The Global Attractors for the Higher-Order Kirchhoff-Type Equation with Nonlinear Strongly Damped Term

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Abstract
We investigate the global well-posedness and the global attractors of the solutions for the Higher-order Kirchhoff-type wave equation with nonlinear strongly damping:

\[ u_t + \sigma \left( \left\| \nabla u \right\|^2 \right) (-\Delta)^m u + \phi \left( \left\| \nabla u \right\|^2 \right) (-\Delta)^m u = f(x). \]

For strong nonlinear damping \( \sigma \) and \( \phi \), we make assumptions (H1) - (H4). Under of the proper assumption, the main results are existence and uniqueness of the solution in \( H^{2m}(\Omega) \times H_0^m(\Omega) \) are proved by Galerkin method, and deal with the global attractors.

Keywords

1. Introduction
We consider the following Higher-order Kirchhoff-type equation:

\[ u_t + \sigma \left( \left\| \nabla u \right\|^2 \right) (-\Delta)^m u + \phi \left( \left\| \nabla u \right\|^2 \right) (-\Delta)^m u = f(x), (x, t) \in \Omega \times [0, +\infty), \]

\[ u(x, t) = 0, \quad \frac{\partial^i u}{\partial \nu^i} = 0, i = 1, 2, \ldots, m-1, x \in \partial \Omega, t \in (0, +\infty), \]

\[ u(x, 0) = u_0(x), u_t(x, 0) = u_t(x), x \in \Omega, \]

where \( m > 1 \) is an integer constant, and \( \Omega \) is a bounded domain of \( \mathbb{R}^n \), with a smooth dirichlet boundary \( \partial \Omega \) and initial value. Moreover, \( \nu \) is the unit outward normal on \( \partial \Omega \). \( \sigma \) and \( \phi \) are scalar functions specified later, \( f \) is a given function.
This kind of wave models goes back to G. Kirchhoff [1] and has been studied by many authors under different types of hypotheses. There have been many researchers on the global attractors existence of Kirchhoff equation, we can refer [2] [3] [4] [5] [6]. What’s more, the global attractors for the Higher-order Kirchhoff-type equation are investigated and we refer to [7] [8] [9].

Zhijian Yang and Pengyan Ding [2] studied the longtime dynamics of the Kirchhoff equation with strong damping and critical nonlinearity on $\mathbb{R}^N$:

$$u_t - \Delta u_t - M \left( \left\| \nabla u \right\|^2 \right) \Delta u + u_t + g(x, u) = f(x).$$ (1.4)

They establish the well-posedness, the existence of the global and exponential attractors in natural energy space $H = H^1 \left( \mathbb{R}^N \right) \times L^2 \left( \mathbb{R}^N \right)$ in critical nonlinearity case. On this basis, they also investigated the global well-posedness and the longtime dynamics of the Kirchhoff equation with fractional damping and supertical nonlinearity [3]:

$$u_t - M \left( \left\| \nabla u \right\|^2 \right) \Delta u + \left( -\Delta \right)^\alpha u_t + f(u) = g(x), \quad \text{with } \alpha \in \left( \frac{1}{2}, 1 \right).$$ (1.5)

The main results are focused on the relationships among the growth exponent $p$ of the nonlinearity $f(u)$, the global well-posedness and the longtime dynamics of the equations. They show that i) even if $p$ is up to the supercritical range, that is,

$$1 \leq p \leq \frac{N + 4\alpha}{(N - 4\alpha)},$$

the well-posedness and the longtime behavior of the solutions of the equation are the characters of the parabolic equation; ii) when

$$\frac{N + 4\alpha}{(N - 4\alpha)} \leq p < \frac{N + 4}{(N - 4)},$$

the corresponding subclass $G$ of the limit solutions exists and possesses a weak global attractors.

Varga Kalantarov and Sergey Zelik [5] present a new method of investigating the so-called quasi-linear strongly damped wave equations:

$$\partial_t^2 u - \gamma \partial_t \Delta u - \Delta u + f(u) = \nabla \cdot \phi (\nabla u) + g.$$ (1.6)

In bounded 3D domains. This method establishes the existence and uniqueness of energy solutions in the case where the growth exponent of the non-linearity $\phi$ is less than 6 and $\gamma$ may have arbitrary polynomial growth rate. Moreover, the existence of a finite-dimensional global and exponential attractors for the solution semigroup associated with that equation and their additional regularity are also established. In a particular case $\phi \equiv 0$ which corresponds to the so-called semi-linear strongly damped wave equation, their result allows to remove the long-standing growth restriction $|f(u)| \leq C \left( 1 + |u|^p \right)$. The Cauchy problem and the boundary value problem for equation under the different assumptions on the nonlinearities $\phi$ and $f$ have been studied in many papers, but the author uses a new method to this equation.

Xiuli Lin and Fushan Li [6] consider the initial-boundary value problem for nonlinear Kirchhoff-type equation:
where \( a,b > 0 \) and \( \beta > 2 \) are constants, \( \varphi \) is a \( C^1 \)-function such that \( \varphi(s) \geq \lambda_0 > 0 \) for all \( s \geq 0 \). Under suitable conditions on the initial data, they show the existence and uniqueness of global solution by means of the Galerkin method and the uniform decay rate of the energy by an integral inequality. Here, \( \varphi(s) \) satisfying \( \varphi(s) \geq m_0 > 1 \) and \( s\varphi(s) \geq \int_0^s \varphi(\tau) d\tau, \forall s \in (0,\infty) \). In this paper, for strong nonlinear damping \( \sigma \) and \( \phi \), we make some similar assumptions. These assumptions will be presented in the following statements.

In 2004, Fucai Li [7] dealt with the higher-order Kirchhoff-type equation with nonlinear dissipation:

\[
\frac{\partial u}{\partial t} + \left( \int_{\Omega} |D^m u|^q \, dx \right)^{\gamma} (-\Delta)^m u + u_t = |u|^{p-2} u, \quad x \in \Omega, t > 0, \tag{1.8}
\]

In a bounded domain, where \( m > 1 \) is a positive integer, and \( q,p,r > 0 \) are positive constants. They obtain that the solution exists global if \( p \leq r \), while if \( p > \max\{r,2q\} \), then for any initial data with negative initial energy, the solution blows up at finite time in \( L^{m+2} \) norm.

