

# Predictive FTF Adaptive Algorithm for Mobile Channels Estimation

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## ABSTRACT

The aim of this research paper is to improve the performance of Fast Transversal Filter (FTF) adaptive algorithm used for mobile channel estimation. A multi-ray Jakes mobile channel model with a Doppler frequency shift is used in the simulation. The channel estimator obtains the sampled channel impulse response (SIR) from the predetermined training sequence. The FTF is a computationally efficient implementation of the recursive least squares (RLS) algorithm of the conventional Kalman filter. A stabilization FTF is used to overcome the problem caused by the accumulation of roundoff errors, and, in addition, degree-one prediction is incorporated into the algorithm (Predictive FTF) to improve the estimation performance and to track changes of the mobile channel. The efficiency of the algorithm is confirmed by simulation results for slow and fast varying mobile channel. The results show about 5 to 15 dB improvement in the Mean Square Error (Deviation) between the estimated taps and the actual ones depending on the speed of channel time variations. Slow and fast vehicular channels with Doppler frequencies 100 Hz and 222 Hz respectively are used in these tests. The predictive FTF (PFTF) algorithm give a better channel SIR estimation performance than the conventional FTF algorithm, and it involves only a small increase in complexity.

**Keywords:** Mobile Channel Estimation; Fast Transversal Filter; Prediction; Adaptive Filtering Algorithms

## 1. Introduction

The time varying multi-path fading channel that exists in mobile communications environment lead to severe Inter-symbol Interference (ISI). In order to achieve high speed reliable communication, channel estimation is necessary to combat ISI [1]. Channel estimators estimates SIR by periodically adjusting an adaptive linear filter according to an algorithm so as to minimize the error between the output of the channel estimator and the received signal [2,3]. There are two major families of adaptive algorithms. The first family is around the stochastic gradient or least mean-square (LMS) algorithm which works well if the communications channel is fixed or slow time varying [4]. LMS is popular because of its low complexity, which is  $2N$  ( $N$  is the number of adaptive filter length) and its robustness. For fast time varying mobile channel, the performance of LMS tracking scheme is poor [4]. The second family is based on recursive least squares (RLS) algorithm that minimizes a deterministic sum of squared error [2]. The RLS algorithm is known to be capable of performing better performance than LMS, but suffers from computational complexity of  $O(N^2)$  operations per symbol [5,6]. When channel estimator has no prior knowledge of the channel, a algorithm gives better convergence rate compared to that of LMS [2]. The RLS

computational complexity restrict its use, so a number of fast RLS algorithms have been presented such as Fast Transversal Filter (FTF) [7,8], and fast a posteriori error sequential technique (FAEST) [9]. They reduce the computational complexity from  $O(N^2)$  to  $O(N)$  operations per symbol by using shifting and avoiding matrix-by-vector multiplications. This paper studies the application of the improved FTF algorithm to mobile channel estimation. A lower computational complexity FTF has been introduced in [10] which reduces the computation to  $O(7N)$ .

The FTF algorithm in its original form is known to exhibit an unstable behavior and a sudden divergence, due to accumulation of roundoff errors in finite precision computation [11]. Methods to overcome these roundoff errors have been suggested in [12-14]. These introduce a limit on a particular parameters in the algorithm which degrade the tracking ability of the FTF [12]. A numerically stable FTF algorithm using redundancy in the calculation of certain parameters and feedback of numerical errors is suggested in [14]. FTF with "leakage correction" stabilization method to overcome the roundoff error accumulation is used in this paper, and a one-step prediction is incorporated into the FTF algorithm that takes into account the rate of change in the estimate of the sampled impulse-response.

Prediction is coupled with LMS to improve channel estimation of the VHF radio links channel taps as in [15] and for also for mobile channels estimation as in [16]. Shimamura *et al.* [17] applied the same technique to design estimation based equalizers. Multistep adaptive algorithm has been presented by Gazor as Second Order LMS (SOLMS) for slow time varying channel to improve the tracking capabilities when some prior information is available on the time variation of the channel [18]. To track time varying channels, he applied a simple smoothing on the increments of the estimated weights to estimate the speed of the weights. The estimated speed is then used to predict the weights of the next iteration [18].

In this paper stabilized FTF with degree-1 Least Square (LS) fading expanded memory prediction (Predictive FTF-PFTF) is proposed for mobile channel estimation. The prediction technique in [19] is applied to update the estimates of the sampled impulse response (SIR) of mobile channel.

The performance of conventional FTF, PFTF is demonstrated by simulations. The results show that PFTF provides superior steady state performance relative to the conventional FTF.

## 2. FTF Algorithm for Mobile Channel Estimation

The mobile channel is assumed to follow Jakes fading Model [20]. The Jakes fading model is a deterministic method for simulating multi-path fading channels. The model assumes that multiple rays arrive at a mobile receiver with uniformly distributed arrival angles ( $\alpha_n$ ). Every ray experiences a Doppler shift of  $f_d = f_m \cos(\alpha_n)$ , where  $f_m = v f_c / c$ ,  $v$  is the speed of the mobile receiver,  $f_c$  is transmitter carrier frequency,  $c$  is the speed of light

$$(c = 3 \times 10^8 \frac{m}{s}).$$

The fading in each path of the channel follows Rayleigh distribution and has U-shaped power spectral density as given by Jakes [20].

$$S(f) = \frac{1}{\pi f_m \sqrt{1 - (f/f_m)^2}}, -f_m \leq f \leq f_m \quad (1)$$

The relative strength of the paths has assumed to have an exponential power delay profile. For simulation purpose, the mobile channel is modeled as three tap finite impulse response (FIR) filter with delay between successive filter taps is assumed to be symbol period. The filter taps or coefficients,  $\mathbf{Y}_i$ , are time varying and generated as complex Gaussian according to the Jakes model for fading channel simulator [20]. The performance of proposed channel estimator algorithm is evaluated for doppler frequencies of 100 Hz and 222 Hz, corresponding to a ve-

hicular channels with speeds of 54 km/hr and 120 km/hr respectively.

