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Adaptation in Stochastic Dynamic Systems—Survey and New Results IV: Seeking Minimum of API in Parameters of Data

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ABSTRACT

This paper investigates the problem of seeking minimum of API (Auxiliary Performance Index) in parameters of Data Model instead of parameters of Adaptive Filter in order to avoid the phenomenon of over parameterization. This problem was stated by Semushin in [2]. The solution to the problem can be considered as the development of API approach to parameter identification in stochastic dynamic systems.

Keywords: Linear Stochastic Systems; Parameter Estimation; Model Identification; Identification for Control; Adaptive Control MSC (2010); 93E10; 93E12; 93E35

1. Introduction

The recent papers [1,2] gave a survey of the field of adaptation in stochastic systems as it has developed over the last four decades. The author's research in this field was summarized and a novel solution for fitting an adaptive model in state space (instead of response space) was given.

In this paper, we further develop the Active Principle of Adaptation for linear time-invariant state-space stochastic MIMO filter systems included into the feedback or considered independently.

We solve the problem of seeking minimum of Auxiliary Performance Index (API) in parameters of Data Source Model (DSM) instead of parameters of Adaptive Filter (AF) in order to avoid difficulties known as Phenomenon of Over Parameterization (PhOP). The PhOP means that the number of parameters to be adjusted in AF is usually much greater than that in DSM. The solution of this problem will enable identification in the space of lower dimension and at the same time provide estimates of the given system state vector according to Original Performance Index (OPI). We verify the obtained theoretical results by two numerical simulation examples.

2. Parameterized Data Models $\mathcal{D}(\theta)$

Following the previous results of [1,2], we assume that all data models $\mathcal{D}(\theta)$ forming a set $\mathcal{D}$ are parameterized by an l-component vector θ. Each particular value of θ (which does not depend on time) specifies a $\mathcal{D}(\theta)$. Hence

$$\mathcal{D}=\left\{\mathcal{D}(\theta)\middle|\theta\in\Theta\subset\mathsf{R}^l\right\} \tag{1}$$

where Θ is the compact subset of R^l. A given physical data model (PhDM) is described by the following equations:

$$\mathcal{D}(\theta):\ \begin{array}{l} x_{t+1}=\Phi(\theta)x_t+\Psi(\theta)u_t+\Gamma(\theta)w_t, t\in\mathsf{Z}_+ \\ y_t=H(\theta)x_t+v_t, t\in\mathsf{Z}_1 \end{array} \tag{2}$$

where Z_+ denotes nonnegative integers, Z_1 strictly positive integers, and Z all integers.

Every model $\mathcal{D}(\theta)$ (2) is assumed to be acting between adjacent switches as long as it is sufficient for accepting as correct the basic theoretical statement (BTS) that all processes related to the $\mathcal{D}(\theta)$ are wide-sense stationary. This statement amounts to the following assumptions. The random x_0 with $\mathsf{E}\{\|x_0\|\}^2<\infty$ is orthogonal [3] to w_t and v_t, the zero-mean mutually orthogonal wide-sense stationary orthogonal sequences with $E\{w_t w_t^T\}=Q(\theta)\geq 0$ and $E\{v_t v_t^T\}=R(\theta)>0$ for all $t\in\mathsf{Z}$; $\left[w^T v^T\right]^T$ is orthogonal to x_j and u_j for all $j \ge t$; u_t is a given signal; it is an "external input" when considering the open-loop case *or* a control strategy function

$$u_t = u_{\mathcal{X}}\left(t, y_1^t, u^{t-1}\right) \tag{3}$$

when considering the closed-loop setup.

Stackable vectors of previous values

$$\mathcal{X}: \begin{array}{l} y_1^t = \left(y_t, y_{t-1}, \cdots, y_1\right) \\ u^{t-1} = \left(u_{t-1}, u_{t-2}, \cdots, u_0\right) \end{array} \tag{4}$$

constitute the experimental condition $\mathcal{X}$ (cf. Ljung [4]).

