International Journal of Modern Nonlinear Theory and Application, 2017, 6, 98-103
http://www.scirp.org/journal/ijmnta
ISSN Online: 2167-9487
ISSN Print: 2167-9479

On Decomposition of New Kinds of Continuity in Bitopological Space

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How to cite this paper: Al-Malki, H. and Al-Blowi, S. (2017) On Decomposition of New Kinds of Continuity in Bitopological Space. *International Journal of Modern Nonlinear Theory and Application*, **6**, 98-103.
https://doi.org/10.4236/ijmnta.2017.63009

Received: May 25, 2017
Accepted: July 30, 2017
Published: August 2, 2017

Abstract

In many papers, new classes of sets had been studied in topological space, then the notion of continuity between any two topological spaces (a function from X to Y is continuous if the inverse image of each open set of Y is open in X) is studied via this new classes of sets. Here the authors also introduce new classes of sets called *pj*-*b*-preopen, *pj*-*b*-*B* set, *pj*-*b*-*t* set, *pj*-*b*-semi-open and *pj*-*sb*-generalized closed set in bitopological space [1] which is a set with two topologies defined on it, then they study the notion of continuity via this set and introduce some of the theories which are studying the decomposition of continuity via this set in bitopological space.

Keywords

pj-*b*-Preopen, *pj*-*b*-Semiopen, *pj*-*b*-*t* Set, *pj*-*b*-*B* Set, *pj*-*sb*-Generalized Closed, Bitopological Space

1. Introduction and Preliminaries

In topological space, there are many classes of generalized open sets given by [2] [3] [4] [5]. Tong [6] introduced the concept of *t*-set and *B*-set in topological space. [7] [8] gave some decomposition of continuity. Decomposition of pairwise continuity was given by Jelice [9] and [10] [11] [12]. In this paper, we introduce decomposition of continuity in bitopological space via new classes of sets called *pj*-*b*-preopen, *pj*-*b*-*B* set, *pj*-*b*-*t* set, *pj*-*b*-semi-open and *pj*-*sb*-generalized closed set with some theories, examples and results.

Definition 1.1. *Let* A *be a subset of a space* X, *then* A *is said to be*:

1) *b-t-set* [7] *if* $Int(A) = Int(bcl(A))$.

2) *b-B-set* [7] *if* $A = U \cap V$, *where* $U \in \tau$ *and* V *is a b-t-set.*

3) *Locally b-closed* [7] *if* $A = U \cap V$, *where* $U \in \tau$ *and* V *is a b-closed set.*

4) *b-preopen* [7] *if* $A \subseteq In(bcl(A))$.

5) *b-semiopen* [7] *if* $A \subseteq cl(bIn(A))$.

Definition 1.2. *Let* A *be a subset of a bitopological space* (X, τ_1, τ_2) *then* A *called pairwise p-open (or p-open)* [11] *if* $A \in \tau_1 \cap \tau_2$. *p-closed is the complement of p-open set. p-interior of* A *(or* $p-\text{int}(A)$*) is the union of all p-open sets of a bitopological space* X *which contained in a subset* A *of* X. *Also, the p-closure of* A *(or* $p-cl(A)$*) is the intersection of all p-closed sets which containing* A.

Definition 1.3. *A subset* A *of a bitopological space* X *is said to be:*

1) *pj-b-open* [10] *if* $A \subset j-cl(p-\text{int}(A)) \cup p-\text{int}(j-cl(A))$.

2) *pj-b-closed* [10] *if* $j-\text{int}(p-cl(A) \cap p-cl(j-\text{int}(A))) \subset A$.

3) *pj-semiopen* [11] *if* $A \subset j-cl(p-\text{int}(A))$.

4) *pj-preopen* [11] *if* $A \subset p-\text{int}(j-cl(A))$.

5) *pj-t-set* [12] *if* $p-\text{int}(j-cl(A)) = p-\text{int}(A)$.

6) *pj-B-set* [12] *if* $A = U \cap V$, *where* U *is p-open and* V *is a pj-t-set.*

7) *jp-regular open* [12] *if* $A = p-\text{int}(j-cl(A))$.

2. pj-b-t-Set, pj-b-B-Set pj-b-Semiopen, pj-b-Preopen and pj-sb-Generalized Closed

In this section, we investigated our new classes of sets *pj-b*-preopen, *pj-b*-semiopen, *pj-b-t* set, *pj-b-B* set and *pj-sb*-generalized closed set and study some of its fundamental properties and examples also we introduce some of important theories which is useful to study the decomposition of continuity via our new classes of sets.