Assume that  $\mathbf{S}_i$  is the transmitted sequence (assumed stationary),  $\mathbf{Y}_i$  is channel sampled impulse response,  $n_i$  is the noise,  $r_i$  is the received symbol,  $r'_i$  is the  $i$ -th estimate of the received symbol and  $\mathbf{Y}'_i$  is the estimate of the channel impulse response. All of the above quantities are measured at  $iT$  time instant, where  $T$  is the sampling time. The received signal ( $r_i$ ) is given by

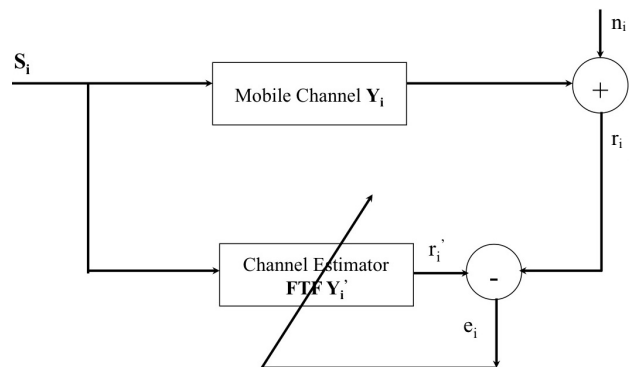
$$\begin{aligned} r_i &= \mathbf{Y}_i \mathbf{S}_i^T + n_i \\ r'_i &= \mathbf{Y}'_i \mathbf{S}_i^T \\ e_i &= r'_i - r_i \end{aligned} \quad (2)$$

where  $\mathbf{Y}_i = [y_{i,0}, y_{i,1}, \dots, y_{i,N-1}]$ , and  $\mathbf{S}_i = [s_i, s_{i-1}, \dots, s_{i-N}]$ .  $\mathbf{Y}_i$ ,  $\mathbf{S}_i$  are  $N$  component vector, where  $N$  is the number of paths in the multi-ray mobile channel.  $\mathbf{S}_i$  is a pseudo-random signal with values of either  $+1$  or  $-1$ . The channel output signal is corrupted by an additive white Gaussian noise (AWGN),  $n_i$ , with variance  $\sigma^2$  and zero mean, which is assumed to be uncorrelated to  $\mathbf{S}_i$ . Adaptive digital filter can be used to estimate the sampled channel impulse response. It consists of Finite Impulse Response (FIR) filter with variable tap weights. These tap weights are adjusted according to FTF method of updating weights as shown in the estimator block diagram in **Figure 1**.

The conventional FTF algorithm forms an estimate of the received sample  $r'_i$ . The estimator next forms the error signal  $e_i$ . The estimated channel sampled impulse response  $\mathbf{Y}'_i$  is obtained recursively in such a way that the cumulative squared error measure  $c_i$  is minimized [2,7].

$$c_i = \sum_{h=1}^i \lambda^{i-h} |e_h|^2 \quad (3)$$

The quantity  $c_i$  is the cumulative sum of the weighted squared errors. The parameter  $\lambda$  is a real-valued constant in the range 0 to 1.  $\lambda$  is a weighting factor that introduces an exponential window into the processed samples and is, therefore, sometimes called the fade factor or the for-



**Figure 1. Mobile channel estimator block diagram.**

getting factor for the filter. It is similar to pre-windowing except that one fixed  $\lambda$  slightly less than unity to track time variations.  $\mathbf{Y}'_i$  is determined recursively as in [7].

$$\mathbf{Y}'_i = \mathbf{Y}'_{i-1} + e_i \tilde{\mathbf{k}}_i \quad (4)$$

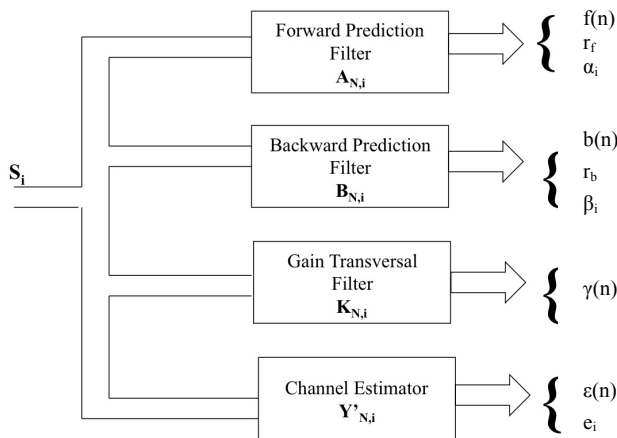
where  $\tilde{\mathbf{k}}_i$ , is the Kalman gain vector. FTF which will be used in this paper as a recursive computation of Kalman gains shows a complexity of  $O(N)$  where  $N$  is the number of taps of the channel ( number of mobile channel rays in this paper) [7]. The FTF algorithm uses four transversal filters in order to obtain the channel estimate [7,8] as shown in **Figure 2**.

One filter gives an estimate of the sampled impulse-response of the HF channel. The other three filters, called the forward predictor filter, the backward predictor filter, and the gain transversal filter are used in the estimation of the Gain Transversal Filter gain vector ( $\tilde{\mathbf{k}}_i$ , in (4)). For each of the four filters an estimation error is first evaluated followed by the updating of the tap coefficients (tap gains) of the filter. All of the four filters share the same input data vector. The complete derivation of the algorithm is beyond the scope of this paper and is given elsewhere [2] and [7].

In [2] and [7], a derivation of fundamental fast transversal filter equations comes to a  $7N$  fast RLS algorithm for system identification. The prediction part of the algorithm uses a system of four different filters that are coupled by the recursions  $\mathbf{A}$ ,  $\mathbf{B}$  are, respectively, forward and backward prediction error filters, and is  $\mathbf{K}$  is Gain Transversal Filter.  $\alpha_i$  and  $\beta_i$  are real valued scalars represent the weighted sum of squares of forward and backward prediction errors.  $\gamma_i$  is a real valued scalar which gives a measure of error in adjustment of gain transversal filter weights.

