Inventories and Mixed Duopoly with State-Owned and Labor-Managed Firms

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ABSTRACT

This paper considers a two-period mixed market model in which a state-owned firm and a labor-managed firm are allowed to hold inventories as a strategic device. The paper then shows that the equilibrium in the second period occurs at the Stackelberg point where the state-owned firm is the leader.

Keywords: Inventory Investment, State-Owned Firm, Labor-Managed Firm

1. Introduction

The analysis of mixed market models that incorporate state-owned public firms has been performed by many researchers, such as [1-16]. However, these studies consider mixed market models in which state-owned firms compete with profit-maximizing capitalist firms, and do not include labor-managed firms.

Mixed market models that incorporate labor-managed firms have also been studied by many researchers, such as [22-34]. However, these studies consider mixed market models in which labor-managed firms compete against profit-maximizing capitalist firms, and do not include state-owned firms.

Some studies examine mixed market models with state-owned and labor-managed firms. For example, Delbono and Rossini [40] explore the creation of 1) a duopoly formed by a labor-managed firm and a state-owned firm in a Cournot-Nash setting, and 2) a horizontal merger between the same agents. In addition, Ohnishi [41] investigates the behaviors of a state-owned firm and a labor-managed firm in a two-stage mixed market model with capacity investment as a strategic instrument. There are few studies that examine mixed market models with state-owned and labor-managed firms.

Therefore, we consider a two-period mixed market model in which a state-owned firm and a labor-managed firm can hold inventories as a strategic device. In the first period, each firm simultaneously and independently chooses its first-period production and its first-period sales. In the second period, the equilibrium occurs at the Stackelberg point where the state-owned firm is the leader.

The remainder of this paper is organized as follows. In Section 2, we describe the model. Section 3 gives supplementary explanations of the model. Section 4 analyzes the equilibrium of the model. Section 5 concludes the paper. All proofs are given in the appendix.

2. The Model

Let us consider a mixed market with one state-owned welfare-maximizing firm (firm 1) and one labor-managed profit-per-worker-maximizing firm (firm 2), producing perfectly substitutable goods. There is no possibility of entry or exit. In the remainder of this paper, subscripts 1 and 2 refer to firms 1 and 2, respectively, and superscripts 1 and 2 refer to periods 1 and 2, respectively. In addition, when and are used to refer to firms in an expression, they should be understood to refer to 1 and 2 with , where is the aggregate sales of each period. We assume that and . The game runs as follows. In the first period, each firm simultaneously and independently chooses its first-period production and its first-period sales.
Firm $i$’s inventory $I_i^t$ becomes $q_1^i - s_1^i$. At the end of the first period, firm $i$ knows $q_1^i$ and $s_1^i$. In the second period, each firm simultaneously and independently chooses its second-period production $q_2^i \in [0, \infty)$. At the end of the second period, each firm sells $s_2^i = I_i^t + q_2^i$ and holds no inventory. For notational simplicity, we consider the game without discounting.

Since $\sum_{i=1}^{2} q_1^i = \sum_{i=1}^{2} s_1^i$, social welfare is

$$W = \sum_{i=1}^{2} \left[ \int_{0}^{q_1^i} P(x)dx - c_i q_1^i - c_2 q_2^i \right]$$

$$= \sum_{i=1}^{2} \left[ \int_{0}^{s_1^i} P(x)dx - c_i s_1^i - c_2 s_2^i \right] \quad (1)$$

where $c_i \in (0, \infty)$ denotes firm $i$’s constant marginal cost. The demand and cost conditions that firms face remain unchanged over time. We assume that firm 1 is less efficient than firm 2, i.e. $c_1 > c_2$. We define

$$w' = \int_{0}^{s_1^i} P(x)dx - c_1 s_1^i - c_2 s_2^i \quad (2)$$

Furthermore, since $\sum_{i=1}^{2} q_2^i = \sum_{i=1}^{2} s_2^i$, firm 2’s profit per worker is