Definition 2.1. *A subset* A *of a bitopological space* X *is said to be:*

1) *pj-b-t-set if* $p-Int(A) = p-Int(j-bcl(A))$.

2) *pj-b-B-set if* $A = U \cap V$, *where* U *is p-open and* V *is a pj-b-t-set.*

3) *pj-b-semiopen if* $A \subset j-cl(p-b\text{int}(A))$.

4) *pj-b-preopen if* $A \subset p-\text{int}(j-bcl(A))$.

Example **2.2.** *Let* $X = \{a,b,c,d\}$, $\tau_1 = \{X, \phi, \{c\}, \{c,d\}\}$ *and* $\tau_2 = \{X, \phi, \{c\}, \{b\}, \{d\}, \{b,c\}, \{c,d\}, \{b,d\}, \{b,c,d\}\}$ *then* $\{c,d\}$ *is a* $p2$*-b-t-set.*

Example **2.3.** *Let* $X = \{a,b,c\}$ *and* $\tau_1 = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$ *and* $\tau_2 = \{X, \phi, \{b\}, \{b,c\}\}$ *then* $\{a,b\}$ *is a* $p1$*-b-B-set.*

Example **2.4.** *Let* $X = \{a,b,c\}$ *and* $\tau_1 = \{X, \phi, \{b\}, \{c\}, \{b,c\}\}$ *and* $\tau_2 = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}\}$ *then* $\{a,b\}$ *it is* $p1$*-b-preopen.*

Proposition **2.5.** *If* A *and* B *are a subsets of a bitopological space* X, *then*

1) A *is a pj-b-t set if and only if* A *is pj-b-semiclosed.*

2) *If* A *is pj-b-closed, then it is a pj-b-t-set.*

3) *If* A *and* B *are pj-b-t-sets, then* $A \cap B$ *is a pj-b-t-set.*

proof. 1) Let A be *pj-b-t* set, then $[p-Int(A) = p-Int(j-bcl(A))]$ that implies $p-Int(j-bcl(A)) \subset p-Int(A) \subset A \Rightarrow p-Int(j-bcl(A)) \subset A \Rightarrow A$ is *pj-b*-semiclosed. conversely, Let A be *pj-b*-semiclosed set, then $p-Int(j-bcl(A))\subset A \Rightarrow p-Int(j-bcl(A))\subset p-Int(A)$. Also, $A\subset j-bcl(A)$ and $p-Int(A)\subset p-Int(j-bcl(A))$. Hence, A is a *pj-b-t* set.

2) Let A be *pj-b*-closed, then $A=j-bcl(A)\Rightarrow p-Int(A)=p-Int(j-bcl(A))$.

3) Let A and B be *pj-b-t*-sets, then we have:

$$\begin{aligned} p-Int(A\cap B) &= p-Int(j-bcl(A\cap B)) \\ &\subset p-Int\left[(j-bcl(A))\cap(j-bcl(B))\right] \\ &= p-Int(j-bcl(A))\cap p-Int(j-bcl(B)) \\ &= p-Int(A)\cap p-Int(B) \\ &= p-Int(A\cap B) \end{aligned}$$

$\Rightarrow p-Int(A\cap B)=p-Int(j-bcl(A\cap B))$, Hence $A\cap B$ is a *pj-b-t*-set.

The following example shows that the converse of (2) is not true in general.

Example 2.6. *From example* 2.2 *it is clear that* $\{c,d\}$ *is a* $p2$*-b-t-set but it is not* $p2$*-b-closed.*

Lemma **2.7.** *Let* A *be p-open subset of a bitopological space* X, *then* $j-bcl(A)=p-Int(j-cl(A))$ *and* $p-Int(j-bcl(A))=p-Int(j-cl(A))$.

proof. *Let* A *be p-open subset of* X, *then*

$$\begin{aligned} j-bcl(A) &= (j-scl(A))\cap(j-pcl(A)) \\ &= A\cup\left[(p-Int(j-cl(A)))\cap(j-cl(p-int(A)))\right] \\ &= A\cup(p-Int(j-cl(A))) \\ &= p-Int(j-cl(A)) \end{aligned}$$