By assumption, $y_t \in \mathrm{R}^m$ is generated by the completely observable PhDM (2), so we can move from the physical state variables $x \in \mathrm{R}^n$ in (2) to another x^* through the similarity transformation $x^* = W_* x$. Such transformation uniquely determines a new state representation

$$\mathcal{D}^*(\theta): \begin{array}{l} x_{t+1}^* = \Phi_*(\theta) x_t^* + \Psi_*(\theta) u_t + \Gamma_*(\theta) w_t, t \in \mathrm{Z}_+ \\ y_t = H_* x_t^* + v_t, t \in \mathrm{Z}_1 \end{array} \tag{5}$$

of the standard observable data model (SODM) (cf. Semushin [2]).

For convenience in the below we shall omit the subscript θ for all the matrices describing PhDM or SODM.

3. Parameterized Innovations

As before, the above data model of a time-invariant data source will be referred to as the *conventional model*, no matter whether it is PhDM (2) or SODM (5). Here we use another *innovation model*, that differs from the time-invariant (due to BTS) innovation model, presented in [2]:

$$\mathcal{M}(\theta): \begin{array}{l} x_{t+1|t} = \Phi x_{t|t-1} + \Psi u_t + G_t \nu_{t|t-1} \\ y_t = H x_{t|t-1} + \nu_{t|t-1} \end{array} \tag{6}$$

with $t \in \mathrm{Z}_1$, the initial $x_{1|0} = \Phi \bar{x}_0 + \Psi u_0$, and $\bar{x}_0 = E\{x_0\}$, which is the well-known (not necessarily steady-state) Kalman filter with the innovation process $\nu_{t|t-1}$, the optimal state predictor $x_{t+1|t}$, the gain

$$\begin{array}{l} K_t = \Sigma_{t|t-1} H^T \left(H \Sigma_{t|t-1} H^T + R\right)^{-1}, \\ G_t = \Phi K_t \end{array} \tag{7}$$

and $\Sigma_{t|t-1}$ satisfying the discrete Riccati iterations [5]

$$\begin{array}{l} \Sigma_{t+1|t} = \Phi \Big[\Sigma_{t|t-1} - \Sigma_{t|t-1} H^T \left(H \Sigma_{t|t-1} H^T + R\right)^{-1} \\ \qquad \times H \Sigma_{t|t-1}\Big] \Phi^T + \Gamma Q \Gamma^T. \end{array} \tag{8}$$

Concurrently, another form

$$\mathcal{M}(\theta): \begin{array}{l} x_{t+1|t} = \Phi x_{t|t} + \Psi u_t, t \in \mathrm{Z}_+ \\ x_{t|t} = x_{t|t-1} + K_t \nu_{t|t-1}, t \in \mathrm{Z}_1 \\ y_t = H x_{t|t-1} + \nu_{t|t-1}, t \in \mathrm{Z}_1 \end{array} \tag{9}$$

with the initial $x_{0|0} = \bar{x}_0$, which is equivalent to (6), can be used where $x_{t|t}$ is the optimal "filtered" estimator for x_t based on experimental condition $\mathcal{X}$ (4). When θ ranges (or switches) over Θ as in (1), we obtain the set of Kalman filters

$$\mathcal{M} = \left\{\mathcal{M}(\theta) \middle| \theta \in \Theta \subset \mathrm{R}^l\right\}. \tag{10}$$

We consider the mean-square criterion

$$J_t^{\mathrm{o}} \overset{\Delta}{=} \frac{1}{2} \mathrm{E}\left\{e_{t+1|t}^T e_{t+1|t}\right\} \tag{11}$$

defined for a one-step predictor $\hat{g}_{t+1|t}$ through its error $e_{t+1|t} = x_{t+1} - \hat{g}_{t+1|t}$ in the Kalman filter. Thus in the basis forming the state-space, $\mathcal{M}(\theta)$ (9) is the model minimizing the Original Performance Index (OPI) J_t^{o} (11) at any t, which is large enough for BTS to hold, so that writing t or $t+1$ as well as any other finitely shifted time in (11) makes no difference.