$$\Phi_2 = \sum_{i=1}^{2} \left[ \frac{P(S')s_2^i - c_2 s_2^i - f_2}{l_2(s_2^i)} \right] = \sum_{i=1}^{2} \left[ \frac{P(S')s_2^i - c_2 s_2^i - f_2}{l_2(s_2^i)} \right] \quad (3)$$

where $f_2 \in (0, \infty)$ is firm 2’s fixed cost, and $l_2$ is the amount of labor in firm 2. We assume that $l_2$ is the function of $s_2^i$ with $l_2' > 0$ and $l_2'' > 0$. This assumption means that the marginal quantity of labor used is increasing. We define

$$\phi_2 = \frac{P(S')s_2^i - c_2 s_2^i - f_2}{l_2(s_2^i)} \quad (4)$$

We analyze the subgame perfect Nash equilibrium of the mixed market model.

### 3. Supplementary Explanation

In this section, we give supplementary explanations of

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1. This assumption is justified in Nett [2,21] and Gunderson [44], and is often used in literature studying mixed markets. See, for instance, George and La Manna [5], Mujumdar and Pal [7], Pal [8], Nishimori and Ogawa [11], Matsumura [13], Ohnishi [14], and Fernández-Ruiz [16]. If firm 1 is equally or more efficient than firm 2, then firm 1 chooses $q_1^i$ and $s_1^i$ such that price equals marginal cost. Therefore, firm 2 has no incentive to operate in the market, and firm 1 supplies the entire market, resulting in a social-welfare-maximizing public monopoly. This assumption is made to eliminate such a trivial solution.

2. The concepts of strategic substitutes and complements are due to Bulow, Geanakoplos, and Klemperer [45].
and the second-order condition is

\[ (P^* s_1' + P - c_2) l_2 - (P s_2' - c_2 s_2' - f_2) l_2' = 0, \]  

(13)

Furthermore, we have

\[ R_2''(s_1') = \frac{P^* s_1' l_2 + P(l_2 - s_2' l_2')}{(P^* s_2' l_2 + 2P^* l_2') - (P s_2' - c_2 s_2' - f_2) l_2''}. \]  

(15)

Since \( l_2'' > 0 \), \( l_2 - s_2' l_2' < 0 \), so that \( P^* s_1' l_2 + P(l_2 - s_2' l_2') \) is positive; that is, \( R_2''(s_1') \) is upward sloping. This means that firm 2 treats \( s_1' \) as strategic complements.

Third, we consider Stackelberg games. If firm 1 is the Stackelberg leader, then firm 1 selects \( s_1' \), and firm 2 selects \( s_2' \) after observing \( s_1' \). Firm 1 maximizes \( w_i(s_2', R_1(s_1')) \) with respect to \( s_1' \). On the other hand, if firm 2 is the Stackelberg leader, then firm 2 selects \( s_2' \), and firm 1 selects \( s_1' \) after observing \( s_2' \). Firm 2 maximizes \( \phi_2(s_2', R_1(s_2')) \) with respect to \( s_2' \). We present the following lemma:

**Lemma 1.** Each firm’s Stackelberg leader sales exceed its Cournot sales without inventory.

Lemma 1 means that each firm prefers sales higher than its Cournot sales without inventory.

### 4. Equilibrium

In this section, we analyze the equilibrium outcomes of the mixed market model. The equilibrium in the first period is stated by the following proposition:

**Proposition 1.** In the first-period of the mixed market model, the equilibrium coincides with the Cournot Nash solution without inventory \( (N_1, N_2) \).

The intuition behind Proposition 1 is as follows. There is no inventory available in the first period, and further \( s_1' \) does not affect \( s_2' \) and \( s_2' \). Since \( w_1^* \) and \( \phi_2^* \) decrease by deviating from the Cournot Nash solution, each firm has no incentive to do so, and therefore the equilibrium is at \( (N_1, N_2) \).