Proposition **2.8.** *Let* A *be a subsets of a bitopological space* X, *then*

1) *If* A *is pj-t-set then it is pj-b-t-set.*

2) *If* A *is pj-b-t-set then it is pj-b-B-set.*

3) *If* A *is pj-B-set then it is pj-b-B-set.*

proof. 1) Let A be *pj-t*-set,then $p-Int(A)=p-Int(j-cl(A))$ from lemma 2.1 $\left[j-bcl(A)=p-Int(j-cl(A))\right] \Rightarrow j-bcl(A)=p-Int(A)$ $\Rightarrow p-Int(j-bcl(A))=p-Int(A)$. Hence A is *pj-b-t*-set.

2) Let A be *pj-b-t*-set. $A=A\cap X$ and X is *p*-open set, then A is *pj-b-B*-set.

3) Let A be *pj-B*-set *i.e.* $A=U\cap V$, where U is *p*-open and V is a *pj-t*-set *i.e.* $\left[p-Int(j-cl(V))=p-Int(V)\right]$ from lemma 2.1 $\left[j-bcl(V)=p-Int(j-cl(V))\right] \Rightarrow p-Int(j-bcl(V))=p-Int(j-cl(V))$ $\Rightarrow p-Int(j-bcl(V))=p-Int(V)$. Hence A is *pj-b-B*-set.

Theorem 2.9. *Let* A *be a subset of a bitopological space* X, *then the following are equivalent*:

1) A is *p*-open set.

2) A is *pj*-*b*-preopen and *pj*-*b*-*B*-set.

proof. (1) $\Rightarrow$ (2) Let A be *p*-open $\Rightarrow A = p - int(A)$ but $A \subset j - bcl(A)$ then $A = p - int(A) \subset p - int(j - bcl(A))$ $\Rightarrow A$ is *pj*-*b*-preopen. Also, $A = A \cap X$ and X is *p*-open and $p - int(j - bcl(A)) = p - int(A) \Rightarrow A$ is *pj*-*b*-*B*-set.

(2) $\Rightarrow$ (1) A be *pj*-*b*-preopen and *pj*-*b*-*B*-set. *i.e.* $A = U \cap V$, where U is *p*-open and $p - int(j - bcl(V)) = p - int(V)$, then we have

$$\begin{aligned} A &\subset p - int(j - bcl(A)) \\ &= p - int(j - bcl(U \cap V)) \\ &= p - int((j - bcl(U)) \cap (j - bcl(V))) \\ &= p - int(j - bcl(U)) \cap p - int(j - bcl(V)) \\ &= p - int(j - bcl(U)) \cap p - int(V) \end{aligned}$$

Hence,

$$\begin{aligned} A &= (U \cap V) \cap U \\ &\subset (p - int(j - bcl(U)) \cap p - int(V)) \cap U \\ &= (p - int(j - bcl(U)) \cap U) \cap p - int(V) \\ &= U \cap p - int(V) \end{aligned}$$

Therefore $A = (U \cap V) = U \cap p - int(V)$ and A is *p*-open.

The following examples show that *pj*-*b*-preopen sets and *pj*-*b*-*B*-sets are independent.

Example 2.10. *From example* 2.3 *it is clear that* $\{a, b\}$ *is a* $p1$-b-B *-set but it is not* $p1$-b*-preopen.*

Example 2.11. *From example* 2.4 *it is clear that* $\{a, b\}$ *it is* $p1$-b*-preopen but it is not a* $p1$-b-B*-set.*

Corollary 2.12. *A subset* A *of a bitopological space* X *is p-open if and only if it is pj-α-open and pj-b-B-set.*

Proposition 2.13. *Let* A *be a subsets of a bitopological space* X, *then the following are equivalent*:

1) A *is jp-regular set.*

2) $A = p - int(j - bcl(A))$

3) A *is pj-b-preopen and pj-b-t-set.*

proof. (1) $\Rightarrow$ (2) Let A be *jp*-regular set.since $j - bcl(A) \subset j - cl(A)$ then $p - int(j - bcl(A)) \subset p - int(j - cl(A)) = A$. Since A is *pj*-*b*-open $\Rightarrow A \subset p - int(j - bcl(A))$. Hence, $A = p - int(j - bcl(A))$

(2) $\Rightarrow$ (3) This is obvious.