4. Uncertainty Parameterization

In contrast to our previous work [2], we do not consider the four levels of uncertainty. Assume that system (2) (and also the SODM (5)) is parameterized by an l-component vector θ of unknown system parameters, which needs to be identified. This means that the entries of the matrices $\Phi(\theta)$, $\Psi(\theta)$, $\Gamma(\theta)$, $Q(\theta)$, $H(\theta)$, $R(\theta)$ are functions of $\theta \in D(\theta)$. However, for the sake of simplicity we will supress the corresponding notations below, *i.e.* instead of $\Phi(\theta)$, $\Psi(\theta)$, $\Gamma(\theta)$, $Q(\theta)$, $H(\theta)$, $R(\theta)$ we will write Φ, Ψ, Γ, Q, H, R. We make the same assumptions about the SODM.

5. The Set $\mathcal{A}$ of Adaptive Models $\mathcal{M}(\hat{\theta})$

Let us consider the set of adaptive models

$$\mathcal{A} = \left\{\mathcal{M}(\hat{\theta}) \middle| \hat{\theta} \in \Theta \subset \mathrm{R}^l\right\} \tag{12}$$

Here we emphasize the fact that we construct adaptive models in the same class as $\mathcal{M}$ belongs to with the only difference that the unknown parameter θ in $\mathcal{M}(\theta)$ is replaced by $\hat{\theta}$ to obtain $\mathcal{M}(\hat{\theta})$. In so doing, each particular value of $\hat{\theta}$, an estimate of θ, leads to a fixed model $\mathcal{M}(\hat{\theta})$. In accordance with The Active Principle of Adaptation (APA) [1], only when $\hat{\theta}$ ranges over Θ in search of $\theta^\dagger$ for *the goal* $\mathcal{M}^*(\theta^\dagger)$ or $\mathcal{M}(\theta^\dagger)$ as governed by a smart, unsupervised helmsman equipped by a vision of the goal in state space and able to pursue it, we obtain a model $\mathcal{M}(\hat{\theta})$ of *active type* within the set $\mathcal{A}$ (12). In this case, $\hat{\theta}$ will act as a self-tuned model parameter and so should be labeled by τ, the time instant of model's inner clock, in order to get thereby the emphasized notations $\hat{\theta}_\tau$ and $\mathcal{M}(\hat{\theta}_\tau)$ in describing parameter identification algorithms (PIAs) to be developed. From this point on $\mathcal{M}(\hat{\theta}_\tau)$ becomes an adaptive estimator.

Remark 1 Note in passing that pace of τ may differ from that of t. We shall need to discriminate between τ and t later when developing a PIA.

Remark 2 If we work in the context of SODM, the set

$$\mathcal{A}^* = \left\{ \mathcal{M}^*(\hat{\theta}) \middle| \hat{\theta} \in \Theta \subset \mathrm{R}^l \right\} \quad (13)$$

instead of (12) should be used.

At this junction, we identify the following tasks as pending:

1) Express $\mathcal{M}^*(\hat{\theta})$ or $\mathcal{M}(\hat{\theta})$ in an explicit form.
2) Build up APIs that could offer vision of the goal.
3) Examine APIs' capacity to visualize the goal.
4) Develop a PIA that could help pursuing the goal.

Consider here the first three points consecutively.

5.1. Parameterized Adaptive Models

Reasoning from (6), (9), we set the adaptive model

$$\mathcal{M}(\hat{\theta}): \begin{aligned} \hat{g}_{t+1|t} &= \Phi(\hat{\theta})\hat{g}_{t|t-1} + \Psi(\hat{\theta})u_t + G_t(\hat{\theta})\eta_{t|t-1} \\ y_t &= H(\hat{\theta})\hat{g}_{t|t-1} + \eta_{t|t-1} \end{aligned} \quad (14)$$

