right)^2\partial^i\partial_i + m^2 \tag{6}$$

In above equation, the left hand is related to the square of energy, and the right hand is related to the square of coordinate. In reference [17], considering the mass-energy shell condition $E^2 = m^2 + P^2$, the generalized expression of energy is,

$$\overline{E} = E\left[1-\beta\left(p^2+m^2\right)\right] \tag{7}$$

Therefore, after we substituted Equations (3), (4) and (7) into Klein-Gordon equation, the modified Klein-Gordon equation are given as [24],

$$-\left(i\hbar\right)^2\partial^t\partial_t\psi = \left[\left(-i\hbar\right)^2\partial^i\partial_i + m^2\right]\left\{1-2\beta\left[\left(-i\hbar\right)^2\partial^i\partial_i + m^2\right]\right\}\psi \tag{8}$$

The modified Klein-Gordon equation tells us that the quantum gravity has an important influence on the dynamic equation of scalar particles. In the following section, we will focus on the tunneling behavior of scalar particles of the Gibbons-Maeda-Dilaton black hole with the corrected Klein-Gordon equation.

3. The Tunneling Radiation of the Gibbons-Maeda-Dilaton Black Hole

In this section, we are devoted to study tunneling radiation of scalar particles of the Gibbons-Maeda-Dilaton black hole by using modified Klein-Gordon equation. In 1991, Garfinkle D. *et al.* obtained the Gibbons-Maeda-Dilaton black hole solution, and the metric is,

$$\begin{aligned} \mathrm{d}s^2 &= -\frac{(r-r_+)(r-r_-)}{R^2}\mathrm{d}t^2 + \frac{R^2}{(r-r_+)(r-r_-)}\mathrm{d}r^2 + R^2\left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2\right) \\ &= -f(r)\mathrm{d}t^2 + \frac{1}{g(r)}\mathrm{d}r^2 + g_{\theta\theta}\mathrm{d}\theta^2 + g_{\varphi\varphi}\mathrm{d}\varphi^2. \end{aligned} \tag{9}$$

where, M, Q, P are the mass, the charge and magnetic charge of the black hole respectively. The symbol $D = \left(P^2 - Q^2\right)/2M$, $R^2 = r^2 - D^2$. D denotes the Dilaton charge of the spacetime. The outer and inner horizon can be expressed as,

$$r_\pm = M \pm \sqrt{M^2 + D^2 - P^2 - Q^2}, \quad A_+ = 4\pi\left(r_+^2 - D^2\right) \tag{10}$$

A_+ is the horizon area. And if the condition $P = Q = 0$ satisfied, the Gibbons-Maeda-Dilaton black hole will degenerate to the Schwarzschild spacetime. Then, we will investigate the tunneling radiation of the Gibbons-Maeda-Dilaton black hole near the event horizon with the modified Klein-Gordon equation. Employing the wave function of the scalar particle as,

$$\psi = \exp\left[\frac{i}{\hbar} I(t, r, \theta, \varphi)\right] \tag{11}$$

where I is the function of (t, r, θ, φ). Substituting the wave function ψ into the modified Klein-Gordon equation,

$$-(i\hbar)^2 \partial^t \partial_t \psi = \left[(-i\hbar)^2 \partial^i \partial_i + m^2\right]\left\{1 - 2\beta\left[(-i\hbar)^2 \partial^i \partial_i + m^2\right]\right\}\psi \tag{12}$$

The equation of motion of scalar particles is obtained,

$$\begin{aligned} &\frac{1}{f(r)}(\partial_t I)^2 \\ &= \left[h(r)(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\varphi\varphi}(\partial_\varphi I)^2 + m^2\right]\left\{1 - 2\beta\left[g(r)(\partial_r I)^2 + g^{\theta\theta}(\partial_\theta I)^2 + g^{\varphi\varphi}(\partial_\varphi I)^2 + m^2\right]\right\}. \end{aligned} \tag{13}$$