In 2007, Salim A. Messaoudi and Belkacem Said Houari [8] improve Li’s result and showed that certain solutions with positive initial energy also blow up in finite time.

Qingyong Gao, Fushan Li, Yanguo Wang [9] obtained the local existence of the solution to the homogeneous Dirichlet boundary value problem for the higher-order nonlinear Kirchhoff-type equation:

\[
\frac{\partial u}{\partial t} + M \left( \int_{\Omega} |D^m u(t)|^p \, dx \right) (-\Delta)^m u + u_t = |u|^{p-2} u, \quad x \in \Omega, t > 0, \tag{1.9}
\]

where \( p > q \geq 2, m \geq 1 \).

At present, most Higher-order Kirchhoff-type equations investigate the blow-up of the solution. We study the global attractor of the solution for Higher-order Kirchhoff-type equations.

Igor Chueshov [4] studied the longtime dynamics of Kirchhoff wave models with strong nonlinear damping:

\[
\frac{\partial u}{\partial t} - \sigma \left( \int_{\Omega} |\nabla u|^2 \right) \Delta u + \phi \left( \int_{\Omega} |\nabla u|^2 \right) \Delta u + f(u) = h(u), x \in \Omega, t > 0. \tag{1.10}
\]

He proves the existence and uniqueness of weak solutions, and established a finite-dimensional global attractor in the sense of partially strong topology.

On the basis of Igor Chueshov, we investigate the global attractor of the higher-order Kirchhoff-type Equation (1.1) with strong nonlinear damping. Such problems have been studied by many authors, but \( \sigma \left( \int_{\Omega} |\nabla u|^2 \right) \) is a definite constant and even
\( \sigma \left( \| \nabla^m u \|^2 \right) = 0. \) Generally, the equation exist a nonlinear \( f(u) \). But in the paper, \( \sigma \left( \| \nabla^m u \|^2 \right) \) is a scalar function and \( f(u) = 0. \) Under of the the proper assume, in section 2, we prove the existence of the solution by priori estimation and the Galerkin method. Therefore, we show that i) the solution \( (u, v) \) of the problem (1.1) - (1.3) satisfies \( (u, v) \in H^m_0(\Omega) \times L^2(\Omega) \); further more, ii) the solution \( (u, v) \) of the problem (1.1) - (1.3) satisfies \( (u, v) \in H^{2m}(\Omega) \times H^m_0(\Omega) \). Then, in section 3, we prove the uniqueness of the solution by using the method that assumption exist two solutions in the same initial value and two solutions are equal. At last, according to define, we obtain to the existence of the global attractor.

2. Preliminaries

For brevity, we denote the simple symbol, \( \| \cdot \| \) represents inner product, and \( H^m = H^m(\Omega), \quad H^m_0 = H^m_0(\Omega), \quad H^{2m} = H^{2m}(\Omega), \quad H = L^2, \quad \| \cdot \|_2 = \| \cdot \|_\infty, \quad \| \cdot \| \), \( f = f(x), \quad c_i (i = 0, 1, \cdots, 7) \) are constants, \( m_i, \mu_i (i = 0, 1) \) are also constants. \( \lambda^m \) is the first eigenvalue of the operator \( \nabla^m \).

In this section, we present some assumptions needed in the proof of our results. For this reason, we assume that

(H1) setting \( \Sigma(s) = \int_0^s \sigma(\xi) d\xi, \quad \Phi(s) = \int s(\xi) d\xi, \quad \gamma(s) = \int \gamma(s) d\xi \), then

\[
\langle s\phi(s) - s\sigma(s) - \eta(\Phi(s) - \Sigma(s)) > s, \quad (2.1)
\]

where \( \forall \varepsilon > 0, \forall \eta > 0 \).

(H2) \[10\]

\[
m_0 < \phi(s) - \varepsilon \sigma(s) < m_1, m = \begin{cases} m_0, & \frac{d}{dt} \| \lambda^m u \|^2 \geq 0; \\ m_1, & \frac{d}{dt} \| \lambda^m u \|^2 < 0. \end{cases} \quad (2.2)
\]

(H3)

\[
\sigma(s), \phi(s) \in C^1(\Omega). \quad (2.3)
\]

(H4)

\[
\mu_0 < \phi(s) + \varepsilon \sigma(s) < \mu_1, \mu = \begin{cases} \mu_0, & \frac{d}{dt} \| \nabla^m w \|^2 \geq 0; \\ \mu_1, & \frac{d}{dt} \| \nabla^m w \|^2 < 0. \end{cases} \quad (2.4)
\]

Now, we can do priori estimates for equation (1.1)

**Lemma 1.** Assume (H1) hold, and \( (u_0, u_1) \in H^m \times H, \quad f \in H \). Then the solution \( (u, v) \) of the problem (1.1) - (1.3) satisfies \( (u, v) \in H^m \times H \), and

\[
\| (u, v) \|_{H^m \times H}^2 = \| \nabla^m u \|^2 + \| v \|^2 \leq W_1(0) e^{-\gamma_1 t} + \frac{c_1}{\gamma_1} \left( 1 - e^{-\gamma_1 t} \right), \quad (2.5)
\]

where \( v = u + \varepsilon u, \quad W_1(0) \equiv \| v_0 \|^2 + 2 \gamma_1 \left( \| \nabla^m u_0 \|^2 \right) - 2 \varepsilon \gamma_1 \left( \| \nabla^m u_0 \|^2 \right), \quad v_0 = u_1 + \varepsilon u_0 \). Thus,
there exists $R_i$ and $t = t_i > 0$, such that

$$\lim_{t \to \infty} \| (u,v) \| \leq \frac{c_i}{\gamma_i} = R_i.$$  \hfill (2.6)

**Proof.** Let $v = u + \varepsilon u$, then we use $v$ multiply with both sides of Equation (1.1) and obtain

$$\left( u_n + \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) (-\Delta)^{m} u_n + \phi \left( \left\| \nabla^{m} u \right\|^2 \right) (-\Delta)^{m} u_n, v \right) = \left( f(x), v \right).$$  \hfill (2.7)