### 3. Stabilized FTF Algorithm

Four main reasons can be given to explain this divergence: erroneous or approximate equation (inclusion of



**Figure 2. FTF estimator transversal filters components.**

the forgetting factor), ill conditioning of the computed correlation matrices, effects of the suboptimal initialization procedure, and accumulation of finite precision errors [21]. There exists a tradeoff exists in the selection of the forgetting factor  $\lambda$ . Decreasing  $\lambda$  stabilizes the RLS algorithm, however, at the cost of increasing noise due to round-off error [21].

Attempts were made to stabilize the FFT by rescue methods [23]. [23] has introduced a rescue method that effectively monitors the rescue variable until a point just prior to the sign reversal, and then rescues the algorithm by restarting with a weighted initial condition. Alternative methods for stabilizing the FTF algorithm have been introduced in [22-25]. These algorithms use a multiplicative leakage factor ( $v_1$ ) in the forward and backward predictor updates, such that the numerical errors associated with their calculations decay to zero over time. **Table 1** shows the FTF algorithm with leakage correction.

The algorithm computes the convergence factor  $\gamma(n)$  directly using the normalized Kalman gain vector  $\tilde{\mathbf{K}}(n)$

**Table 1. FTF algorithm with leakage correction.**

Stabilized FTF	No. Multi.
$\eta(n) = s(n) - \mathbf{S}_T(\mathbf{n}-1)\mathbf{A}(\mathbf{n}-1)$	$N$
$f(n) = \gamma(n-1)\eta(n)$	1
$\tilde{r}_f(n) = \eta(n)/\lambda\alpha(n-1)$	$1 + 1(\div)$
$\tilde{\mathbf{k}}_{N+1}(n) = \begin{bmatrix} \tilde{r}_f(n) \\ v_1 \tilde{\mathbf{k}}(n-1) - v_1 \tilde{r}_f(n)\mathbf{A}(\mathbf{n}-1) \end{bmatrix}$	$2N + 1$
$\alpha(n) = \lambda\alpha(n-1) + f(n)\eta(n)$	2
$\mathbf{A}(n) = v_1\mathbf{A}(n-1) + v_1 f(n)\mathbf{k}(n-1)$	$2N + 1$
$\begin{bmatrix} \tilde{\mathbf{k}}_{N+1}(n-1) \\ \tilde{r}_b(n) \end{bmatrix} = \tilde{\mathbf{k}}_{N+1}(n)$	
$\psi(n) = \lambda\beta(n-1)\tilde{r}_b(n)$	2
$\tilde{\mathbf{k}}(n) = \tilde{\mathbf{k}}_{N+1}(n) + v_1 \tilde{r}_b(n)\mathbf{B}(n-1)$	$N + 1$
$\gamma(n) = \frac{1}{1 + \mathbf{S}_T(n)\mathbf{k}(n)}$	$N + 1 (\div)$
$b(n) = \psi(n)\gamma(n)$	1
$\beta(n) = \lambda\beta(n-1) + b(n)\psi(n)$	2
$\mathbf{B}(n) = v_1\mathbf{B}(n-1) + v_1 b(n)\mathbf{k}(n)$	$2N + 1$
$\varepsilon(n) = r(n) - \mathbf{S}_T(n)\mathbf{Y}'(n-1)$	$N$
$e(n) = \varepsilon(n)\gamma(n)$	1
$\mathbf{Y}'(n) = \mathbf{Y}'(n-1) + e(n)\tilde{\mathbf{k}}(n)$	$N$
Total	$11N + 14 + 2 (\div)$

and input signal vector  $\mathbf{X}(\mathbf{n})$ . The multiplicative leakage factor ( $v_1$ ) has to be in the range  $0 \leq v_1 \leq \lambda$  for stable operation. This algorithm has  $O(11N)$  computational complexity if leakage is employed at each time instant. While simulations in [25] indicate the useful behavior of the new stabilized algorithm. [24] analyze the stabilized FTF algorithm and show that the FTF algorithm employing the leakage-based update in **Table 1** is numerically stable if  $v_1$  is chosen in the range  $0 < v_1 < \lambda$ . Computer simulations indicate the level of accuracy and show the usefulness of the stabilized FTF algorithm with leakage correction to track mobile channel estimation.

#### 4. Proposed Predictive FTF (PFTF) Algorithm

To improve the performance of FTF algorithm for tracking time varying channel a prediction scheme is used. In [19], so called "degree-1 Least Square fading memory prediction" was employed to take *a priori* information about the channel into the estimation scheme. The method of least square fading memory prediction is based on the fact that a better prediction of  $\mathbf{Y}'_{i+1}$  from the sequence of vectors  $\mathbf{Y}'_i, \mathbf{Y}'_{i-1}, \dots$ , is obtained by determining the set of  $n+1$  polynomials of given degree (0, 1 or 2) each of which gives the LS fit to the components in the corresponding locations in the vectors  $\mathbf{Y}'_i, \mathbf{Y}'_{i-1}, \dots$ , and then using the values of the polynomial at time  $t = (i+1)T$  to determine the  $i$ -th component of  $\mathbf{Y}'_{i+1}$ . Each chosen polynomial is such that it gives the best fit to the sequence of past measurements, in the sense that the exponentially weighted sum of the squares of the prediction errors is minimized, for the given degree of the polynomial [26,27].

Extensive tests have shown that degree-1 polynomial filters generally give the best overall performance, and, in spite of the additional coupling (feedback) introduced here by using updated estimates in place of independent measurements, the technique substantially improves the overall performance of the estimator, without any sign of instability. Further details of the polynomial filters are given elsewhere [26,27].

