Unsupervised Segmentation Method of Multicomponent Images Based on Fuzzy Connectivity Analysis in the Multidimensional Histograms

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Abstract

Image segmentation denotes a process for partitioning an image into distinct regions, it plays an important role in interpretation and decision making. A large variety of segmentation methods has been developed; among them, multidimensional histogram methods have been investigated but their implementation stays difficult due to the big size of histograms. We present an original method for segmenting \( n \)-D (where \( n \) is the number of components in image) images or multidimensional images in an unsupervised way using a fuzzy neighbourhood model. It is based on the hierarchical analysis of full \( n \)-D compact histograms integrating a fuzzy connected components labelling algorithm that we have realized in this work. Each peak of the histogram constitutes a class kernel, as soon as it encloses a number of pixels greater than or equal to a secondary arbitrary threshold knowing that a first threshold was set to define the degree of binary fuzzy similarity between pixels. The use of a lossless compact \( n \)-D histogram allows a drastic reduction of the memory space necessary for coding it. As a consequence, the segmentation can be achieved without reducing the colors population of images in the classification step. It is shown that using \( n \)-D compact histograms, instead of 1-D and 2-D ones, leads to better segmentation results. Various images were segmented; the evaluation of the quality of segmentation in supervised and unsupervised of segmentation method proposed compare to the classification method k-means gives better results. It thus highlights the relevance of our approach, which can be used for solving many problems of segmentation.

Keywords: Multicomponent Images, Unsupervised Segmentation, \( n \)-D Histogram, Fuzzy Connected Components Labelling, \( n \)-D Compact Histogram, Evaluation of Segmentation Quality

1. Introduction

In many fields such as medicine, food, robotics, security systems, etc, the acquisition and the analysis of images are essential processes for taking decisions [1-4], as it is often difficult to perform them manually, it is necessary to automate certain tasks by computing. Thus in this paper, we present, a new vectorial segmentation method based more particularly on the full \( n \)-D compact histograms by a fuzzy connectivity analysis.

Segmentation is an important step in image processing and automatic pattern recognition processes based on image analysis as subsequent extracted data are highly dependent on the accuracy of this operation. In general, the automated segmentation is one of the most difficult tasks in image analysis [5], because a false segmentation will cause degradation of the measurement process and therefore the interpretation may fail. The segmentation methods based on the analysis of color histograms are facing the difficulty of treating a huge quantity of information: for a color image of resolution \( N \times M \) with each component coded on 8 bits, a classical 3-D histogram is an array of \( 2^{24} \) cells, the number in each cell being coded on at least \( \log_2(MN) \) bits in order to store the number of pixels of each color allowed. In the case where \( M = N = 256 \), the standard 3-D histogram requires 128 Mo.
A few years ago, we have proposed a new way of coding the nD histograms, leading to the so-called compact histogram [6], in which only the occupied cells of the classical histogram are memorised. This reduces the memory space required drastically: it is lowered to a value of 500 ko for a 256 × 256 image with color components coded on 8 bits, without any loss of color information, it shows its efficiency in previous articles [7,8].

Using histograms for classifying colour pixels can be achieved through four different strategies.

The first strategy proceeds in a marginal way. Each component of the histogram is examined separately [9,10]. The method is easy to implement, but it does not take into account the correlation between colorimetric components.

In the second strategy, each colorimetric component is requantized on q bits (q < 8) in order to reduce the histogram size [11]. The method is efficient, but it makes an a priori color classification.

The third strategy that many authors have developed proceeds by projection of the 3-D histogram on two of the three colorimetric axes. W have developed this strategy in a previous paper [12] and used also in other articles such as [13-15]: The histogram, with populations requantized on 256 levels, is considered as a gray level image and processed by a watershed algorithm. The correlation between components is partially taken into account, but the requantization of the populations alters the true histogram, and the projection can result in ignoring some significant classes.

