Reliability Optimization of Entropy Based Series-Parallel System Using Global Criterion Method

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Abstract: In this paper, we have considered a series-parallel system to find out optimum system reliability with an additional entropy objective function. Maximum system reliability of series-parallel system is depending on proper allocation of redundancy component in different stage. The goal of entropy based reliability redundancy allocation problem is to find optimal number of redundancy component in each stage such a manner that maximize the system reliability subject to available total system cost. Global criterion method is used to analyze entropy based reliability optimization problem with different weight function of objective functions. Numerical examples have been provided to illustrate the model.

Keywords: reliability, series-parallel system, redundancy, entropy, global criterion method

1. Introduction

The problem of reliability redundancy allocation is to find out optimal system reliability by optimal allocation of redundancy components in series-parallel system. Reliability of a multi-stage system can be improved by adding similar or some different components redundancy to each sub-system as design alternatives. The design of a reliable system was improved by Hikita et al. [3] by the addition of redundant components. Several researcher [1, 2,4,5,12,13,17,20–22] presented redundancy optimization with multiple objective functions of system reliability, system cost and system weight etc. and solve that objective redundancy allocation problem by different algorithm and nonlinear optimization techniques for multi-objective system reliability design optimization in fuzzy and crisp environments. Tillman et al. [19] presented a comprehensive survey of previous works for system reliability with redundancy. Singh and Misra [18], Kuo and Prasad [7], Kuo et al. [8], and Misra [10] presented reliability redundant allocation problem to increase the system reliability, which is important in reliability engineering.


Here, we have considered a multi-objective entropy based reliability redundancy problem to finding the optimum number of redundant components, which maximize the system reliability with entropy as an additional objective function subject to available system cost. The redundancy reliability optimization problem is considered with two objective functions such as maximum system reliability and maximum entropy amount simultaneously with restriction on system cost. Numerical example is presented using global criterion method.

2. Reliability Redundancy Allocation Model

2.1. Notations

Series Parallel system, reliability redundancy allocation model is developed under the following notations.

- \( R_i \) reliability of each component of reliability model in the \( i^{th} \) stage,
- \( C_i \) cost of each component of reliability model in the \( i^{th} \) stage,
- \( C \) available system cost of the reliability model,
- \( x_i \) number of redundancy components in the \( i^{th} \) stage (decision variables),
system reliability function of the reliability model,
system cost function of the reliability model,
entropy function of the reliability model.

2.2. Reliability Redundancy Allocation Problem

It is to be considered that an n stage series system and at each stage added \((x_i-1)\) redundant components in parallel, the objectives are to determine the number of redundant components at each stage such that the system reliability will be maximize subject to related cost constraints.

Therefore the maximization of \(R_s\) subject to the limited available cost \(C\) has to be found.

Therefore the problem becomes

\[
\text{Maximize } R_s(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} \left[1 - (1 - R_i)^{x_i} \right] \quad (1)
\]

subject to

\[
C_s(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} C_i \left( x_i + \exp \left( \frac{X_i}{4} \right) \right) \leq C
\]

\(x_i > 1\) for \(i = 1, 2, \ldots, n\).

2.3. Entropy in Series Parallel System

Entropy has important physical implications as the amount of “disorder” of a system. A more abstract definition is used in mathematics.

The Shannon entropy of a variable \(X\) is defined as

\[
\text{En}(X) = -\sum_{x} p(x) \ln p(x) \quad (2)
\]

where \(p(x)\) is the probability that \(X\) is the state \(x\), and \(p(x) \log p(x)\) is defined as \(0\) if \(p(x) = 0\).