After a computation (2.7) one by one, as follow

$$\left( u_n, v \right) = (v, v) = (v, v) - \varepsilon (u, v) = (v, v) - \varepsilon (v - \varepsilon u, v)$$

$$\geq \frac{1}{2} \frac{d}{dt} \left\| v \right\|^2 - \varepsilon \left\| v \right\|^2 - \frac{\varepsilon^2}{2\lambda} \left\| \nabla^{m} u \right\|^2 - \frac{\varepsilon^2}{2} \left\| v \right\|^2. \hfill (2.8)$$

$$\left( \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) (-\Delta)^{m} u_n, v \right)$$

$$= \left( \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) (-\Delta)^{m} (v - \varepsilon u), v \right)$$

$$= \left( \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) (-\Delta)^{m} v, v \right) - \varepsilon \left( \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) (-\Delta)^{m} u_n, v \right)$$

$$= \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) \left\| \nabla^{m} v \right\|^2 - \varepsilon \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) \left\| \nabla^{m} u \right\|^2$$

$$\geq \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) \left\| \nabla^{m} v \right\|^2 - \varepsilon \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) \left\| \nabla^{m} u \right\|^2$$

$$\geq \lambda^m \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) \left\| \nabla^{m} u \right\|^2 \frac{d}{dt} \left\| \nabla^{m} u \right\|^2 - \varepsilon^2 \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) \left\| \nabla^{m} u \right\|^2.$$  \hfill (2.9)

Because $f \in H$, by using Holder inequality, Young’s inequality, we obtain

$$\left( f(x), v \right) \leq \left\| f \right\| \left\| v \right\| \leq \frac{1}{2\varepsilon^2} \left\| f \right\|^2 + \frac{\varepsilon^2}{2} \left\| v \right\|^2. \hfill (2.10)$$

From the above, we have

$$\frac{d}{dt} \left\| v \right\|^2 + 2\Phi \left( \left\| \nabla^{m} u \right\|^2 \right) - 2\varepsilon \left( \left\| \nabla^{m} u \right\|^2 \right)$$

$$+ \left( 2\lambda^m \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) - 2\varepsilon - 2\varepsilon^2 \right) \left\| v \right\|^2$$

$$+ \left( 2\varepsilon \left( \left\| \nabla^{m} u \right\|^2 \right) - 2\varepsilon \sigma \left( \left\| \nabla^{m} u \right\|^2 \right) \left( -\Delta \right)^m u_n \right) \left\| \nabla^{m} u \right\|^2 \leq \frac{1}{\varepsilon^2} \left\| f \right\|^2.$$  \hfill (2.11)

According to (2.1), we have
\[
\begin{aligned}
&\left( 2\epsilon \phi \left( \| \nabla u \|^{2} \right) - 2\epsilon^{2} \sigma \left( \| \nabla u \|^{2} - \frac{\epsilon^{2}}{\lambda u} \right) \| \nabla u \|^{2} \\
&\geq \left( 2\epsilon \phi \left( \| \nabla u \|^{2} \right) - 2\epsilon^{2} \sigma \left( \| \nabla u \|^{2} - 2\epsilon \right) \| \nabla u \|^{2} \\
&\geq 2\epsilon \eta \left( \Phi \left( \| \nabla u \|^{2} \right) - \Sigma \left( \| \nabla u \|^{2} \right) \right),
\end{aligned}
\]

where \( \epsilon < 2\lambda u \).

Substitution (2.13) into (2.12), we receive
\[
\begin{aligned}
&\frac{d}{dt} \left( \| v \|^{2} + 2\Phi \left( \| \nabla u \|^{2} \right) - 2\epsilon \Sigma \left( \| \nabla u \|^{2} \right) \right) \\
&+ \left( 2\lambda u \sigma \left( \| \nabla u \|^{2} \right) - 2\epsilon - 2\epsilon^{2} \right) \| v \|^{2} \\
&+ 2\epsilon \eta \left( \Phi \left( \| \nabla u \|^{2} \right) - \Sigma \left( \| \nabla u \|^{2} \right) \right) \leq \frac{1}{\epsilon^{2}} \| f \|^{2}.
\end{aligned}
\]

We deal with the items, we have
\[
\begin{aligned}
&\left( 2\lambda u \sigma \left( \| \nabla u \|^{2} \right) - 2\epsilon - 2\epsilon^{2} \right) \| v \|^{2} + 2\epsilon \eta \left( \Phi \left( \| \nabla u \|^{2} \right) - \Sigma \left( \| \nabla u \|^{2} \right) \right) \\
&= \left( 2\lambda u \sigma \left( \| \nabla u \|^{2} \right) - 2\epsilon - 2\epsilon^{2} \right) \| v \|^{2} + 2\epsilon \eta \Phi \left( \| \nabla u \|^{2} \right) - 2\epsilon \eta \Sigma \left( \| \nabla u \|^{2} \right) \\
&= \left( 2\lambda u \sigma \left( \| \nabla u \|^{2} \right) - 2\epsilon - 2\epsilon^{2} \right) \| v \|^{2} + 2\epsilon \Phi \left( \| \nabla u \|^{2} \right) - 2\epsilon \Sigma \left( \| \nabla u \|^{2} \right) \\
&+ \left( \epsilon^{2} \eta - 2\epsilon \Sigma \right) \| \nabla u \|^{2} \\
&\geq \gamma_{1} \left( \| v \|^{2} + 2\Phi \left( \| \nabla u \|^{2} \right) - 2\epsilon \Sigma \left( \| \nabla u \|^{2} \right) \right) - \gamma_{2} \Sigma \left( \| \nabla u \|^{2} \right) \\
&= \gamma_{1} \left( \| v \|^{2} + 2\Phi \left( \| \nabla u \|^{2} \right) - 2\epsilon \Sigma \left( \| \nabla u \|^{2} \right) \right),
\end{aligned}
\]

where we take a proper constant \( \gamma_{i} \), such that
\[
2\lambda u \sigma \left( \| \nabla u \|^{2} \right) - 2\epsilon - 2\epsilon^{2} > 0, \gamma_{1} = \min \left\{ 2\lambda u \sigma \left( \| \nabla u \|^{2} \right) - 2\epsilon - 2\epsilon^{2}, \epsilon \eta \right\},
\]
\[
\eta = \frac{\gamma_{1}}{2 - \epsilon}.
\]

Then, we get
\[
\frac{d}{dt} W_{t} (t) + \gamma_{1} W_{t} (t) \leq c_{t},
\]

where
\[
W_{t} (t) = \| v \|^{2} + 2\Phi \left( \| \nabla u \|^{2} \right) - 2\epsilon \Sigma \left( \| \nabla u \|^{2} \right),
\]
\[
c_{t} = \frac{1}{\epsilon^{2}} \| f \|^{2}.
\]