Thus, it behaves as a coefficient prediction filter. So, FTF with LS expanded fading memory prediction algorithm (PFTF) which is proposed in this paper to improve mobile channel estimation is given by the following set of equations [16,26,27]:

$$\begin{aligned} \mathbf{E}_i &= \mathbf{Y}'_i - \mathbf{Y}'_{i-1} \\ \mathbf{Y}''_i &= \mathbf{Y}''_{i-1} + \alpha \mathbf{E}_i \\ \mathbf{Y}'_i &= \mathbf{Y}'_{i-1} + \mathbf{Y}''_i + \beta \mathbf{E}_i \end{aligned} \quad (5)$$

where  $\alpha = (1 - \theta^2)$  and  $\beta = (1 - \theta^2)$  are real-valued scalars, and  $\theta$  is the smoothing constant (should be between 0 and 1) that controls the forgetting amount of the past in a

compromise with an accurate estimate.  $\theta$  has to be tuned depending on signal to noise ratio (SNR) and channel behavior (speed of variation). PFTF algorithm assumes that the sampled impulse-response of the channel varies linearly with time.

Computer-simulation tests, on the accuracy of the one step prediction given by equation 5, for use with the FTF algorithm, have shown a useful improvement in the performance of the channel estimator without any sign of instability. The additional complexity is only  $2N$  operations per symbol.

#### 5. Simulation Results

Extensive computer simulations were carried out to compare the performance of the conventional FTF, proposed predictive FTF (PFTF) for tracking mobile channels. The input to the channel is a pseudo random sequence of +1, -1 and white Gaussian noise of different variances is used through out the test. In these simulations, slow and fast vehicular mobile channels with doppler frequency of 100 Hz, and 222 Hz respectively is used (equivalent to the mobile speed of 54 km/hr, and 120 km/hr at 2 GHz carrier frequency and signal rate of 15 ksymbol/sec). The mobile channels are assumed to have three paths. The variations of these taps against time is shown in **Figure 3**. The Signal to Noise Ratio (SNR) used is  $10 \log_{10}(1/\sigma^2)$  where  $\sigma^2$  is the noise variance. Each transmission burst consist of 10000 symbols and the Mean Square Error (MSE) between the estimated channel taps and the actual once is obtained as average of 10 independent trials. The MSE is a measure of actual error in channel estimation. During the first 500 of these tests, the estimator is in startup, so no measurements is done s it is in a transient period. MSE is defined as

$$\text{MSE} = 10 \log_{10} \left( \sum_{50}^{10000} |\mathbf{Y}_i - \mathbf{Y}'_{i-1}|^2 \right) \quad (6)$$

**Figure 4** shows and for SNR = 40 dB and fast varying mobile channel, the conventional and stabilized FTF performance. It is clear that and after few thousands of received symbols, the unstabilized estimators got a divergence operation when used for such channels.

The MSE has been collected for different values of  $\lambda$  (weighting factor) and SNRs is shown in **Figures 5** and **6**. The purpose of these tests is to collect the optimum values of  $\lambda$ . These values which correspond to minimum MSE will be used in comparisons with PFTF performance. It is clear that optimum values of  $\lambda$  decrease as the speed of channel variations is increases.

Using these optimum values, the MSE of PFTF is plotted against different values of  $\theta$  (smoothing constant) as shown in **Figures 7** and **8**. The optimum value of  $\theta$  depends on the input SNR and the speed of channel variations. The optimum value for  $\lambda$  and  $\theta$  will be used

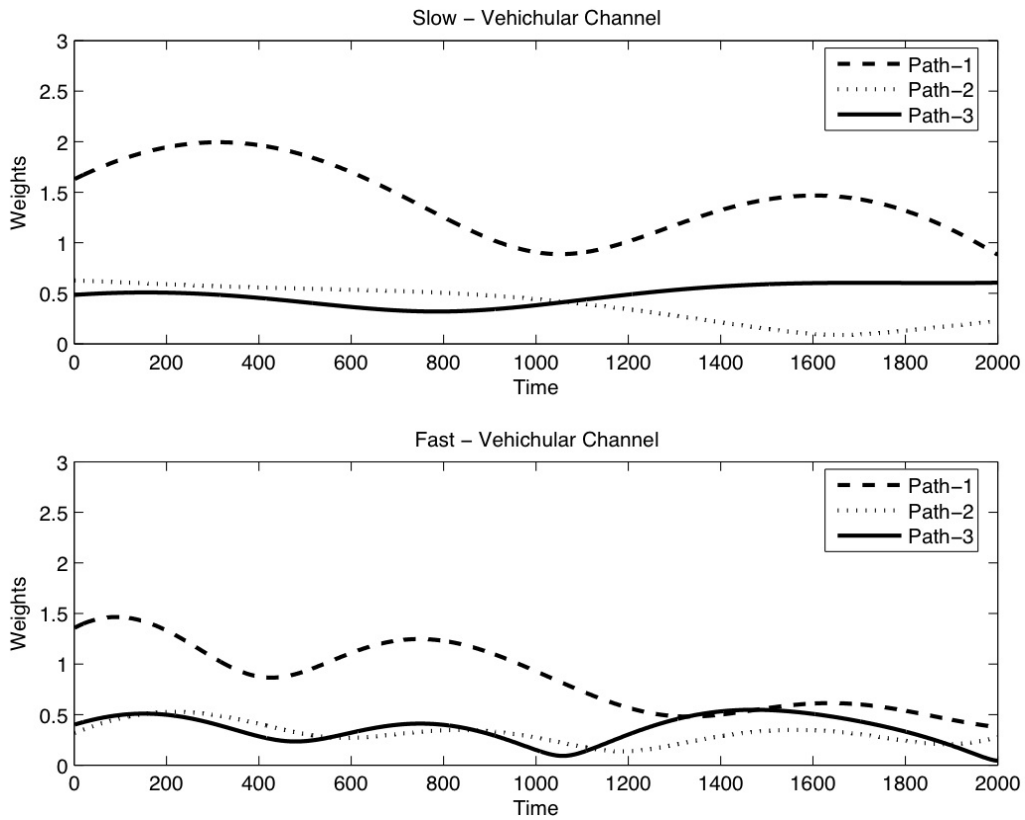


Figure 3. Mobile channel taps weights variations according to Jakes' Model.