Reliability redundancy allocation problem is the redundancy distribution of each stage of a series-parallel system. To determine a suitable measure of allocation, let us consider a \(n\)-stage series-parallel system with \(x_i\) (\(i = 1, 2, \ldots, n\)) number of redundant components of each \(i^{th}\) stage of the system. It is known that \(x_i\) are positive integer and total number of components is \(\sum_i x_i\). Redundancy allocation of components share of \(i^{th}\) stage is the share of the total number of redundant component is \(p_i = \frac{x_i}{\sum_i x_i}\). Normalizes the redundancy numbers \(x_i\) by dividing them by the total number of redundant components \(\sum_i x_i\) then the probability distribution \(p_i = \frac{x_i}{\sum_i x_i}\) is found.

The measure of allocation shall be defined as the expected information of the message which transforms the system shares into the share of each stage.

\[
\text{So } \text{En}(x_1, x_2, \ldots, x_n) = -\sum_{i=1}^{n} p_i \ln p_i \quad (3)
\]

where \(p_i = \frac{x_i}{\sum_i x_i}\) satisfying the condition \(p_i \geq 0 (\forall i=1,2,\ldots,n)\) and \(\sum_i p_i = 1\) defines a probability distribution and the Shannon-entropy measure the diversity of the probability distribution \(\{p_1, p_2, \ldots, p_n\}\). Maximum is reached when \(p_1 = p_2 = \ldots = p_n = 1/n\) i.e. when allocation of all stage have the same no of redundant components. Since increasing of \(x_i\), maximizing \(\ln p_i\) is equivalent to maximizing entropy as defined above. This is one of the reasons why the entropy optimization model is particularly suitable for the redundancy allocation problem. In redundancy allocation problem, entropy acts as a measure of dispersal of allocation between stages. So it will be more potential if we would like to have maximum system reliability as well as maximize entropy measure.

2.4. Multi-Objective Entropy Redundancy Allocation Problem

Taking entropy function as additional objective function the problem (1) becomes

\[
\text{Maximize } R_s(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} \left[1 - (1 - R_i)^{x_i} \right] \quad (4)
\]

\[
\text{Maximize } \text{En}(x_1, x_2, \ldots, x_n) = -\sum_{i=1}^{n} \left( \frac{x_i}{\sum_i x_i} \right) \log \left( \frac{x_i}{\sum_i x_i} \right)
\]

Subject to same constraint and restriction as in (1)

3. Method of Global Criterion

A multi-objective non-linear problem may be taken in the following form

Maximize/Minimize \(f(x) = [f_1(x), f_2(x), \ldots, f_k(x)]^T\) \( (5)\)

subject to \(x \in X = \{x \in \mathbb{R}^n : g_j(x) \leq b_j \text{ for } j=1,\ldots,m; l_i \leq x_i \leq u_i \text{ for } i=1,\ldots,n\}\).

Solve the multi-objective non-linear problem (5) as a
single objective non-linear problem $k$ times for each problem by taking one of the objective at a time and ignoring the others. From the result, determine the corresponding values for every objective for each derived solution. For each objective $f_r(x)$, find lower bound (minimum) $f_r^-$ and the upper bound (maximum) $f_r^+$. 

In the global criterion method [9], the distance between some reference point and the feasible objective region is minimized. The analyst has to select the reference point and the metric for measuring the distances. Suppose that the weighting coefficients $w_r$ are real numbers such that $w_r \geq 0, \forall r = 1,2,...,k$ and $\sum_{r=1}^{k} w_r = 1$.

Here we examine the method where the ideal objective vector is used as a reference point and $0, 1, 2,...,n$ are chosen for every objective for each derived solution. For each objective $f_r(x)$, find lower bound (minimum) $f_r^-$ and the upper bound (maximum) $f_r^*$. 

The solution obtained depends greatly on the value chosen for $p$, commonly used choices are $p=1,2$ or $\infty$.

For $p=1$, $L_1(f(x)) = \sum_{r=1}^{k} w_r \left| \frac{f_r(x) - f_r^-}{f_r^* - f_r^-} \right|^p$ (7)

The objective function $L_1(f(x))$ is the sum of the normalized weighted deviations, which is to be minimized.