By using Gronwall inequality, we obtain
\[ W_1(t) \leq W_1(0)e^{-\gamma t} + \frac{C_1}{\gamma_1}(1 - e^{-\gamma t}), \]  
\text{(2.18)}

where
\[ W_1(0) = \left\| v_0 \right\|^2 + 2\Phi\left( \left\| \nabla^m u_0 \right\|^2 \right) - 2\varepsilon\Xi\left( \left\| \nabla^m u_0 \right\|^2 \right), \]
\text{(2.19)}

\[ v_0 = u_1 + \varepsilon u_0. \]

So, we have
\[ \left\| (u,v) \right\|^2_{H^m_{\gamma,t}} = \left\| \nabla^m u \right\|^2 + \left\| \nabla^m v \right\|^2 \leq W_1(0)e^{-\gamma t} + \frac{C_1}{\gamma_1}(1 - e^{-\gamma t}), \]
\text{(2.20)}

and
\[ \lim_{t \to \infty} \left\| (u,v) \right\|^2_{H^m_{\gamma,t}} \leq \frac{C_1}{\gamma_1}. \]
\text{(2.21)}

Thus, there exist \( t = t_1(\Omega) \) and \( t = t_2(\Omega) \), such that
\[ \left\| (u,v) \right\|^2_{H^m_{\gamma,t}} \leq \frac{C_1}{\gamma_1} = R_1(t > t_1). \]
\text{(2.22)}

**Remark 1.** Assumption (H_1) imply
\[ \Phi\left( \left\| \nabla^m u \right\|^2 \right) > \varepsilon \Xi\left( \left\| \nabla^m u \right\|^2 \right) > \varepsilon \left\| \nabla^m u \right\|^2 + c_0, \]
\text{(2.23)}

such that (2.20) hold.

**Lemma 2.** Assume (H_2) hold, \( f \in H^m_0 \), and \( (u_0, u_1) \in H^{2m} \times H^m_0 \). Then the solution \( (u,v) \) of the problem (1.1) - (1.3) satisfies \( (u,v) \in H^{2m} \times H^m_0 \), and
\[ \left\| \nabla^m v \right\|^2 + \left\| \Delta^m u \right\|^2 \leq \frac{W_2(0)}{L}e^{-\gamma_2 t} + \frac{C_2}{L\gamma_2}(1 - e^{-\gamma_2 t}), \]
\text{(2.24)}

where \( \varepsilon = u_1 + \varepsilon u \), \( 0 < L < \min \left\{ 1, m \right\} \), \( W_2(0) = \left\| \nabla^m v_0 \right\|^2 + m \left\| \Delta^m u_0 \right\|^2 \), \( v_0 = u_1 + \varepsilon u_0 \).

There exist \( t = t_2(\Omega) \) and \( R_2 \), such that
\[ \lim_{t \to \infty} \left\| \nabla^m u \right\|^2_{H^m_{\gamma,t}} \leq \frac{C_2}{L\gamma_2} = R_2. \]
\text{(2.25)}

**Proof.** Let \( (-\Delta)^m v = (-\Delta)^m u_1 + \varepsilon(-\Delta)^m v \), we use \( (-\Delta)^m v \) multiply sides of equation (1.1) and obtain
\[ \left( u_1 + \sigma\left( \left\| \nabla^m u \right\|^2 \right)(-\Delta)^m u_1 + \phi\left( \left\| \nabla^m u \right\|^2 \right)(-\Delta)^m u, (-\Delta)^m v \right) = \left( f(x), (-\Delta)^m v \right). \]
\text{(2.26)}

After a computation (2.26) one by one, as follow
\[ \left( u_1, (-\Delta)^m v \right) = \left( v_1 - \varepsilon u_1, (-\Delta)^m v \right) \]
\[ = \frac{1}{2} \frac{d}{dt} \left\| \nabla^m v \right\|^2 - \varepsilon \left( v - \varepsilon u_1, (-\Delta)^m v \right) \]
\[ = \frac{1}{2} \frac{d}{dt} \left\| \nabla^m v \right\|^2 - \varepsilon \left\| \nabla^m v \right\|^2 + \varepsilon^2 \left( u_1, (-\Delta)^m v \right) \]
\[ \geq \frac{1}{2} \frac{d}{dt} \left\| \nabla^m v \right\|^2 - \varepsilon \left\| \nabla^m v \right\|^2 - \frac{\varepsilon^2}{2\lambda^n} \left\| \Delta^m u \right\|^2 - \frac{\varepsilon^2}{2} \left\| \nabla^m v \right\|^2. \]
\text{(2.27)}
\[
\left( \sigma \left( \| \nabla^m u \|^2 \right) \right) (-\Delta)^m u, (-\Delta)^m v
\]

\[= \left( \sigma \left( \| \nabla^m u \|^2 \right) \right) (-\Delta)^m v - \varepsilon \left( -\Delta \right)^m u, (-\Delta)^m v \]

\[= \sigma \left( \| \nabla^m u \|^2 \right) \| \Delta^m u \|^2 - \left( \sigma \left( \| \nabla^m u \|^2 \right) \varepsilon \left( -\Delta \right)^m u, (-\Delta)^m u + \varepsilon (-\Delta)^m u \right) \]

\[= \sigma \left( \| \nabla^m u \|^2 \right) \| \Delta^m u \|^2 - \frac{1}{2} \varepsilon \sigma \left( \| \nabla^m u \|^2 \right) \frac{d}{dt} \| \Delta^m u \|^2 - \varepsilon^2 \sigma \left( \| \nabla^m u \|^2 \right) \| \Delta^m u \|^2 \]

\[\geq \lambda^m \sigma \left( \| \nabla^m u \|^2 \right) \| \nabla^m v \|^2 - \frac{1}{2} \varepsilon \sigma \left( \| \nabla^m u \|^2 \right) \frac{d}{dt} \| \Delta^m u \|^2 - \varepsilon^2 \sigma \left( \| \nabla^m u \|^2 \right) \| \Delta^m u \|^2 . \]

\[
\left( \phi \left( \| \nabla^m u \|^2 \right) \right) (-\Delta)^m u, (-\Delta)^m v
\]

\[= \left( \phi \left( \| \nabla^m u \|^2 \right) \right) (-\Delta)^m u, (-\Delta)^m u + \varepsilon (-\Delta)^m u \right) \]

\[= \frac{1}{2} \phi \left( \| \nabla^m u \|^2 \right) \frac{d}{dt} \| \Delta^m u \|^2 + \varepsilon \phi \left( \| \nabla^m u \|^2 \right) \| \Delta^m u \|^2 . \]