We now consider the equilibrium of the second period. It is thought that the equilibrium of the second period is decided by the level of \( I_t^{1I} \). We discuss the following three cases:

1) The case in which only firm 1 can hold inventory
2) The case in which only firm 2 can hold inventory
3) The case in which each firm can hold inventory

We discuss these cases in order:

1) The case in which only firm 1 can hold inventory

First, consider Figure 1, where \( R_1^2 \) denotes firm \( i \)'s second-period reaction curve without inventory. \( R_2^1 \) is downward sloping, whereas \( R_2^1 \) is upward sloping. Suppose that firm 1 holds \( I_1^{1I} \) in the second period. By holding inventory, firm 1’s best response becomes (7). Firm 1’s inventory investment thus creates a kink in its reaction curve at the level of \( I_1^{1I} \). That is, firm 1’s reaction curve becomes the kinked bold lines as drawn in the figure. The equilibrium is decided in a Cournot fashion, i.e., the intersection of firm 1’s and firm 2’s reaction curves gives us the equilibrium of the game. Figure 1 shows that the intersection of new reaction curves is not affected by the kink. Hence, the equilibrium occurs at \( N \).

Next, consider Figure 2. Suppose that firm 1 holds \( I_1^{1B} \). From (7), firm 1’s reaction curve becomes the kinked bold lines. The intersection of firm 1’s and firm 2’s reaction curves gives us the equilibrium of the game.
From Figure 2, we see that the inventory level of $I_1^B$ changes the equilibrium of the game. The intersection of the new reaction curves is the equilibrium sales in the second period. That is, if firm 1 holds $I_1^B$, then the equilibrium occurs at $B$.

We can now state the following proposition:

**Proposition 2.** Suppose that only firm 1 can hold inventory. Then the equilibrium coincides with the Stackelberg solution where firm 1 is the leader.

2) The case in which only firm 2 can hold inventory

First, consider Figure 3. Suppose that firm 2 holds $I_2^{D}$. From (12), firm 2’s reaction curve becomes the kinked bold broken lines. In this figure, the reaction curves of both firms do not cross each other. That is, if firm 2 maintains the inventory level of $I_2^{D}$, then there is no solution.

Next, consider Figure 4. Suppose that firm 2 holds $I_2^{E}$. From (12), firm 2’s reaction curve becomes the kinked bold broken lines. The reaction curves of both firms cross twice as in Figure 4. We can see easily that $N$ and $E$ are stable solutions. That is, there are two stable solutions. However, we see that firm 2’s profit per worker is higher at $N$ than at $E$.

We can now state the following proposition:

**Proposition 3.** Suppose that only firm 2 can hold inventory. Then the equilibrium coincides with the Cournot Nash solution without inventory $(N_1, N_2)$.

3) The case in which each firm can hold inventory

First, consider Figure 5. Suppose that firms 1 and 2 hold $I_1^{F}$ and $I_2^{G}$, respectively. In this figure, the reaction curves of both firms do not cross each other. That is, if firms 1 and 2 maintain the inventory levels of $I_1^{F}$ and $I_2^{G}$, respectively, then there is no solution.

Next, consider Figure 6. Suppose that each firm holds $I_i^{N}$. Firm 1’s reaction curve becomes the kinked bold
lines, and firm 2’s reaction curve becomes the kinked bold broken lines. The new reaction curves of both firms cross twice. We see easily that $H$ and $J$ are stable solutions. That is, there are two stable solutions. However, we see that firm 2’s profit per worker is higher at $N$ than at these points.

The main result of this study is described by the following proposition:

**Proposition 4.** In the second period of the mixed market model, the equilibrium coincides with the Stackelberg solution where firm 1 is the leader. At equilibrium, firm 2’s profit per worker is lower than in the Cournot mixed duopoly game without inventory.