Here we propose a fourth strategy. It is fully vectorial, which is allowed by the use of the compact n-D histogram. Its principle consists in finding the peaks of classic multidimensional histogram using the compact multidimensional histogram by the consideration of a model of fuzzy neighbourhood. The kernels of the built classes correspond to the peaks retained using the algorithm for labelling connected components with binary fuzzy neighbourhood that we realized in this work, it accepts two input parameters which are the degree of fuzzy connectedness expressing the similarity between the vector points of the compact n-D histogram and a population threshold limiting the class size. The choice of the fuzzy connectedness here is justified by the difficulty of defining a net similarity between the vector points attribute or “colors” of the pixels of the image [16,17]. This leads to different results of segmentation of the same number of classes for different values of pairs of thresholds. Assessing the quality of segmentation can choose the most relevant segmentation. The results of our multidimensional segmentation method are compared to those of the method of classification K-means [13,18, 19] in order to evidence the performance of our approach using supervised and unsupervised [20-23] evaluation methods of the segmentation quality.

2. A Fast and Compact Multidimensional Histogram

We consider multicomponent images (for example multispectral images) with n components \( I_i(x_i, x_j) \), for \( i = 1 \) to \( n \), each \( I_i \) has a tonal resolution of \( Q \) possible values and each pixel of spatial coordinate \((x_i, x_j)\) takes a value among the \( Q \) possibilities. A n-dimensional histogram of such an image would comprise \( Q^n \) cells. For an image with \( N \) pixels of resolution, only at most \( N \) of these \( Q^n \) cells can be occupied, meaning, as \( n \) grows, most of the cells of the n-dimensional histogram are in fact empty. For example, for a commun 512 × 512 RGB color image with \( n = 3 \) and \( Q = 256 \), there are \( Q^3 = 256^3 \approx 16 \times 10^6 \) colorimetric cells of n-tuples with at most only \( 3N = 512^3 = 262144 \) of them which can be occupied.

The idea of a compact representation of the n-dimensional histogram [6], where only those cells that are occupied are coded. The n-dimensional histogram is coded as a linear array where the entries are the n-tuples (the colors) present in the image and arranged in lexicographic order of their components \((I_1, I_2, \cdots, I_n)\). To each entry (in number \( \leq N \)) is associated the number of pixels in the image having this n-value (this color). An example of this compact representation of the n-dimensional histogram is shown in Table 1.

The practical calculation of such a compact histogram starts with the lexicographic ordering of the \( N \) n-tuples corresponding to the \( N \) pixels of the image. The result is a linear array of the \( N \) ordered n-tuples with their population. With a dichotomic quick sort algorithm to realize the lexicographic ordering, the whole process of calculating the compact multidimensional histogram can be achieved with an average complexity of \( O(N \log N) \), independent of the dimension \( n \). Therefore, both compact

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>population</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>201</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>255</td>
<td>40</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>255</td>
<td>251</td>
<td>254</td>
<td>78</td>
</tr>
</tbody>
</table>
representation and its fast calculation are afforded by the process for the multidimensional histogram. For example, for a 9-component $838 \times 762$ satellite image with $Q = 256$, the compact histogram was calculated in about 5 s on a standard 1 GHz clock desktop computer, while the classic histogram would take $3.60 \times 10^{16}$ Moctets completely unmanageable by today’s computers.

The advantages of the compact multidimensional applied to multicomponent images (see Figure 1) are shown in Table 2.

3. Fuzzy Connected Components Labeling of Compact Histogram

3.1. Formalism of Fuzzy Logic and Concept of Fuzzy Neighbourhood

We present here some aspects of the theory of fuzzy logic, fundamental to understanding the algorithm of Fuzzy Connected Component Labeling (FCCL) compact multidimensional histograms, developed in following sections. The reader will find a complete overview in [24].