In above equation, the higher order terms of $\hbar$ in above equation were neglected. Then, the standard separation of variables are taken into account [24],

$$I = -\omega t + w(r, \theta) + j\varphi \tag{14}$$

Here, ω is the energy of outgoing scalar particles. j is the angular momentum of the emitted particle, and our concern is the radial component $w(r)$. It is worth to note that $w(r, \theta)$ cannot be separated as $w(r)\Theta(\theta)$. For the convenience, we fix the angle θ at a certain value of θ_0. Substituting Equation (14) into Equation (13),

$$\frac{1}{f(r)}\omega^2 = \left[g(r)(\partial_r w)^2 + g^{\theta\theta}(\partial_\theta w)^2 + g^{\varphi\varphi} j^2 + m^2\right]\left\{1 - 2\beta\left[g(r)(\partial_r w)^2 + g^{\theta\theta}(\partial_\theta w)^2 + g^{\varphi\varphi} j^2 + m^2\right]\right\} \tag{15}$$

Therefore, the Equation (15) can be simplified as,

$$\mathbf{A}(\partial_r w)^4 + \mathbf{B}(\partial_r w)^2 + \mathbf{C} = 0 \tag{16}$$

where,

$$\mathbf{A} = -2\beta g(r)^2 \tag{17}$$

$$\mathbf{B} = g(r)\left(1 - 4\beta g^{\varphi\varphi} j^2 - 4\beta m^2\right) \tag{18}$$

$$\mathbf{C} = m^2 + g^{\varphi\varphi} j^2 - 2\beta\left(g^{\varphi\varphi}\right)^2 j^4 - 4g^{\varphi\varphi} j^2 \beta m^2 - 2\beta m^4 - \frac{\omega^2}{f(r)} \tag{19}$$

Then, we considered the condition $\mathbf{B}^2 - 4\mathbf{AC} > 0$. So,

$$\partial_r w_\pm = \pm\sqrt{\frac{-\mathbf{B} + \sqrt{\mathbf{B}^2 - 4\mathbf{AC}}}{2\mathbf{A}}} \tag{20}$$

After we substituted Equations (9) and (10) into Equation (20), so the solution of this quartic equation at the horizon is,

$$w_\pm = \pm\int \mathrm{d}r \frac{1}{\sqrt{f(r)h(r)}} \sqrt{\omega^2 - m^2 f(r) - g^{\varphi\varphi} j^2 f(r) + 2\beta\Pi} \left[1 + \beta\left(m^2 + \frac{\omega^2}{f(r)}\right)\right] \tag{21}$$

Here,

$$\Pi = \left(g^{\varphi\varphi}\right)^2 j^4 f(r) + 2m^2 f(r) g^{\varphi\varphi} j^2 - m^4 f(r) \tag{22}$$

With the path integral, substituting the metric of the Gibbons-Maeda-Dilaton black hole into the above equation, the value $w_\pm$ of the Gibbons-Maeda-Dilaton black hole is,

$$w_\pm = \pm i\pi\omega \frac{R^2}{r_+ - r_-}\left[1 + \beta\Xi\right] = \pm i\pi\omega \frac{r_+^2 - D^2}{r_+ - r_-}\left[1 + \beta\Xi\right] \tag{23}$$

And, $\pm$ can be related to outgoing/ingoing particles of the Gibbons-Maeda-Dilaton black hole, the symbol Ξ in Equation (23) can be express as,

$$\Xi = \frac{m^2\left(r_+ - r_-\right)^2 - 4R^2\omega^2 - \dfrac{j^2\left(r_+ - r_-\right)^2}{\mathrm{Sin}^2\theta R^2}}{2\left(r_+ - r_-\right)^2} \tag{24}$$

In this paper, the relation between the tunneling rate and action can be written as,

$$\Gamma = \frac{P_{(\text{emission})}}{P_{(\text{absorption})}} = \frac{\exp\left[-2\left(\mathrm{Im}W_+\right)\right]}{\exp\left[-2\left(\mathrm{Im}W_-\right)\right]} \tag{25}$$