(3) $\Rightarrow$ (1) Let A be *pj*-*b*-preopen and *pj*-*b*-*t*-set.Then $A \subset p - int(j - bcl(A)) = p - int(cl(A)) \subset A$ and A is *p*-open by lemma 2.1 $A = p - Int(j - bcl(A)) = p - Int(j - cl(A))$ Hence, A is *jp*-regular set.

Definition 2.14. *A subset* A *of a bitopological space* X *is called pj-sb-generalized closed if pj-* $s(bCl(A)) \subset U$, *whenever* $A \subset U$ *and* U *is pj-b-*

preopen.

Definition 2.15. *pj-* $s(bCl(A))$ *is the intersection of all pj-semiclosed sets which containing* A.

Theorem 2.16. *Let* A *be a subset of a bitopological space* X, *the following properties are equivalent:*

1) A *is jp-regular open set.*
2) A *is pj-b-preopen and pj-sb-generalized closed set.*

proof. (1) $\Rightarrow$ (2) Let A be *jp*-regular open.Then A is *pj-b*-open. $A \subset p-Int(j-bCl(A))$. Moreover, by Lemma 2.1 *pj-* $s(bCl(A)) = A \cup (p-Int(j-bCl(A))) = p-Int(j-bCl(A)) = p-Int(j-Cl(A))$ $= A$. Hence, A is *pj-sb*-generalized closed.

(2) $\Rightarrow$ (1) Let A be *pj-b*-preopen and *pj-sb*-generalized closed. $\Rightarrow pj-s(bCl(A)) \subset A \Rightarrow A$ is *pj-b*-semiclosed. Then $p-Int(j-b(Cl(A))) = A$. Therefore by Proposition 2.3 A is *jp*-regular open.

Corollary 2.17. *A subset* A *of a bitopological space* X *is jp-regular open if and only if it is pj-α-open and pj-b-t-set.*

3. Decompositions of New Kinds of Continuity

After we had been defined and studied the propriety of our new classes of sets we are ready to study the concept of continuity between any two bitopological spaces via our new classes of sets.

Definition 3.1. *A function* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is called pj-b-continuous* [10] (*resp. pj-Locally b-closed continuous* [10], *pj-D(c,b)-continuous* [10], *pj-α-continuous* [11] *pj-semi continuous* [11], *jp-semi continuous* [11], *pj-B-continuous* [12], *pj-Locally closed continuous* [12], *jp-regular continuous* [13]) *if* $f^{-1}(V)$ *is pj-b-set* (*resp. pj-Locally b-closed set, pj-D(c,b)-set, pj-α-open, pj-semiopen, jp-semiopen, pj-B-set, pj-Locally closed,, jp-rgular*) *in* X *for each p-open set V of Y.*

Theorem 3.2. *A function* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is called pj-B-continuous if and only if it is locally pj-b-closed-continuous and pj-semi-continuous.*

proof. It is following from lemma 3.4 in [10]

Definition 3.3. *Afunction* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is called pj-b-pre-continuous* (*resp. pj-b-B-continuous, pj-b-t-continuous, pj-b-semi-continuous*) *if* $f^{-1}(V)$ *is pj-b-preopen* (*resp. pj-b-B-set, pj-b-t-set, pj-b-semiopen*) *in* X *for each p-open set* V *of* Y.

Theorem 3.4. *A function* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is called p-continuous if and only if it is pj-α-continuous and pj-b-B-continuous.*

proof. It is follows from theorem 2.1.

Theorem 3.5. *A function* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is called p-continuous if and only if it is pj-b-pre-continuous and pj-b-B-continuous.*

proof. It is follows from corollary 2.1.

Definition 3.6. *Afunction* $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ *is called contra pj-sb-continuous if* $f^{-1}(V)$ *is pj-sb-generalized closed in* X *for each p-open set*

V *of* Y.

Theorem 3.7. *A function* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is called completely p-continuous if and only if it is pj-b-pre-continuous and pj-b-t-continuous.*

proof. It is follows from proposition 2.3.

Theorem 3.8 *A function* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is called completely p-continuous if and only if it is pj-b-pre-continuous and contra pj-sb-continuous.*

proof. It is follows from theorem 2.2.

Theorem 3.9 *A function* $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *is called completely p-continuous if and only if it is pj-α-continuous and pj-b-t-continuous.*

proof. It is follows from corollary 2.2.

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