$X$ denote the universe of reference, consisting of elements $x$, and place us first in the theory of classical ensembles, that is to say net. Then any subset $A$ of $X$ net is completely defined by its characteristic function $\mu_A$.

\[
\mu_A(x) = \begin{cases} 
1 & \text{ssi } x \in A \\
0 & \text{sinon}
\end{cases}
\]  

(1)

If the set score is now the continuum $[0,1]$, $A$ becomes a fuzzy subset of $X$, and $\mu_A$ is its membership function. The subset $A$ is then defined by:

\[
A = \{[x, \mu_A(x)], x \in X\}
\]  

(2)

An $\alpha$-cut of $A$ is the net subset of items with a membership degree to $A$ greater than or equal to $\alpha$. It is noted $C_\alpha(A)$:

\[
C_\alpha(A) = \{x \in X | \mu_A(x) \geq \alpha\}
\]  

(3)

The concept of fuzzy relation is a generalization with the fuzzy domain of the concept of equivalence relation defined in the net case. A fuzzy relation can be measured by a scalar in the interval $[0,1]$, the degree to which a logical proposal is verified. With a fuzzy relation $R$ is associated membership function, denoted by $\mu_R$. Let $X$ and $Y$ be two universes of reference, the respective elements $x$ and $y$. A fuzzy relation between the elements of these two worlds is formally defined as:

\[
R : X \times Y \rightarrow [0,1] \\
(x, y) \mapsto \mu_R(x, y)
\]  

(4)

<table>
<thead>
<tr>
<th>Image</th>
<th>Dimensions</th>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forsythia 1</td>
<td>652 × 578</td>
<td>n = 3</td>
</tr>
<tr>
<td>Raison</td>
<td>384 × 287</td>
<td>n = 3</td>
</tr>
<tr>
<td>House</td>
<td>256 × 256</td>
<td>n = 3</td>
</tr>
<tr>
<td>Mandrill</td>
<td>256 × 256</td>
<td>n = 3</td>
</tr>
<tr>
<td>Peppers</td>
<td>256 × 256</td>
<td>n = 3</td>
</tr>
<tr>
<td>IRM</td>
<td>512 × 512</td>
<td>n = 4</td>
</tr>
<tr>
<td>M4</td>
<td>838 × 762</td>
<td>n = 9</td>
</tr>
<tr>
<td>Orge</td>
<td>512 × 512</td>
<td>n = 10</td>
</tr>
</tbody>
</table>

Figure 1. Example of multicomponent images.
When \( X = Y \), the fuzzy relation is known as binary.

In one of the following sections will be built a fuzzy connected component labeling of compact multidimensional histogram (it is him which plays the role of universe of reference \( X \)). The labelling will use a binary fuzzy relation, which we call fuzzy similarity, assessing the degree of similarity between two spells \( x \) and \( y \) of compact histogram multidimensional. The membership function of this relationship will be expressed as:

\[
\mu_R(x, y) = \begin{cases} 
1 & d(x, y) \leq M \\
\frac{1}{1 + d(x, y)} & \text{ sinon}
\end{cases}
\] (5)

This model of fuzzy neighborhood is used in many studies [25,26]. In our case, the threshold \( M \) is set at 7 which is the maximum distance allowed so that two spells resembles itself, and the distance \( d \) considered will be that of Chebychev or again call Queen-wise distance, given by the following equation:

\[
d(x, y) = \max_{i=1}^{n} |x_i - y_i|
\] (6)

### 3.2. Classical Connected Components Labelling

We have recently achieved [8] the connected components labelling (CCL) of \( n \)-D compact histograms. It consists in sweeping all the \( n \)-tuples present in the compact histogram, in order to gather, under the same label, the \( n \)-uplets which are neighboring in the \( n \)-D colorimetric space.

Since the \( n \)-uplets are ordered in lexicographical order inside the compact histogram, labelling a \( n \)-tuple is achieved by sweeping only the \((3^n-1)/2\) \( n \)-tuples connected neighbours preceding it. For example, Table 3 shows the four doublets of \((i, j)\) to be swept for labelling the doublet \((i, j)\) in the case of 2-D compact histograms.