5. Conclusions

We have considered a two-period mixed market model in which a state-owned firm and a labor-managed firm are allowed to hold inventories as a strategic device. We have then shown that the equilibrium in the second period occurs at the Stackelberg point where the state-owned firm is the leader and at equilibrium the labor-managed firm’s profit per worker is lower than in the Cournot mixed duopoly game without inventory. As a result, we see that the introduction of inventory investment into the analysis of mixed market competition with state-owned and labor-managed firms is profitable for the state-owned firm while it is not profitable for the labor-managed firm.

**REFERENCES**


Appendix

Proof of Lemma 1
First, we prove that firm 1’s Stackelberg leader sales are higher than its Cournot sales without inventory. Firm 1 maximizes $w'(s_1^*, R_1(s_1^*))$ with respect to $s_1^*$. Therefore, firm 1’s Stackelberg leader sales satisfy the first-order condition:

$$\frac{\partial w}{\partial s_1} + \frac{\partial w}{\partial R_1} \frac{\partial R_1}{\partial s_1} = 0,$$

where $\frac{\partial w}{\partial s_1} = P - c_2$ is positive from (8) and $c_1 > c_2 > 0$, and $\frac{\partial R_1}{\partial s_1}$ is also positive from (10), (11) and (15). To satisfy (16), $\frac{\partial w}{\partial s_1}$ must be negative.

Second, we prove that firm 2’s Stackelberg leader sales are higher than its Cournot sales without inventory. Firm 2 maximizes $\phi_2(s_2^*, R_2(s_2^*))$ with respect to $s_2^*$. Therefore, firm 2’s Stackelberg leader sales satisfy the first-order condition:

$$\frac{\partial \phi_2}{\partial s_2} + \frac{\partial \phi_2}{\partial R_2} \frac{\partial R_2}{\partial s_2} = 0,$$

where $\frac{\partial \phi_2}{\partial s_2} = P' s_2$ is negative from $P' < 0$, and $\frac{\partial R_2}{\partial s_2}$ is also negative from (5), (6) and (9). To satisfy (17), $\frac{\partial \phi_2}{\partial s_2}$ must be negative. Thus, the lemma follows. Q. E. D.

Proof of Proposition 1
Since $s_1^*$ does not affect $s_2^*$ and $s_2^*$, $s_1^*$ has no strategic value. Thus, the result follows easily from (5) and (10). Q. E. D.

Proof of Proposition 2
The equilibrium is decided in a Cournot fashion, i.e., the intersection of firm 1’s and firm 2’s reaction functions gives us the equilibrium of the game. In $R_2^*$, social welfare is the highest at firm 1’s Stackelberg leader point. Lemma 1 states that firm 1’s Stackelberg leader sales exceed its Cournot sales without inventory. $R_2^*$ is downward sloping, whereas $R_1^*$ is upward sloping. From (12), firm 2 cannot choose its Stackelberg leader point. Let $\phi_2^*$ be assumed to be continuous and concave in $s_2^*$. The further a point on $R_2^*$ gets from firm 2’s Stackelberg leader point, the more firm 2’s profit per worker decreases. Thus, Proposition 3 follows Q. E. D.

Proof of Proposition 4
First, consider the possibility that firm 2 holds inventory as a strategic device. Lemma 1 states that firm 2’s Stackelberg leader sales exceed its Cournot sales without inventory. $R_2^*$ is downward sloping, whereas $R_1^*$ is upward sloping. From (12), firm 2 cannot choose its Stackelberg leader point. Let $\phi_2^*$ be assumed to be continuous and concave in $s_2^*$. The further a point on $R_2^*$ gets from firm 2’s Stackelberg leader point, the more firm 2’s profit per worker decreases. Hence, firm 2 does not choose neither $I_1^*$ nor $I_2^*$, then it is impossible for firm 2 to change $s_2^*$ in equilibrium. Thus, Proposition 3 follows Q. E. D.