or equivalently (due to $G_t = \Phi K_t$) the model

$$\mathcal{M}(\hat{\theta}): \begin{aligned} \hat{g}_{t+1|t} &= \Phi(\hat{\theta})\hat{g}_{t|t} + \Psi(\hat{\theta})u_t \\ \hat{g}_{t|t} &= \hat{g}_{t|t-1} + K_t(\hat{\theta})\eta_{t|t-1} \\ y_t &= H(\hat{\theta})\hat{g}_{t|t-1} + \eta_{t|t-1} \end{aligned} \quad (15)$$

as a member of $\mathcal{A}$ (12). Here $\hat{\theta}$ is the self-tuned parameter intended to estimate (in one-to-one corresponddence) parameter θ. In parallel, reasoning from $\mathcal{M}^*(\theta)$, we build the adaptive model

$$\mathcal{M}^*(\hat{\theta}): \begin{aligned} \hat{g}_{t+1|t} &= \Phi_*(\hat{\theta})\hat{g}_{t|t-1} + \Psi_*(\hat{\theta})u_t + G_t^*(\hat{\theta})\eta_{t|t-1} \\ y_t &= H_*\hat{g}_{t|t-1} + \eta_{t|t-1} \end{aligned} \quad (16)$$

or equivalently (due to $G_t^* = \Phi_* K_t^*$) the model

$$\mathcal{M}^*(\hat{\theta}): \begin{aligned} \hat{g}_{t+1|t} &= \Phi_*(\hat{\theta})\hat{g}_{t|t} + \Psi_*(\hat{\theta})u_t \\ \hat{g}_{t|t} &= \hat{g}_{t|t-1} + K_t^*(\hat{\theta})\eta_{t|t-1} \\ y_t &= H_*\hat{g}_{t|t-1} + \eta_{t|t-1} \end{aligned} \quad (17)$$

where H_* does not depend on $\hat{\theta}$. Matrices $G_t^*(\hat{\theta})$ and $K_t^*(\hat{\theta})$ are evaluated according to (7), (8).

Adaptor $\mathcal{A}(\hat{\theta})$ using (14)-(15) (or alternatively, $\mathcal{A}^*(\hat{\theta})$ using (16)-(17)) is supposed to contain a PIA to offer the prospect of convergence. For convergence in parameter space, we anticipate almost surely (a. s.) convergence, as it is the case for MPE identification methods [3,4]. It actuates either or both of the two other types of convergence. The type of convergence in state space, as well as in response space, is induced by the type of Proximity Criterion, PC (cf. [2], **Figures 1-3**). As seen from (11), we are oriented to the PC, which is quadratic in error; this being so, it would appear reasonable that these convergences would be in mean square (m. s.). Thus we anticipate the following properties of our estimators:

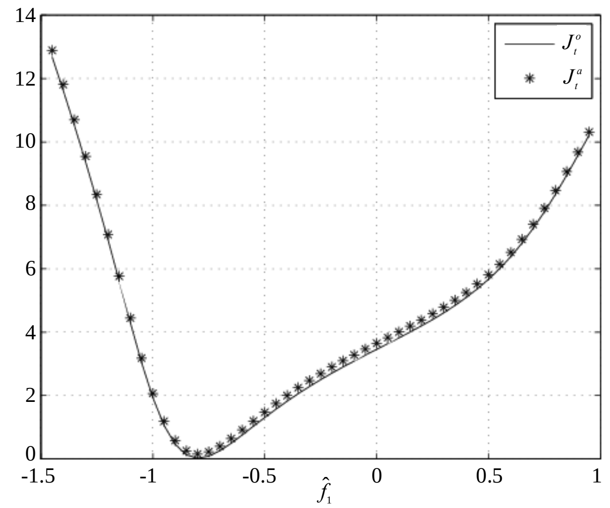

Figure 1. The values of J_t^o and J_t^a versus the estimates of parameter f_1 (*Example E*1).