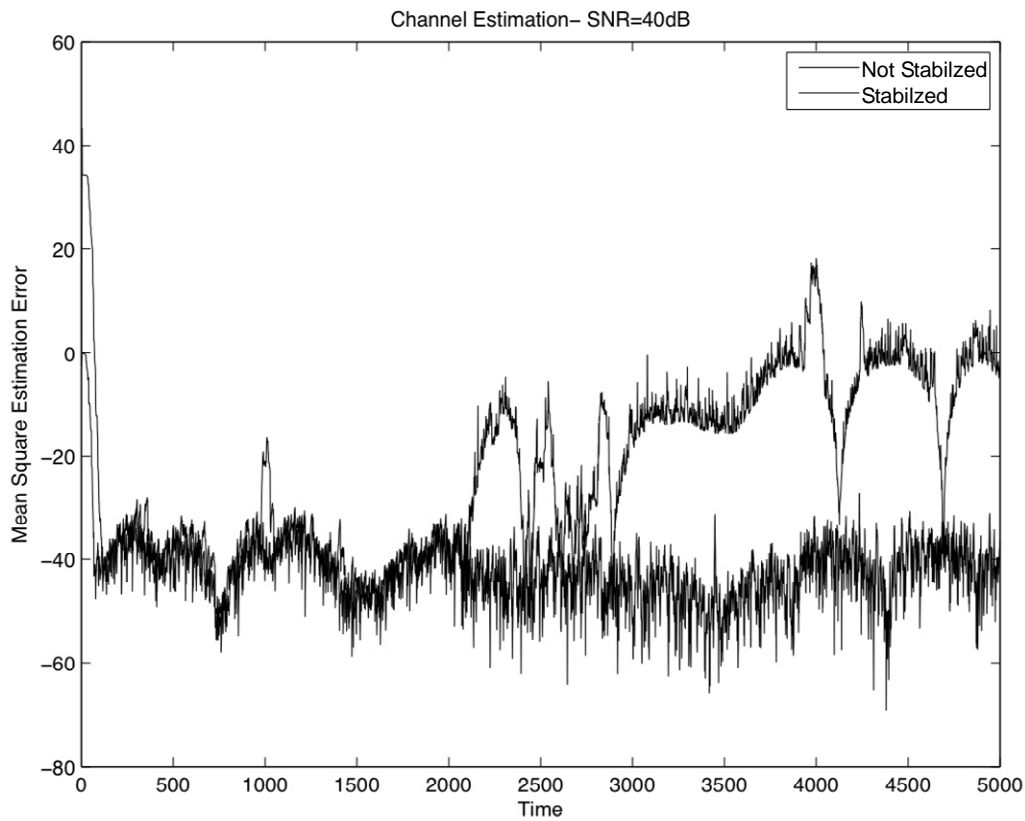


Figure 4. Stabilized and un-stabilized FTF channel estimation performance.

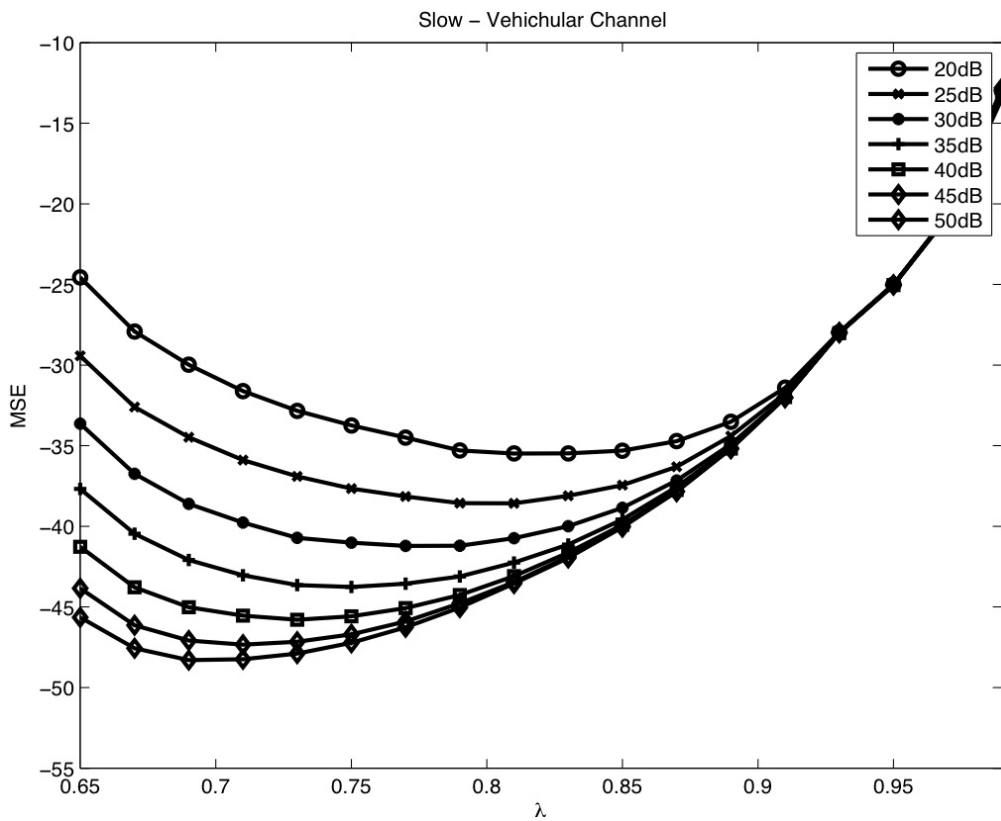


Figure 5. FTF performance against weighting factor ( $\lambda$ ) for slow channel.