For $p=2$, $L_2(f(x)) = \sum_{r=1}^{k} w_r \left| \frac{f_r(x) - f_r^-}{f_r^* - f_r^-} \right|^2$ (8)

When $p$ becomes larger, the minimization of the deviation becomes more and more important.

If $p=\infty$, GCM (6) is of the form

Minimize $\max_{r=1,2,...,k} w_r \left| \frac{f_r(x) - f_r^-}{f_r^* - f_r^-} \right|$ (9)

Subject to $x \in X$.

The problem (9) can be transformed into the following form

Minimize $\lambda$ (10)

Subject to $w_r \left| \frac{f_r(x) - f_r^-}{f_r^* - f_r^-} \right| \leq \lambda$ for all $r=1,2,...,k$.

$x \in X$.

where both $x \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$ are variables.

4. Global Criterion Method on Entropy Based Reliability Redundancy Problem

In entropy based reliability optimization of series-parallel system, maximum system reliability $R_s(x)$ and maximum entropy $En(x)$ has to be found, having subject to the system cost constraint $C_s(x)$ with goal of system cost is $C$. So the problem is a multi-objective entropy reliability redundancy allocation problem as follows

Maximize $R_s(x)$ (11)

Maximize $En(x)$

Subject to $C_s(x) \leq C$

Where $x = (x_1, x_2, ..., x_n)$ and $x_i > 1$ for $i = 1,2,...,n$.

To solve the above multi-objective reliability optimization problem (11), according to section 3 pay-off matrix is formulated as follows:

$$R_s(x) [\begin{array}{l} R_s'(x') \cr En(x) \end{array}]$$

$$x^2 [\begin{array}{l} R_s(x) \cr En'(x') \end{array}]$$

Now lower and upper bounds of $R_s(x)$ and $En(x)$ are identified and denoted as $R^l_s, R^u_s$ and $En^l, En^u$ respectively.

Using Global criterion method for the problem (11), the weighted $L_p$-problem for minimizing distances is stated as

Minimize $L_p(R_s(x), En(x)) = \left( \sum_{r=1}^{k} w_r \left| \frac{R_s(x) - R_s^l}{R_s^u - R_s^l} \right|^p \right)^{1/p}$ (12)

Subject to $C_s(x) \leq C$, for $1 \leq p < \infty$

Putting different value of $p$ (1,2 or $\infty$) in (12), we get as follows

For $p=1$,

$L_1(R_s(x), En(x)) = \left( \sum_{r=1}^{k} w_r \left| \frac{R_s(x) - R_s^l}{R_s^u - R_s^l} \right|^p \right)^{1/p}$ (13)

For $p=2$,

$L_2(R_s(x), En(x)) = \left( \sum_{r=1}^{k} w_r \left| \frac{R_s(x) - R_s^l}{R_s^u - R_s^l} \right|^2 \right)^{1/2}$ (14)
For $p=\infty$, (12) is of the form

\[
\text{Minimize } \lambda
\]
\[\text{Subject to } \begin{cases}
w_1 \left( \frac{R_s(x) - R^L_s}{R^U_s - R^L_s} \right) \leq \lambda \\
w_2 \left( \frac{En(x) - En^L}{En^U - En^L} \right) \leq \lambda \\
\lambda \in \mathbb{R}
\end{cases}
\]

To solve the entropy based reliability redundancy allocation problem (11) using GCM, we have to solve (13), (14), (15) with same constraints as in Equation (10) for different weight.