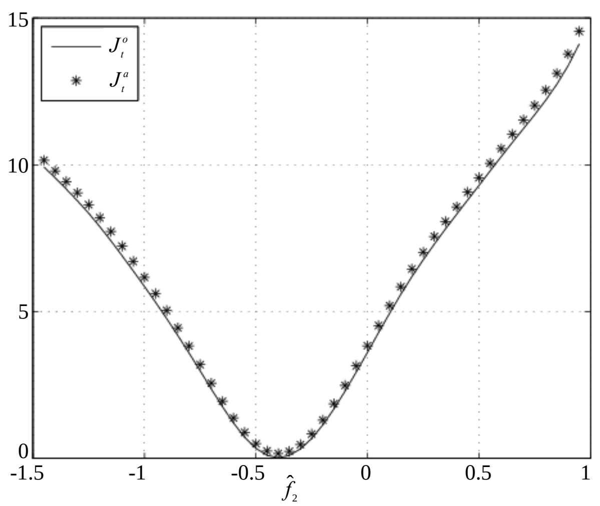

Figure 2. The values of J_t^o and J_t^a versus the estimates of parameter f_2 (*Example E*1).

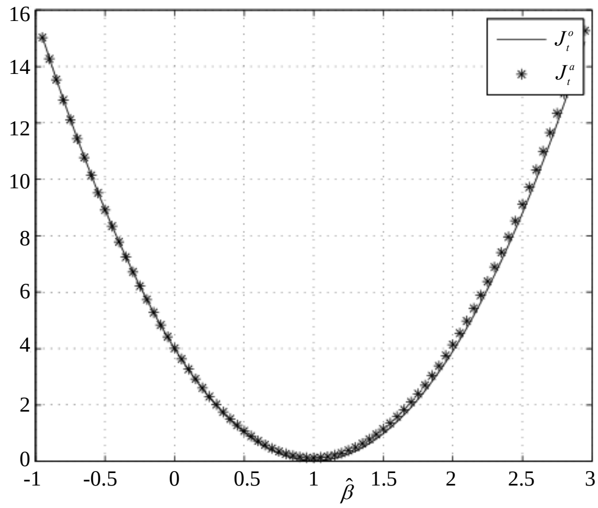

Figure 3. The values of J_t^o and J_t^a versus the estimates of parameter β (*Example E*1).

$$\left.\begin{array}{l} \mathcal{M}\left(\hat{\theta}_\tau\right) \xrightarrow{\text{m.s.}} \mathcal{M}\left(\theta^\dagger\right) \\ \mathcal{M}^*\left(\hat{\theta}_\tau\right) \xrightarrow{\text{m.s.}} \mathcal{M}^*\left(\theta^\dagger\right) \end{array} : \begin{array}{ll} \hat{\theta}_\tau & \xrightarrow{\text{a.s.}} \theta^\dagger \\ \hat{g}_{t+1|t} & \xrightarrow{\text{m.s.}} x_{t+1|t} \\ \hat{g}_{t|t} & \xrightarrow{\text{m.s.}} x_{t|t} \end{array} \right|_{t\to\infty} \tag{18}$$

With the understanding that errors for PC

$$\begin{array}{l} e_{t+1|t} \overset{\Delta}{=} x_{t+1} - \hat{g}_{t+1|t}, e_{t|t} \overset{\Delta}{=} x_t - \hat{g}_{t|t} \\ r_{t+1|t} \overset{\Delta}{=} x_{t+1|t} - \hat{g}_{t+1|t}, r_{t|t} \overset{\Delta}{=} x_{t|t} - \hat{g}_{t|t} \end{array} \tag{19}$$

are fundamentally invisible for any measurement, we search for a function

$$\varepsilon_t\left(\hat{\theta}\right) = f\left(y_{t+1}^{t+s} - \hat{y}_{t+1|t}^{t+s|t}\right) \in \mathsf{R}^n \tag{20}$$

of the difference in two terms: outputs y_{t+1}^{t+s} generated by Data Source described in any appropriate form (2), (5), (6), or (9), and their estimates $\hat{y}_{t+1|t}^{t+s|t}$ generated by the adaptive model $\mathcal{M}\left(\hat{\theta}\right)$ (or $\mathcal{M}^*\left(\hat{\theta}\right)$). For $\varepsilon_t\left(\hat{\theta}\right)$ in (20), we will also use notations $\varepsilon_{t+1|t}$ or $\varepsilon_{t|t}$, thus bringing them into correlation with $e_{t+1|t}$ or $e_{t|t}$ (correspondingly, with $r_{t+1|t}$ or $r_{t|t}$) from (19). Then