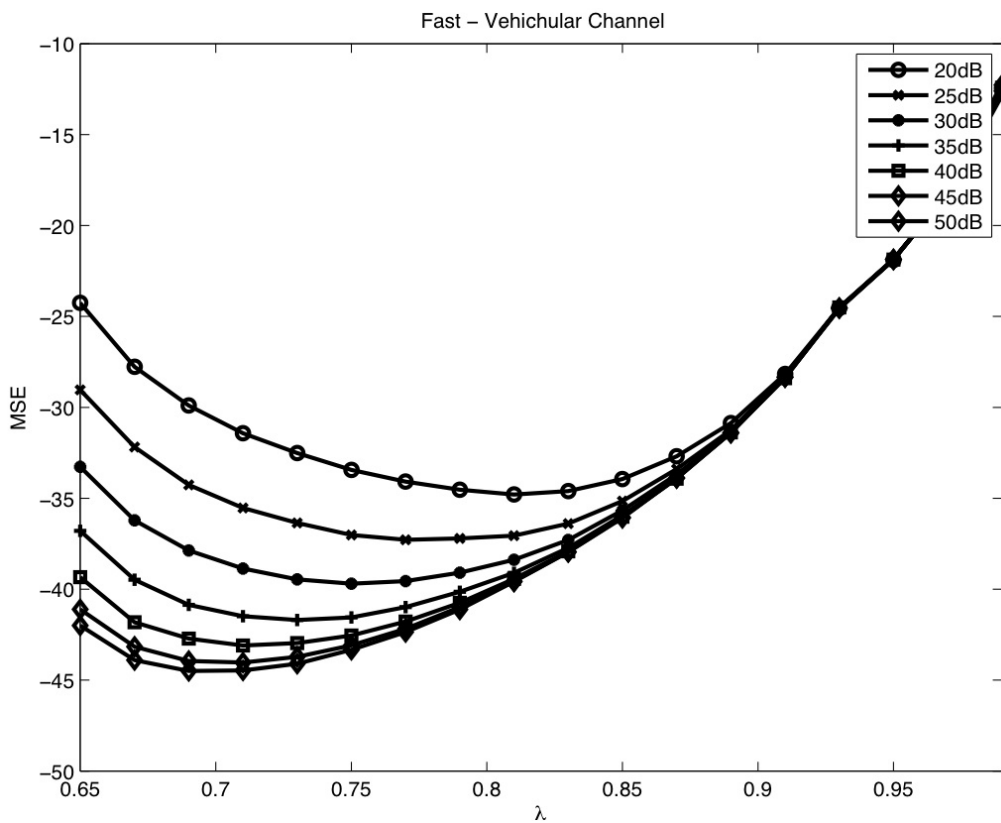


Figure 6. FTF performance against weighting factor ( $\lambda$ ) for fast channel.

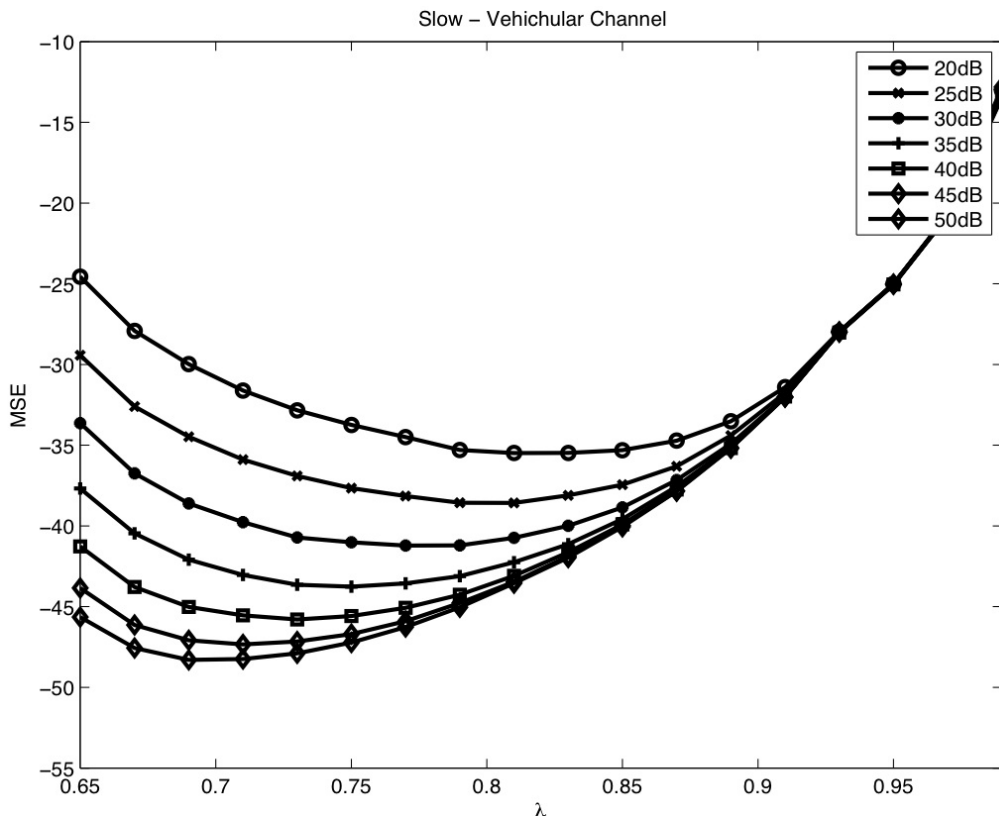


Figure 7. PFTF performance against smoothing constant ( $\theta$ ) for slow channel.