Due to \( f \in H^m_0 \), by using Holder inequality, Young’s inequality, we obtain

\[
\left( f (x), (-\Delta)^m v \right) \leq \frac{\varepsilon^2}{2} \| \nabla^m v \|^2 + \frac{1}{2\varepsilon} \| \nabla^m f \|^2 .
\]

From the above, we obtain

\[
\frac{d}{dt} \| \nabla^m v \|^2 + \left( \phi \left( \| \nabla^m u \|^2 \right) - \varepsilon \sigma \left( \| \nabla^m u \|^2 \right) \right) \frac{d}{dt} \| \Delta^m u \|^2
\]

\[+ \left( \frac{2 \lambda^m \sigma \left( \| \nabla^m u \|^2 \right) - 2 \varepsilon - 2 \varepsilon^2 \right) \| \nabla^m v \|^2
\]

\[+ \left( 2 \varepsilon \phi \left( \| \nabla^m u \|^2 \right) - 2 \varepsilon^2 \sigma \left( \| \nabla^m u \|^2 \right) - \frac{\varepsilon^2}{\lambda^m} \right) \| \Delta^m u \|^2 \leq \frac{1}{2 \varepsilon} \| \nabla^m f \|^2 .
\]

According to (2.2), we have

\[
\left( \phi \left( \| \nabla^m u \|^2 \right) - \varepsilon \sigma \left( \| \nabla^m u \|^2 \right) \right) \frac{d}{dt} \| \Delta^m u \|^2
\]

\[+ \left( 2 \varepsilon \phi \left( \| \nabla^m u \|^2 \right) - 2 \varepsilon^2 \sigma \left( \| \nabla^m u \|^2 \right) - \frac{\varepsilon^2}{\lambda^m} \right) \| \Delta^m u \|^2 \geq m \frac{d}{dt} \| \Delta^m u \|^2 + \left( 2 \varepsilon m_0 - \frac{\varepsilon^2}{\lambda^m} \right) \| \Delta^m u \|^2 .
\]

Collecting with (2.32), we obtain from (2.31) that

\[
\frac{d}{dt} \left( \| \nabla^m v \|^2 + m \| \Delta^m u \|^2 \right) + \left( \frac{2 \lambda^m \sigma \left( \| \nabla^m u \|^2 \right) - 2 \varepsilon - 2 \varepsilon^2 \right) \| \nabla^m v \|^2
\]

\[+ \left( 2 \varepsilon m_0 - \frac{\varepsilon^2}{\lambda^m} \right) \| \Delta^m u \|^2 \leq \frac{1}{2 \varepsilon} \| \nabla^m f \|^2 .
\]

Noticing \( 0 < \varepsilon < \lambda^m \left( 2m_0 - m \right) \), this will imply

\[
\left( 2 \varepsilon m_0 - \frac{\varepsilon^2}{\lambda^m} \right) \| \Delta^m u \|^2 \geq \varepsilon m \| \Delta^m u \|^2 .
\]
Substituting (2.34) into (2.33), we can get the following inequality
\[
\frac{d}{dt}(\|\nabla u\|^2 + m\|\Delta u\|^2) + \left(2\lambda^* \sigma(\|\nabla u\|^2) - 2\epsilon - 2\epsilon^2\right)\|\nabla u\|^2 + \frac{\epsilon}{m}\|\Delta u\|^2 \leq \frac{1}{\epsilon^2}\|\nabla f\|^2.
\] (2.35)

Hence, we take a proper constant \(\epsilon\), such that \(2\lambda^* \sigma(\|\nabla u\|^2) - 2\epsilon - 2\epsilon^2 > 0\), we get
\[
\frac{d}{dt}W_2(t) + \gamma_2 W_2(t) \leq c_2,
\] (2.36)

where
\[
W_2(t) = \|\nabla u\|^2 + m\|\Delta u\|^2,
\]
\[
\gamma_2 = \min \left\{2\lambda^* \sigma(\|\nabla u\|^2) - 2\epsilon - 2\epsilon^2, \epsilon\right\},
\]
\[
c_2 = \frac{1}{\epsilon^2}\|\nabla f\|^2.
\] (2.37)

By using Gronwall inequality, we end up with
\[
W_2(t) \leq W_2(0)e^{-\gamma_2 t} + \frac{c_2}{\gamma_2}(1 - e^{-\gamma_2 t}),
\] (2.38)

where
\[
W_2(0) = \|\nabla u_0\|^2 + m\|\Delta u_0\|^2,
\]
\[
\nu_0 = u_1 + \epsilon u_0.
\] (2.39)

Taking \(L = \min \{1, m\}\), we have
\[
\|\nabla u\|^2 + \|\Delta u\|^2 \leq \frac{W_2(0)}{L}e^{-\gamma_2 t} + \frac{c_2}{L\gamma_2}(1 - e^{-\gamma_2 t}),
\] (2.40)

and
\[
\lim_{t \to \infty}\|u(t), v\|_{H^{2m} \times H^m_0} \leq \frac{c_2}{L\gamma_2}.
\] (2.41)

Thus, there exist \(t = t_2(\Omega)\) and \(R_2\), such that
\[
\|u(t), v\|_{H^{2m} \times H^m_0} \leq R_2(t > t_2).
\] (2.42)

### 3. Global Attractor

#### 3.1. The Existence and Uniqueness of Solution

**Theorem 3.1.** Assume (H1) - (H4) hold, and \((u_0, u_1) \in H^{2m} \times H^m_0, \ f(x) \in H^m_0\), \(v = u_1 + \epsilon u\). So equality (1.1) exists a unique smooth solution \((u, v) \in L^2((0, +\infty), H^{2m} \times H^m_0)\).

**Remark 2.** We denote the solution in Theorem 3.1 by \(S(t)(u_0, u_1) = (u(t), u(t))\). Then \(S(t)\) composes a continuous semigroup in \(H^{2m} \times H^m_0\).