$$J_t^{\text{a}} = J_t^{\text{a}}\left(\hat{\theta}\right) \overset{\Delta}{=} \frac{1}{2} E\left\{\varepsilon_t\left(\hat{\theta}\right)^T \varepsilon_t\left(\hat{\theta}\right)\right\} \tag{21}$$

will be taken as the PC and determined with the key aim:
True (Unbiased) System Identifiability

$$\min_{\hat{\theta}} J_t^{\text{a}}\left(\hat{\theta}\right) \lhd\!\!=\!\!\rhd \mathcal{M}\left(\hat{\theta}\right) \equiv \mathcal{M}\left(\theta^\dagger\right) \vee \mathcal{M}^*\left(\hat{\theta}\right) \equiv \mathcal{M}^*\left(\theta^\dagger\right)$$

Here, the equivalence symbol $\equiv$ needs clarification. Its sense correlates with the above concept of convergence (18). Necessary refinements will be done (in Theorem 1).

5.2. API Identifiability of $\mathcal{M}^*\left(\theta^\dagger\right)$

Let the auxiliary process (20) be built for the API (21) as

$$\varepsilon_{t+1|t} = S\left(y_{t+1}^{t+s}\right) - \hat{g}_{t+1|t} - \mathcal{F}\left(\Psi_*\left(\hat{\theta}\right)\right) u_{t+1}^{t+s} \tag{22}$$

or, equivalently, as

$$\varepsilon_{t|t} = S\left(y_{t+1}^{t+s}\right) - \Phi_*\left(\hat{\theta}\right)\hat{g}_{t|t} - \mathcal{B}\left(\Psi_*\left(\hat{\theta}\right)\right) u_t^{t+s-1} \tag{23}$$

where special matrix transformations are used (see the section "Ancillary Matrix Transformations" of [2]).

Theorem 1 Let $\varepsilon_t\left(\hat{\theta}\right)$ (20) be a vector-valued n-component function of $\eta_{t+1|t}^{t+s|t}$. If $\varepsilon_t\left(\hat{\theta}\right)$ is defined by (22) or (equivalently) (23) in order to form the API (21), then minimum in $\hat{\theta}$ of the API fixed out at any instant *t* is the necessary and sufficient condition for adaptive model $\mathcal{M}^*\left(\hat{\theta}\right)$ to be consistent estimator of $\mathcal{M}^*\left(\theta^\dagger\right)$ in mean *square*,

$$\forall t \in \mathsf{Z}_+ : \mathcal{M}^*\left(\hat{\theta}\right) \overset{\text{m.s.}}{\equiv} \mathcal{M}^*\left(\theta^\dagger\right), \text{ that is}$$

True (Unbiased) *m.s.* System Identifiability

$$\min_{\hat{\theta}} J_t^{\text{a}}\left(\hat{\theta}\right) \lhd\!\!=\!\!\rhd E\left\{\left\|x_{t+1|t} - \hat{g}_{t+1|t}\right\|^2\right\} = 0$$

in the following three setups:

Setup 1 (*Random Control Input*) $\{u_t\}$ is a preassigned zero-mean orthogonal wide-sence stationary process orthogonal to $\{w_t, v_t\}$ but in contrast to $\{w_t\}$ and $\{v_t\}$, known and serving as a testing signal;

Setup 2 (*Pure Filtering*) $\forall t \in \mathsf{Z}_+ : u_t = 0$, *and*

Setup 3 (*Close-loop Control*) with Ψ_*, which does not depend on θ.

Proof is similar to the proof of Theorem 2 in [2].