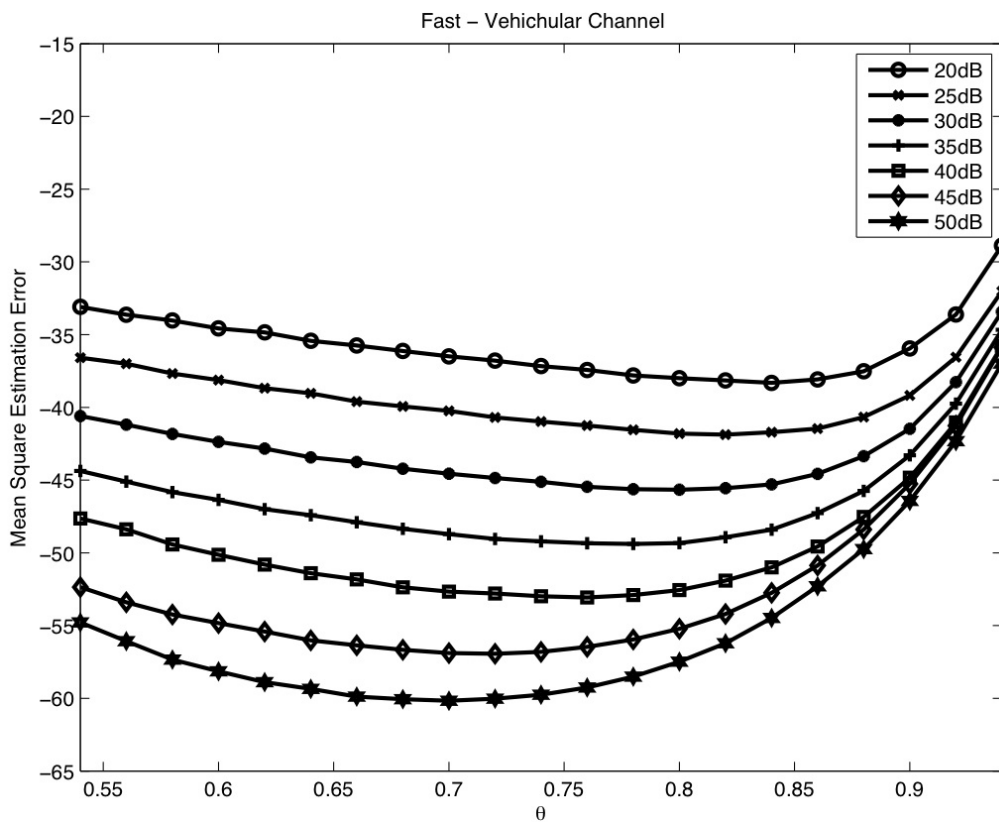


Figure 8. PFTF performance against smoothing constant ( $\theta$ ) for fast channel.

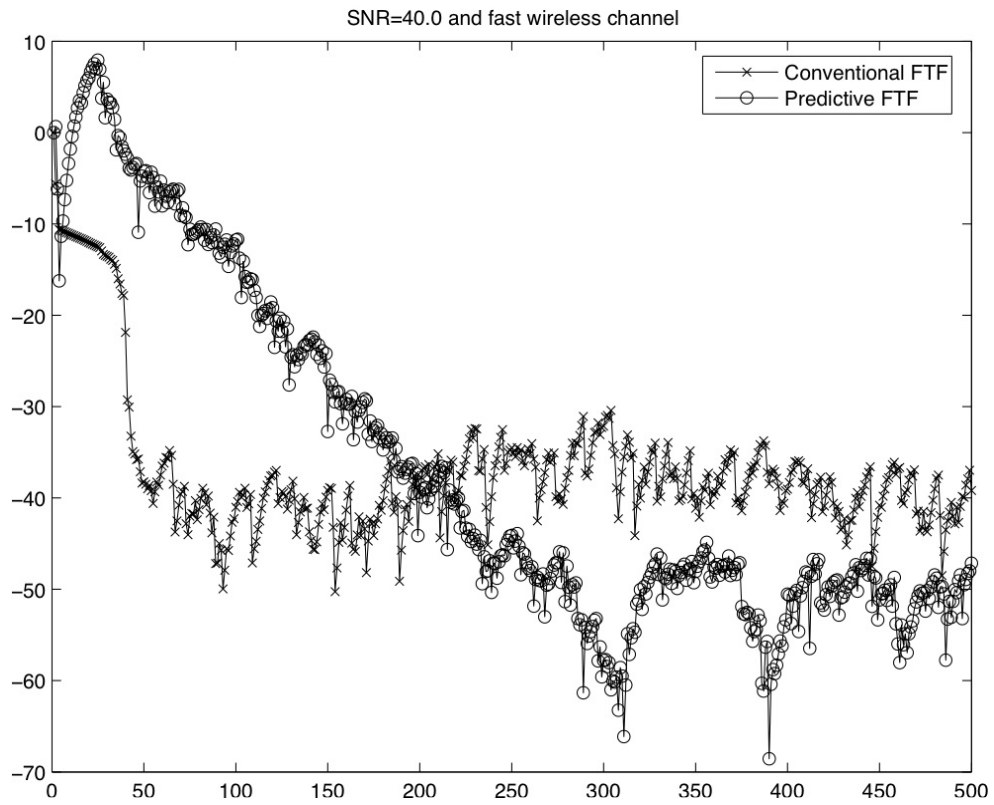


Figure 9. PFTF and FTF transient channel estimation performance.