The CCL is applied to the full \( n \)-D compact histograms of five multicomponent images (see Figure 1)

<table>
<thead>
<tr>
<th>Images</th>
<th>Classical histogram data size (Mo)</th>
<th>Compact histogram data size (Mo)</th>
<th>Number of spels of compact histogram (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forsythia 1</td>
<td>128</td>
<td>0,28</td>
<td>73278</td>
</tr>
<tr>
<td>Raisin</td>
<td>128</td>
<td>0,35</td>
<td>90802</td>
</tr>
<tr>
<td>House</td>
<td>128</td>
<td>0,36</td>
<td>33925</td>
</tr>
<tr>
<td>Mandrill</td>
<td>128</td>
<td>0,65</td>
<td>61662</td>
</tr>
<tr>
<td>Peppers</td>
<td>128</td>
<td>0,56</td>
<td>53488</td>
</tr>
<tr>
<td>IRM</td>
<td>3,60.10 (10^3)</td>
<td>1,61</td>
<td>140881</td>
</tr>
<tr>
<td>M4</td>
<td>9,22.10 (10^7)</td>
<td>1,89</td>
<td>116425</td>
</tr>
<tr>
<td>Orge</td>
<td>9,22.10 (10^8)</td>
<td>4,22</td>
<td>246104</td>
</tr>
</tbody>
</table>

Table 3. Connected neighbours to be swept for CCL of doublet \((i, j)\) in the case of a 2-D compact histogram (colorimetric axes \( I \) and \( J \)).

<table>
<thead>
<tr>
<th>Plan I</th>
<th>Plan J</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>i-1</td>
<td>j-1</td>
</tr>
<tr>
<td>i-1</td>
<td>j</td>
</tr>
<tr>
<td>i-1</td>
<td>J+1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>i</td>
<td>j-1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>2^0-1</td>
<td>2^0-1</td>
</tr>
</tbody>
</table>

taken on the web site of gdr-isis. Table 4 shows the number of connected components (CC) of their histograms and, the minimum and maximum size of CC are expressed as number of pixels compared to the size of the images.

### 3.3. Principle of Fuzzy Connected Components Labelling

The connected component labeling with fuzzy neighborhood (CCLF) is based on the notion of fuzzy connectedness and requires the definition of a fuzzy similarity relation between the spells. Contrary to the notion of net neighborhood, a fuzzy neighborhood is defined on a fuzzy subset. In the literature, several studies take into account the spatial information in the definition of fuzzy similarity relation [27,28]. Our neighborhood fuzzy model disregards the fact that we work solely on the histogram. The model of neighborhood fuzzy (fuzzy similarity) selected is defined by Equation (5). The cost of a path between two related spells \( c \) and \( d \) in the histogram...
can be defined by the fuzzy relation $\xi$ as:
$$\mu_\xi(P) = \min\left[\mu_1(C^1, C^2), \ldots, \mu_m(C^{m-1}, C^m)\right], m \geq 2$$

(7)

ou $P = \{C^1, \ldots, C^m\}$ est un chemin entre les sorties $C^1 = c$ et $C^m = d$. Plusieurs chemins peuvent exister entre les sorties $c$ et $d$, le coût global du chemin est défini par la valeur maximum de tous les coûts calculés sur le chemin. Le coût global du chemin entre deux sorties $c$ et $d$ dans le histogramme peut alors être défini par la relation floue $\psi$ comme :
$$\mu_\psi(P) = \max_{P \in PC} \mu_\xi(P)$$

(8)

La relation floue $\psi$ est une relation d'équivalence. La définition d'une relation $\psi$-voisinage entre deux sorties requiert de fixer un seuil minimum de similitude qui nous est dénoté $\alpha$. Ce seuil étant trouvé, trouver les prédécesseurs d'une sortie $c$ en cherchant ses voisins de la valeur minimum $\alpha$ c'est à dire les sorties de la net set $\alpha$-cut de la relation floue $\psi$.

Soit $d$ un élément du set des sorties précédentes de $c$ dans le histogramme $H_c$.

$$\text{Predecessors}(c) = \{d \in H_c | \mu_\psi(c, d) \geq \alpha\}$$

(9)

Le principe du CCLF est similaire à celui du CCL, avec pour seule différence la recherche des prédécesseurs d'une sortie dans le histogramme compact comme indiqué par l'équation (9). En pratique, nous varierons la valeur $\alpha$ pour étudier son influence sur le nombre de composants connectés. Deux valeurs nous ramèneront à des cas particuliers :

- lorsque $\alpha = 1$, les sorties du histogramme compact sont considérées comme une seule sortie comme un composant connecté, le nombre de composants connectés est égal au nombre de sorties ;
- lorsque $\alpha = 0.5$ le CCLF correspond à la CLF.