So the corrected tunneling rate of the Gibbons-Maeda-Dilaton black hole near the event horizon can be express as,

$$\Gamma = \exp\left[-4\pi\omega \frac{r_+^2 - D^2}{r_+ - r_-}\left(1 + \beta\Xi\right)\right] \tag{26}$$

Therefore, the corrected Hawking temperature is,

$$T = \frac{r_+ - r_-}{4\pi\left(r_+^2 - D^2\right)\left(1 + \beta\Xi\right)} \tag{27}$$

The corrected Hawking temperature of the Gibbons-Maeda-Dilaton black hole near the horizon can be rewritten as,

$$T = \frac{r_+ - r_-}{4\pi\left(r_+^2 - D^2\right)}\left(1 - \beta\Xi\right) = T_{\text{original}}\left(1 - \beta\Xi\right) \tag{28}$$

where, the T_{original} is,

$$T_{\text{original}} = \frac{r_+ - r_-}{4\pi\left(r_+^2 - D^2\right)} \tag{29}$$

The expression of Equations (26)-(29) is the corrected Hawking temperature and corrected tunneling rate of Gibbons-Maeda-Dilaton black hole. Carefully analysis on the Equations (26)-(29), we can find that the corrected Hawking temperature is not related to the mass of the black hole, but related to the mass and energy of the outgoing particles. And this is due to the influence of quantum gravity, the Hawking radiation of the Gibbons-Maeda-Dilaton black hole are corrected. Further studies on the tunneling results, we can get the conclusion that the quantum correction slows down the increase of temperature during the tunneling radiation, and the tunneling radiation will be stopped at some particular temperature.

4. Residuum of the Gibbons-Maeda-Dilaton Black Hole

In [24], Wang has obtained the residuum of Schwarzschild black holes. Considering the massless particle, the Hawking temperature stops increasing when,

$$\left(M - \mathrm{d}M\right)\left(1 + \beta\Xi\right) = M \tag{30}$$

The residue mass and the upper limit value of temperature in black hole can be express as,

$$M_{\text{Res}} \simeq M_P^2 / \left(\beta_0 \omega\right) \geq M_P / \beta_0 \tag{31}$$

$$T_{\text{Res}} \geq \beta_0 / 8\pi M_p \tag{32}$$

Here, $\mathrm{d}M = \omega$, $\beta = \beta_0 / M_p^2$, $\omega \simeq M_P$, M_P is the Planck mass. In this way, we neglect the higher order terms of ω. So, Equation (31) is the expression of the residuum in Schwarzschild black hole. Now, let's focus on the residuum of the Gibbons-Maeda-Dilaton black hole. With the same method, we can get,

$$M_{\text{Res}} \simeq \frac{\omega}{\beta\Xi} - \omega = \frac{\omega M_p^2}{\beta_0 \Xi} - \omega \tag{33}$$

Equation (31) is the expression of the residuum of the Gibbons-Maeda-Dilaton black hole. In the calculation of Equation (31), the condition which Ξ is a constant is considered.

5. Conclusions

In this letter, we investigated the quantum tunneling radiation of scalar particles of the Gibbons-Maeda-Dilaton black hole. The results indicate that the tunneling radiation is not only related to the mass of the Gibbons-Maeda-Dilaton black hole, but also related to the mass and energy of the outgoing particle. So we can realize that the Hawking radiation is effected by the quantum gravity. In Equation (31), the parameters (β, m) slow down the increase of temperature during the tunneling radiation, and the Hawking radiation will stop at some particular temperature. According to the careful calculation, the residue mass in Gibbons-Maeda-Dilaton black hole can be obtained which is $M_{\text{Res}} \simeq \frac{\omega}{\beta\Xi} - \omega = \frac{\omega M_p^2}{\beta_0 \Xi} - \omega$. On the other hand, the Hawking radiation is replenished once again by this conclusion.

In a conclusion, the quantum gravity has attracted more and more attention of physicists. In this paper, we only calculated the tunneling behavior of scalar particles with effect of the quantum gravity. In future, we will focus on the other fields of the quantum gravity.

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