5. Numerical Example

A four stage reliability redundancy allocation problem with entropy objective function with cost constraints is considered for numerical exposure. The problem becomes as follows:

\[
\text{Maximize } \prod_{i=1}^4 \left( 1 - (1 - R_i)^x \right)
\]

Maximize $En(x_1, x_2, x_3, x_4) = -\sum_{i=1}^4 \left( \sum_{x} \log \left( \frac{\sum_{x} x}{\sum_{x}} \right) \right)$

subject to

$C_i(x_1, x_2, \ldots, x_n) = \sum_{i=1}^n C_i \left( x_i + \exp \left( \frac{x_i}{4} \right) \right) \leq C_i,$

$x_i > 1$ for $i=1,2,3,4$.

Input parameters of the problem (5.1) are given in Table 1.

Solution:

Table 1. Input data for model (4)

<table>
<thead>
<tr>
<th>Case</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$R_4$</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.85</td>
<td>0.9</td>
<td>0.8</td>
<td>0.95</td>
<td>8</td>
<td>9</td>
<td>7</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>II</td>
<td>0.9</td>
<td>0.8</td>
<td>0.5</td>
<td>0.5</td>
<td>8</td>
<td>9</td>
<td>7</td>
<td>10</td>
<td>200</td>
</tr>
</tbody>
</table>

The pay-off matrix is formulated as follows:

$R_i(x) \quad En(x)$

\[
x^1 \left[ 0.9976021, 1.366073 \right]
\]

\[
x^2 \left[ 0.8627854, 1.386294 \right]
\]

Here $R^U_s = 0.9976021, R^L_s = 0.8627854, En^U = 1.386294, En^L = 1.366073$ are identified and using these bounds construct the objective functions. The optimal solutions of the multi-objective reliability optimization model (16) using global criterion method (following (13), (14) and (15)) are given in Table 2 for different preference values of the objective functions.

In case-I, Table 2 shows different optimal solutions when the decision maker supplies more preference to the entropy function than the reliability function. Here $R_i^*(x^*)$ is maximum when $p=2$ or $\infty$, whereas $En^*(x^*)$ is maximum when $p=2$ or $\infty$.

In Table 2, case-II gives different optimal solutions when the decision maker supplies equal preferences to the reliability function and entropy function. Here $R_i^*(x^*)$ is maximum when $p=1$, whereas $En^*(x^*)$ is remains unaltered for $p$.

In case-III, Table 2 shows different optimal solutions when the decision maker supplies more preference to the reliability function than the entropy function. Here $R_i^*(x^*)$ and $En^*(x^*)$ remains unaltered for $p$.

Table 2. Optimal solution for different weigntages of system reliability ($w_1$) and entropy functions ($w_2$) by GCM

<table>
<thead>
<tr>
<th>Case</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$p$</th>
<th>$x^*_1$</th>
<th>$x^*_2$</th>
<th>$x^*_3$</th>
<th>$x^*_4$</th>
<th>$R_i^<em>(x^</em>)$</th>
<th>$En^<em>(x^</em>)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.2</td>
<td>0.8</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>0.9373444</td>
<td>1.366159</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\infty$</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0.9351179</td>
<td>1.368922</td>
</tr>
<tr>
<td>II</td>
<td>0.5</td>
<td>0.5</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>0.9575832</td>
<td>1.368922</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\infty$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0.9266935</td>
<td>1.386294</td>
</tr>
<tr>
<td>III</td>
<td>0.8</td>
<td>0.2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0.9266935</td>
<td>1.386294</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\infty$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0.9266935</td>
<td>1.386294</td>
</tr>
</tbody>
</table>
6. Conclusions

Here redundancy allocation problem of series-parallel system with reliability and entropy objectives is presented. Global criterion method is used to solve the problem of multi-objective reliability redundancy allocation problem with entropy function. Two objective functions are combined into a single objective function by the global criterion method. The optimal solutions for different preferences on objective functions are presented. Decision-maker may obtain the Pareto optimal results according to his expectation of system cost.

Here it is considered that the problem as to finding the optimum number of redundancies, which maximize the system reliability and entropy subject to the limited system cost. The system reliability increases with increases of redundancies and entropy of the system decreases which is expected.

REFERENCES


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