### 3.4. Notion de Fuzzy Connected Components Labelling

Le programme d'intégration des prédécesseurs flous, leur nombre étant limité par la fixation du coût global $\alpha$. Le programme retourne comme output le vecteur $E$ contenant les labels des sorties qui apparaissent dans le histogramme compact, et le nombre de composants connectés ($N_{berCc}$) du histogramme.

**fonction** [E, NberCc] = FCCL(Hc, $\alpha$)

//Hc: histogramme compact n-D
//E: tableau de sorties label
//NberCc: nombre de composants connectés
//Tequiv: tableau de correspondances

**Label**

//taille: fonction qui retourne le nombre de rows dans une table
//elimineRedondance: retire les éléments redondants d'un tableau
//max: fonction qui retourne la valeur maximum d'un tableau
//min: fonction qui retourne la valeur minimum d'un tableau

$$N = \text{taille}(Hc)$$ //nombre de sorties du n-D compact histogram

$E(1) = 1$ //label de la première sortie de $Hc$ à 1

**For** $i = 2: N$

$P = Hc(i,:)$ //current sortie labeling

$j = i - 1$

$\text{IndexPred} = \emptyset$

//Prédécesseur de $c$

**While** $(Hc(j,1) - Hc(i,1) \leq d_i)$

$c = Hc(j,:)$

$$\text{If} (\mu_\psi(c, d) \geq \alpha)$$

$\text{IndexPred} = [\text{IndexPred}; j]$

**EndIf**

**EndWhile**

**EndFor**

**If** $(\text{taille} \text{IndexPred} \neq 0)$

$E\text{label} = E(\text{IndexPred})$//retourne les labels

**EndIf**

**If** $(taille(Elabel) = 0)$

$E(i) = 1 + \max(E)$

$E\text{equiv}(E(i)) = E(i)$

**ElseIf** $(\text{taille}(Elabel) = 1)$

$E(i) = Elabel$

**Else**

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ENG
\[ E(i) = \min(E_{\text{tig}}) \]
\[ \text{Tequiv}(E(\text{IndexPred})) = E(i) \]

\textbf{EndIf}

\textbf{EndFor}

\textbf{For} \quad i = 2:N

\[ E(i) = \text{Tequiv}(E(i)) \] //global update of labels

\textbf{EndFor}

3.5. Example of Results

The FCCL algorithm is applied on the compact \( n \)-D histograms of multicomponent images in Figure 1. Table 5 illustrates the results for different values of \( \alpha \). It shows that when the degree of similarity \( \alpha \) decrease, the number of connected components (CC) decrease.

4. Fuzzy Vectorial Classification of Tuples

4.1. Principle

The classification of colours is carried out in two steps: the learning step and the decision step.

The learning step is a hierarchical decomposition of populations in the compact \( n \)-D histogram. For each level of population \( p_n \), peaks \( P_i \) are identified by the FCCL algorithm for a given value of \( \alpha \), which retains the connected components whose populations are greater than or equal to \( p_n \). Each peak is then iteratively decomposed into narrower peaks, beginning from population 0. A peak is labelled as significant if it represents a population greater than or equal to a threshold \( S \) (expressed in percent of the total population in the histogram). The procedure is illustrated in part a of Figure 2 (drawn in one dimension for clarity). We shall name kernels \( K_i \) the peaks corresponding to circled leaves in part b of Figure 2. In other words, kernels are significant peaks (part a of Figure 2) without descendants in the hierarchical decomposition tree (part b of Figure 2) (e.g., Figure 2 shows five significant peaks \( P_i \) (\( i = 0 \) to 4) and three kernels \( K_i \) \( (i = 2, 3, 4) \)). The number of classes \( N_c \) is taken equal to the number of kernels (the class corresponding to kernel \( K_i \) is noted \( C_i \)). Therefore \( N_c \) depends