**Proof.** By the Galerkin method, Lemma 1 and Lemma 2, we can easily obtain the existence of Solutions, the procedure is omitted. Next, we prove the uniqueness of Solu-
tions in detail. Let $u, v$ are two solutions of the problems (1.1) - (1.3), we denote $w = u - v$, then $w(x, 0) = w_0(x) = 0$, $w_i(x, 0) = w_i(x) = 0$ and the two equations subtract and obtain

$$w_x + \sigma \left( \|\nabla^m u\|^2 \right) (\Delta)^m u - \sigma \left( \|\nabla^m v\|^2 \right) (\Delta)^m v$$

$$+ \phi \left( \|\nabla^m u\|^2 \right) (\Delta)^m u - \phi \left( \|\nabla^m v\|^2 \right) (\Delta)^m v = 0. \quad (3.1)$$

By using $w_i + \varepsilon w$ to inner product of the equation (3.1), and we have

$$\left( w_x + \sigma \left( \|\nabla^m u\|^2 \right) (\Delta)^m u - \sigma \left( \|\nabla^m v\|^2 \right) (\Delta)^m v, w_i + \varepsilon w \right)$$

$$= \left( \sigma \left( \|\nabla^m u\|^2 \right) (\Delta)^m u - \sigma \left( \|\nabla^m v\|^2 \right) (\Delta)^m v, w_i + \varepsilon w \right)$$

$$= \left( \sigma \left( \|\nabla^m u\|^2 \right) (\Delta)^m u - \sigma \left( \|\nabla^m v\|^2 \right) (\Delta)^m v, w_i \right)$$

$$+ \varepsilon \left( \sigma \left( \|\nabla^m u\|^2 \right) (\Delta)^m u - \sigma \left( \|\nabla^m v\|^2 \right) (\Delta)^m v, w \right)$$

$$= I_1 + \varepsilon I_2, \quad (3.2)$$

Next, we process each item in turn

$$I_1 = \sigma \left( \|\nabla^m u\|^2 \right) (\Delta)^m u - \sigma \left( \|\nabla^m v\|^2 \right) (\Delta)^m v$$

$$+ \sigma \left( \|\nabla^m u\|^2 \right) (\Delta)^m v - \sigma \left( \|\nabla^m v\|^2 \right) (\Delta)^m v, w_i$$

$$= \sigma \left( \|\nabla^m u\|^2 \right) ((\Delta)^m w_i, w_i) + \left( \sigma \left( \|\nabla^m u\|^2 \right) - \sigma \left( \|\nabla^m v\|^2 \right) \right) ((\Delta)^m v, w_i)$$

$$\geq \sigma \left( \|\nabla^m u\|^2 \right) \|\nabla^m w_i\|^2 + \sigma' (\|x\|) \left( \|\nabla^m u\| + \|\nabla^m v\| \right) \|\nabla^m u\| - \|\nabla^m v\| \|\nabla^m v, \nabla^m w_i\|$$

$$\geq \sigma \left( \|\nabla^m u\|^2 \right) \|\nabla^m w_i\|^2 - \|x\| \left( \|\nabla^m u\| + \|\nabla^m v\| \right) \|\nabla^m u\| \|\nabla^m w, \nabla^m w_i\|$$

$$= \sigma \left( \|\nabla^m u\|^2 \right) \|\nabla^m w_i\|^2 - c_1 \|\nabla^m w\| \|\nabla^m w_i\|$$

$$= \sigma \left( \|\nabla^m u\|^2 \right) \|\nabla^m w_i\|^2 - \frac{c_1}{2\varepsilon} \|\nabla^m w\|^2 - \frac{c_2}{2\varepsilon} \|\nabla^m w_i\|^2$$

$$= \sigma \left( \|\nabla^m u\|^2 \right) \|\nabla^m w_i\|^2 - \frac{c_1}{2\varepsilon} \|\nabla^m w\|^2.$$ 

Analogous to $I_1$, we deal with $I_2$
\[ I_2 = \left( \sigma \left( \|\nabla^w w\| \right) (\Delta)^w u - \sigma \left( \|\nabla^w v\| \right) (\Delta)^w v \right) \\
+ \sigma \left( \|\nabla^w u\| \right) (\Delta)^w v - \sigma \left( \|\nabla^w v\| \right) (\Delta)^w v, w \] \\
= \frac{\sigma \left( \|\nabla^w u\| \right)}{2} \frac{d}{dt} \|\nabla^w w\|^2 + \sigma' (\xi) \left( \|\nabla^w u\| + \|\nabla^w v\| \right) \left( \|\nabla^w u\| - \|\nabla^w v\| \right) (\Delta)^w v, w \] 
(3.6)
\[ \geq \frac{\sigma \left( \|\nabla^w u\| \right)}{2} \frac{d}{dt} \|\nabla^w w\|^2 - \sigma' (\xi) \left( \|\nabla^w u\| + \|\nabla^w v\| \right) \|\nabla^w v, \|\nabla^w w\|^2 \] \\
= \frac{\sigma \left( \|\nabla^w u\| \right)}{2} \frac{d}{dt} \|\nabla^w w\|^2 - c_4 \|\nabla^w w\|^2 .

Combining with (3.5) - (3.6), we obtain from (3.4) that

\[ \left( \sigma \left( \|\nabla^w u\| \right) (\Delta)^w u - \sigma \left( \|\nabla^w v\| \right) (\Delta)^w v, w, ++ \varepsilon w \right) \] \\
\[ \geq \left( \sigma \left( \|\nabla^w u\| \right) - \frac{c_2^2}{2\varepsilon} \right) \|\nabla^w w\|^2 - \frac{\varepsilon}{2} \|\nabla^w w\|^2 + \frac{\varepsilon}{2} \sigma \left( \|\nabla^w u\| \right) \frac{d}{dt} \|\nabla^w w\|^2 - \varepsilon c_4 \|\nabla^w w\|^2 \] 
(3.7)
\[ = \left( \sigma \left( \|\nabla^w u\| \right) - \frac{c_2^2}{2\varepsilon} \right) \|\nabla^w w\|^2 + \frac{\varepsilon}{2} \sigma \left( \|\nabla^w u\| \right) \frac{d}{dt} \|\nabla^w w\|^2 - \left( \varepsilon c_4 + \frac{\varepsilon}{2} \right) \|\nabla^w w\|^2 .