Remark 3 Our main goal is to identify the vector θ of unknown parameters. The minimization of API by some PIA allows us to determine the optimal value $\theta^\dagger$. Then it must be substituted into (14)-(15) (or (16)-(17)) to get the optimal model $\mathcal{M}\left(\hat{\theta}\right)$ (*or* $\mathcal{M}^*\left(\hat{\theta}\right)$). At the same time, we obtain the optimal estimates $\hat{x}_{t+1|t}$ (or $\hat{x}_{t+1|t}^*$) according to OPI.

5.3. Main Conceptual Novelty

Seeking minimum of API in parameters of Data Model instead of parameters of Adaptive Filter is more profitable, as:

- It takes into account the dynamics of the discrete Riccati equations, which positively affects the quality of parameter and state vector estimates.

- The number of unknown parameters can be substantially (an order of magnitude or more) reduced thus helping avoid the difficulties of PhOP.
- API gradient is calculated easily—without the construction of sensitivity model of adaptive filter.
- It can be implemented in the case of non-stationary systems, which is critical, for example, to handle the navigation data.

Thus, the proposed variant of API method is a new, thanks to the solution of its important tasks:

1) Numerical construction of API, which has the same minimizing argument that the OPI does;

2) The numerical minimization of the API by conventional optimization methods such as Newton-Raphson method, and

3) The combination of parameter identification of the system with the process of adaptive estimation of its states.

6. Simulation Examples

We take two examples to simulate:

E1 Second order open-loop system with unknown parameters $(f_1, f_2, \beta, \alpha)$ is given by

$$x_{t+1} = \begin{bmatrix} 0 & 1 \\ f_1 & f_2 \end{bmatrix} x_t + \begin{bmatrix} 0 \\ \beta \end{bmatrix} u_t + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} w_t$$

$$y_t = \begin{bmatrix} 1 & 0 \end{bmatrix} x_t + v_t$$

Unknown parameters should be identified. Adaptive model parameter is the four-component vector

$$\hat{\theta} = \left(\hat{f}_1, \hat{f}_2, \hat{\beta}, \hat{\alpha}\right).$$

Its true value is

$$\theta^{\dagger} = (-0.8,\ 0.1,\ 1.0,\ 0.4).$$

Covariances Q and R of the noises w_t and v_t are equal to 0.04 and 0.06, correspondingly.

E2 The same (but closed-loop) system as in E1. The system is designed to operate with a minimum expected control cost

$$J_c = E\left\{\frac{1}{500}\sum_{t=1}^{500}\left[\left(x_t^1\right)^2 + 5u_{t-1}^2\right]\right\}.$$

Unknown parameters α, β, f_1 and f_2 should be identified. The true values of parameters are the same as in E1.

Simulation results of **Figures 1-4** and **5-7** obtained from Julia Tsyganova's MATLAB programs demonstrate equimodality (coincidence of the minimizing arguments) of the auxiliary performance index with the original performance index. It is seen that the minimums of OPI and API coincide near $\theta^{\dagger}$. Thus, the obtained results confirm applicability of the presented method.

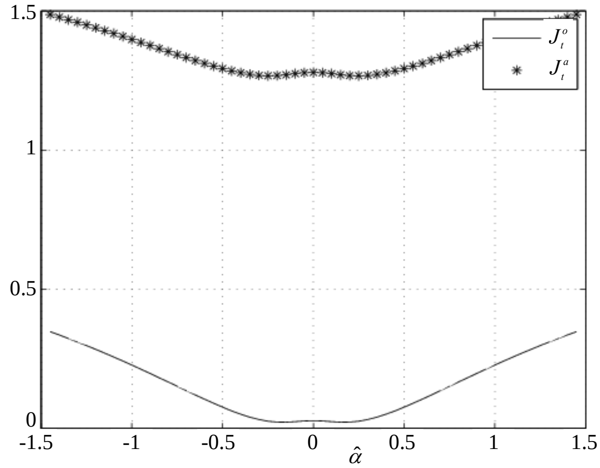

Figure 4. The values of J_t^o and J_t^a versus the estimates of parameter α (*Example E*1).