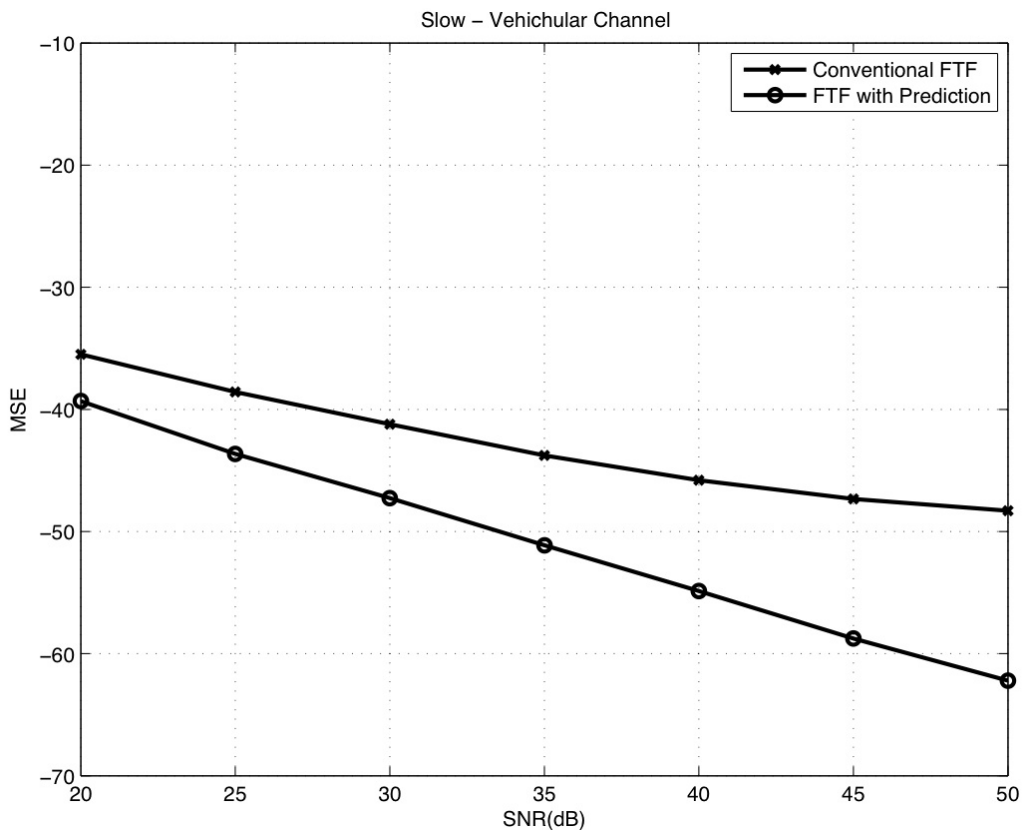


Figure 10. PFTF and FTF performance against SNR for slow channel.



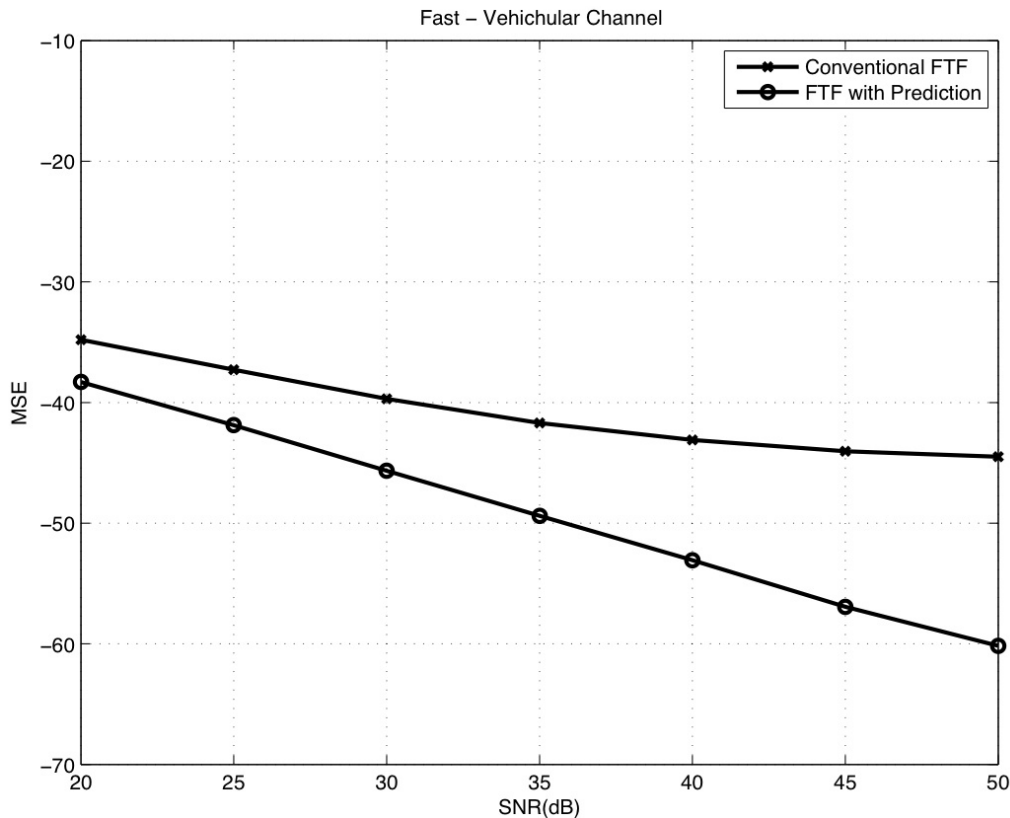


Figure 11. PFTF and FTF performance against SNR for slow channel.

for performance comparisons. The setup time for PFTF is longer than for FTF as shown in Figure 9, so it is recommended that the receiver starts with FTF for a few hundreds and then switch to PFTF.

Figures 10 and 11 compares the performance of mobile channel estimation using conventional FTF and PFTF schemes. It can be seen that the steady state performance of the proposed PFTF compared with the conventional FTF is improved by about 5 dB for poor SNR (20 dB) while it gains around 15 dB for a SNR of 50 dB. PFTF achieve considerable improvements in mobile channel estimation performance compared to conventional FTF, this due to prediction used in PFTF within the channel estimator operation.

## 6. Conclusion

Mobile Channel estimation based on FTF with degree-1 Least Square fading memory prediction (PFTF) has been explored. Based on a steady state mean performance PFTF offers a quite distinct benefit in comparison with the conventional FTF—based method. Simulation results show that under the well accepted Jakes' fading channel model, the PFTF based offers about 5 dB to 15 dB benefit vehicular mobile channel estimation over FTF based algorithm when SNR is 20 dB and 50 dB respectively. It is shown that the algorithm has the capability of tracking

slow and fast time varying mobile channels. Also PFTF does not add any substantial computation complexity.

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