Similarly,

\[ \left( \phi \left( \|\nabla^w u\| \right) (\Delta)^w u - \phi \left( \|\nabla^w v\| \right) (\Delta)^w v, w, + \varepsilon w \right) \] \\
\[ = \left( \phi \left( \|\nabla^w u\| \right) (\Delta)^w u - \phi \left( \|\nabla^w v\| \right) (\Delta)^w v, w, \right) \] \\
\[ + \varepsilon \left( \phi \left( \|\nabla^w u\| \right) (\Delta)^w u - \phi \left( \|\nabla^w v\| \right) (\Delta)^w v, \right) \] \\
\[ \geq \frac{\phi \left( \|\nabla^w u\| \right)}{2} \frac{d}{dt} \|\nabla^w w\|^2 - \frac{\varepsilon}{2} \|\nabla^w w\|^2 - \frac{c_2^2}{2\varepsilon} \|\nabla^w w\|^2 \] \\
+ \varepsilon \phi \left( \|\nabla^w u\| \right) \|\nabla^w w\|^2 - \varepsilon c_4 \|\nabla^w w\|^2 \] 
(3.8)
\[ = \frac{\phi \left( \|\nabla^w u\| \right)}{2} \frac{d}{dt} \|\nabla^w w\|^2 - \frac{c_2^2}{2\varepsilon} \|\nabla^w w\|^2 + \left( \varepsilon \phi \left( \|\nabla^w u\| \right) - \frac{\varepsilon}{2} - \varepsilon c_4 \right) \|\nabla^w w\|^2 .

Therefore, by the above inequality

\[ \frac{d}{dt} \|\nabla^w \|^2 + 2\varepsilon \left( \|\nabla^w w\|, w \right) + \left( \phi \left( \|\nabla^w u\| \right) + \varepsilon \sigma \left( \|\nabla^w u\| \right) \right) \frac{d}{dt} \|\nabla^w w\|^2 \] \\
+ \left( 2\sigma \left( \|\nabla^w u\| \right) - \frac{c_2^2 + c_3^2}{\varepsilon} \right) \|\nabla^w w\|^2 - 2\varepsilon \|\nabla^w w\|^2 \] \\
+ \left( 2\varepsilon \phi \left( \|\nabla^w u\| \right) - 2\varepsilon - 2\varepsilon c_4 - 2\varepsilon c_5 \right) \|\nabla^w w\|^2 \leq 0 .

(3.9)

when \( \sigma \left( \|\nabla^w u\| \right) > \frac{c_2^2 + c_3^2}{2\varepsilon} \), we get
\[
\frac{d}{dr}(\|w\|^2 + 2\varepsilon(w, w)) + \phi(\|\nabla^m u\|^2) + \varepsilon\sigma(\|\nabla^m u\|^2) \frac{d}{dr}\|\nabla^m w\|^2 \\
\leq 2\varepsilon\|w\|^2 + 2\varepsilon(1 + c_4 + c_6 - \phi(\|\nabla^m u\|^2))\|\nabla^m w\|^2.
\] (3.10)

In view of (H4), there exist constant \(\mu\), and let \(c_7 = 1 + c_4 + c_6\), such that
\[
\frac{d}{dr}(\|w\|^2 + 2\varepsilon(w, w) + \mu\|\nabla^m w\|^2) \leq 2\varepsilon\|w\|^2 + 2\varepsilon(c_7 - \phi(\|\nabla^m u\|^2))\|\nabla^m w\|^2.
\] (3.11)

According to Hölder inequality, Young’s inequality and Poincaré inequality, we obtain
\[
\varepsilon^2 (w, w) \geq -\frac{\varepsilon^2}{2}\|w\|^2 - \frac{\varepsilon^2}{2}\|w\|^2 \geq -\frac{\varepsilon^2}{2}\|w\|^2 - \frac{\varepsilon^2}{2\lambda}\|\nabla^m w\|^2.
\] (3.12)

Combining with (3.11) - (3.12), we receive
\[
\frac{d}{dr}(\|w\|^2 + 2\varepsilon(w, w) + \mu\|\nabla^m w\|^2) \\
\leq (2\varepsilon + e^2)\|w\|^2 + 2\varepsilon\left(c_7 + \frac{\varepsilon^2}{2\lambda}\phi(\|\nabla^m u\|^2)\right)\|\nabla^m w\|^2 + 2\varepsilon^2 (w, w)
\] (3.13)
\[
= \varepsilon \left[(2 + \varepsilon)\|w\|^2 + 2\left(c_7 + \frac{\varepsilon^2}{2\lambda}\phi(\|\nabla^m u\|^2)\right)\|\nabla^m w\|^2 + 2\varepsilon (w, w)\right].
\]

Next, we prove that there is a constant \(K\) large enough, such that
\[
(2 + \varepsilon)\|w\|^2 + 2\varepsilon(w, w) + \mu\|\nabla^m w\|^2 \\
\leq K\left(\|w\|^2 + 2\varepsilon(w, w) + \mu\|\nabla^m w\|^2\right).
\] (3.14)

Supposing there is a constant \(K\) large enough, we have
\[
(2 + \varepsilon - K)\|w\|^2 + 2\left(c_7 + \frac{\varepsilon^2}{2\lambda}\phi(\|\nabla^m u\|^2)\right)\|\nabla^m w\|^2 + (2\varepsilon - 2\varepsilon K)(w, w) \\
\leq (2 + \varepsilon - K)\|w\|^2 + 2\left(c_7 + \frac{\varepsilon^2}{2\lambda}\phi(\|\nabla^m u\|^2)\right)\|\nabla^m w\|^2 + (2\varepsilon - 2\varepsilon K)(w, w) \\
\leq (2 + \varepsilon - K)\|w\|^2 + 2\left(c_7 + \frac{\varepsilon^2}{2\lambda}\phi(\|\nabla^m u\|^2)\right)\|\nabla^m w\|^2 + 2\varepsilon (K - 1)\|w\|^2 \\
\leq (2 + \varepsilon - K)\|w\|^2 + 2\left(c_7 + \frac{\varepsilon^2}{2\lambda}\phi(\|\nabla^m u\|^2)\right)\|\nabla^m w\|^2 \\
+ \varepsilon (K - 1)\|w\|^2 + \frac{\varepsilon (K - 1)\lambda}{\lambda}\|\nabla^m w\|^2 \\
= [2 + (\varepsilon - K)]\|w\|^2 + 2\left[c_7 - \phi(\|\nabla^m u\|^2) + \left(\frac{\varepsilon^2}{2\lambda} - \frac{\varepsilon}{2}\mu\right)K\right]\|\nabla^m w\|^2 \\
\leq 0,
\] (3.15)