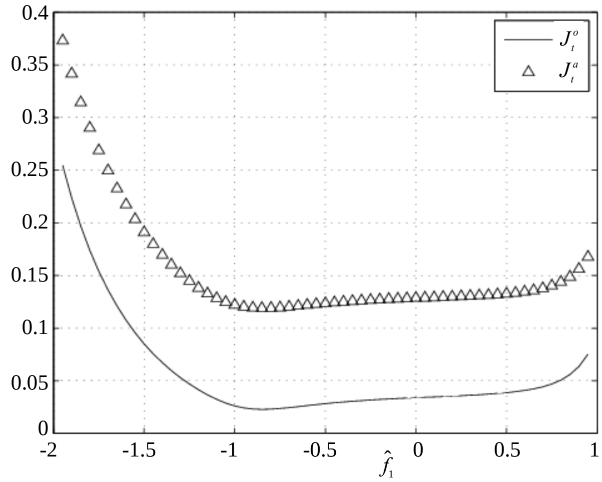

Figure 5. The values of J_t^o and J_t^a versus the estimates of parameter f_1 (*Example E*2).

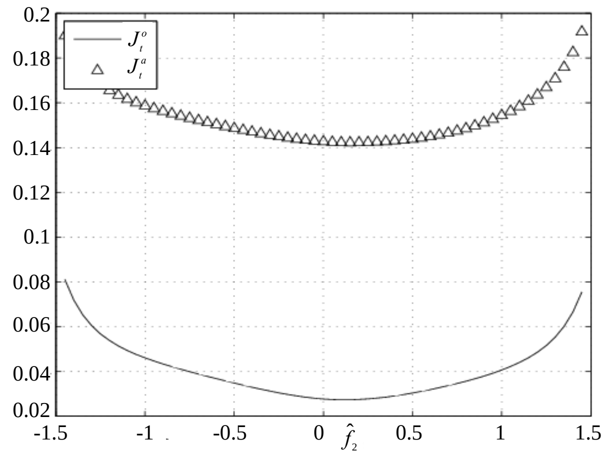

Figure 6. The values of J_t^o and J_t^a versus the estimates of parameter f_2 (*Example E*2).

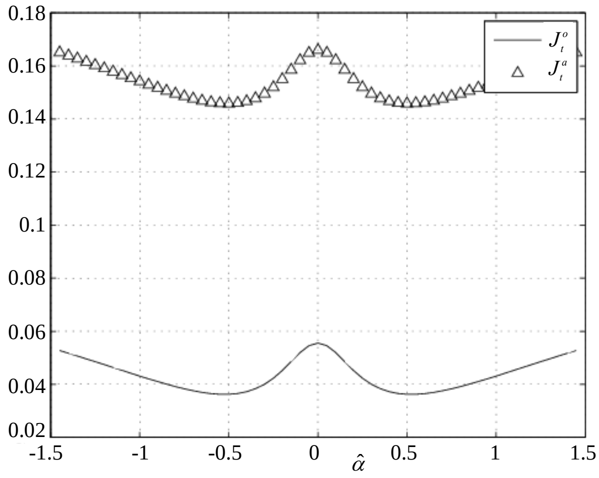

Figure 7. The values of J_t^o and J_t^a versus the estimates of parameter α (*Example E*2).

7. Conclusion

The present paper gives a comprehensive solution to the problem of seeking minimum of $J_t^a(\hat{\theta})$ in parameters $\hat{\theta}$ of Data Model $\mathcal{D}(\hat{\theta})$ or $\mathcal{D}^*(\hat{\theta})$ instead of parameters of Adaptive Filter $\mathcal{M}(\hat{\theta})$ or $\mathcal{M}^*(\hat{\theta})$. The obtained results were verified by two numerical simulation examples.

Our further research is aimed at obtaining solutions to the following issues:

- Economic feasibility, numeric stability and convergence reliability of each proposed parameter identification algorithm.
- Numerical testing of the approach and determining the scope of its appropriate use in real life problems, for example, in Health Care field [6].

8. Acknowledgements

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