where \(\varepsilon = \min \{1, \lambda^\mu\}, \phi(\|\nabla^m u\|^2) < c_7 + \frac{\varepsilon^2}{2\lambda}\).
Hence, there is a constant $K$ large enough, such that (3.14) hold. Due to (3.14), we have
\[
\frac{d}{dt} Y(t) \leq \varepsilon KY(t),
\]
where
\[
Y(t) = \|w(t)\|^2 + 2\varepsilon (w_t, w) + \mu \|\nabla w\|^2.
\]
Therefore,
\[
0 \leq Y(t) \leq Y(0) e^{\varepsilon K t} = 0,
\]
where
\[
Y(0) = \|w_{(0)}\|^2 + 2\varepsilon (w_{(0)}, w(0)) + \mu \|\nabla w(0)\|^2.
\]
So, we can get
\[
\|w\|^2 + 2\varepsilon (w, w) + \mu \|\nabla w\|^2 = 0.
\]
According to (3.12), we get
\[
(1-\varepsilon)\|w\|^2 + \left(\mu - \frac{\varepsilon}{\lambda m}\right)\|\nabla w\|^2 \leq 0.
\]
That shows that
\[
\|w\|^2 = 0, \|\nabla w\|^2 = 0.
\]
That is
\[
w(s, t) = 0.
\]
Therefore,
\[
u = v.
\]
So we prove the uniqueness of the solution.

### 3.2. Global Attractor

**Theorem 3.2.** [11] Let $E$ be a Banach space, and \( \{S(t)\}_{t \geq 0} \) are the semigroup operator on $E$. \( S(t) : E \to E, \ S(t + \tau) = S(t)S(\tau) (\forall t, \tau \geq 0), \ S(0) = I, \) here $I$ is a unit operator. Set $S(t)$ satisfy the follow conditions:

1) $S(t)$ is uniformly bounded, namely $\forall R > 0, \|w\| \leq R$, it exists a constant $C(R)$, so that
\[
\|S(t)w\| \leq C(R)(t \in [0, +\infty));
\]

2) It exists a bounded absorbing set $B_0 \subset E$, namely, $\forall B \subset E$, it exists a constant $t_0$, so that
\[
S(t)B \subset B_0 (t \geq t_0);
\]

where $B_0$ and $B$ are bounded sets.

3) When $t > 0$, $S(t)$ is a completely continuous operator $A$. 
Therefore, the semigroup operators \( S(t) \) exists a compact global attractor \( A \).

**Theorem 3.3.** Under the assume of Lemma 1, Lemma 2 and Theorem 3.1, equations have global attractor

\[
A = \omega(B_0) = \bigcap_{t \geq 0} \overline{\bigcup_{s \geq t} S(s)B_0},
\]

where

\[
B_0 = \left\{ (u, v) \in H^{2m} \times H^m_0 : \left\| (u, v) \right\|_{H^{2m} \times H^m_0} = \left\| u \right\|_{H^{2m}} + \left\| v \right\|_{H^m_0} \leq R_1 + R_2 \right\},
\]

\( B_0 \) is the bounded absorbing set of \( H^{2m} \times H^m_0 \) and satisfies.

1) \( S(t)A = A, t > 0 \);
2) \( \lim_{t \to \infty} \text{dist}(S(t)B, A) = 0 \), here \( B \subset H^{2m} \times H^m_0 \) and it is a bounded set,

\[
\text{dist}(S(t)B, A) = \sup_{x \in A} \left( \inf_{y \in B} \| S(t)x - y \|_{H^{2m} \times H^m_0} \right) \to 0, t \to \infty.
\]

**Proof.** Under the conditions of Theorem 3.1, it exists the solution semigroup \( S(t) \), \( S(t) : H^{2m} \times H^m_0 \to H^{2m} \times H^m_0 \), here \( E = H^{2m} \times H^m_0 \).

1) From Lemma 1 to Lemma 2, we can get that \( \forall B \subset H^{2m} \times H^m_0 \) is a bounded set that includes in the ball \( \left\{ (u, v) : \left\| (u, v) \right\|_{H^{2m} \times H^m_0} \leq R \right\} \),

\[
\left\| S(t)(u_0, v_0) \right\|_{H^{2m} \times H^m_0} = \left\| u_0 \right\|_{H^{2m}} + \left\| v_0 \right\|_{H^m_0} + C
\leq \left\| u_0 \right\|_{H^{2m}} + \left\| v_0 \right\|_{H^m_0} + C
\leq R^2 + C, \quad (t \geq 0, (u_0, v_0) \in B)
\]

This shows that \( S(t)(t \geq 0) \) is uniformly bounded in \( H^{2m} \times H^m_0 \).

2) Furthermore, for any \( (u_0, v_0) \in H^{2m} \times H^m_0 \), when \( t \geq \max \{ t_1, t_2 \} \), we have

\[
\left\| S(t)(u_0, v_0) \right\|_{H^{2m} \times H^m_0} = \left\| u_0 \right\|_{H^{2m}} + \left\| v_0 \right\|_{H^m_0} \leq R_1 + R_2.
\]

So we get \( B_0 \) is the bounded absorbing set.

3) Since \( H^{2m} \times H^m_0 \hookrightarrow H^m \times H \) is compact embedded, which means that the bounded set in \( H^{2m} \times H^m \) is the compact set in \( H^m \times H \), so the semigroup operator \( S(t) \) exist a compact global attractor \( A \).

The prove is completed.

4. Conclusion

The paper’s main results deal with global attractors. At first, we prove the existence and uniqueness of the solution. Then we establish the existence of the global attractors. Therefore, we show that i) the solution \( (u, v) \) of the problem (1.1) - (1.3) satisfies \( (u, v) \in H^m(\Omega) \times L^2(\Omega) \); furthermore, ii) the solution \( (u, v) \) of the problem (1.1) - (1.3) satisfies \( (u, v) \in H^{2m}(\Omega) \times H^m_0(\Omega) \). Then, we prove the uniqueness of the solution. At last, according to define and theorem, we obtain to the existence of the global attractor.

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**References**


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