<table>
<thead>
<tr>
<th>Images</th>
<th>( a = 0.50 )</th>
<th>( a = 0.33 )</th>
<th>( a = 0.25 )</th>
<th>( a = 0.20 )</th>
<th>( a = 0.17 )</th>
<th>( a = 0.14 )</th>
<th>( a = 0.125 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>House</td>
<td>6812</td>
<td>2000</td>
<td>684</td>
<td>252</td>
<td>108</td>
<td>59</td>
<td>28</td>
</tr>
<tr>
<td>Mandrill</td>
<td>21859</td>
<td>5939</td>
<td>2016</td>
<td>736</td>
<td>279</td>
<td>106</td>
<td>57</td>
</tr>
<tr>
<td>Peppers</td>
<td>12443</td>
<td>3277</td>
<td>1130</td>
<td>514</td>
<td>261</td>
<td>153</td>
<td>110</td>
</tr>
<tr>
<td>IRM</td>
<td>66538</td>
<td>33594</td>
<td>16572</td>
<td>7687</td>
<td>3718</td>
<td>1704</td>
<td>837</td>
</tr>
<tr>
<td>M4</td>
<td>94443</td>
<td>30373</td>
<td>8583</td>
<td>3020</td>
<td>1307</td>
<td>678</td>
<td>379</td>
</tr>
<tr>
<td>Orge</td>
<td>87558</td>
<td>55312</td>
<td>43431</td>
<td>35493</td>
<td>29236</td>
<td>24808</td>
<td>21423</td>
</tr>
</tbody>
</table>

Figure 2. An example of hierarchical decomposition with \( \alpha = 0.5 \). The circled leaves (part (b)) correspond to significant peaks as obtained at the end of the iterative decomposition (solid lines in part (a)), whereas leaves marked \( < S \) (part (b)) correspond to insignificant peaks (dotted lines in part (a)).
on the threshold $S$, i.e. on the precision the image colors are analyzed with and the value of $\alpha$ the degree of similarity between the spels.

In the decision step, the mass center $\mu(K_i)$ of each kernel $K_i$ is calculated in the feature multidimensional space. Let us denote by $\beta$ the color corresponding to the point of coordinates $(g_1, g_2, \ldots, g_n)$ in the feature space. Two cases appear: if $(g_1, g_2, \ldots, g_n)$ belongs to $K_i$, color $\beta$ is attributed to class $C_i$; if not, let us denote by $P_k$ the peak which belong to $(g_1, g_2, \ldots, g_n)$; color $\beta$ is attributed to class $C_i$ corresponding to kernel $K_i$, son of $P_k$, such that $d[\mu(K_i), (g_1, g_2, \ldots, g_n)]$ is minimum, where $d[y, z]$ is the Euclidean distance between $y$ and $z$.

We give a name of this method that we call HierarchieFuzzy_nD.

4.2. Results of Segmentation

The following figure denote Figure 3 shows an example of segmented images by HierarchieFuzzy_nD and K-means methods.

5. Methods for Assessing the Quality of Segmentation

Image segmentation is a fundamental process in image and video analysis. Several approaches have been put forward in the literature [22,29]. We have the region, contour and texture approaches, but this work interests the region approach. It is often used to partition an image into separate regions, which ideally correspond to different

<table>
<thead>
<tr>
<th>Segmented images</th>
<th>IRM (5 classes)</th>
<th>M4 (8 classes)</th>
<th>House (5 classes)</th>
<th>Peppers (6 classes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a = 0.50)</td>
<td><img src="image1" alt="IRM" /></td>
<td><img src="image2" alt="M4" /></td>
<td><img src="image3" alt="House" /></td>
<td><img src="image4" alt="Peppers" /></td>
</tr>
<tr>
<td>(a = 0.33)</td>
<td><img src="image5" alt="IRM" /></td>
<td><img src="image6" alt="M4" /></td>
<td><img src="image7" alt="House" /></td>
<td><img src="image8" alt="Peppers" /></td>
</tr>
<tr>
<td>(a = 0.125)</td>
<td><img src="image9" alt="IRM" /></td>
<td><img src="image10" alt="M4" /></td>
<td><img src="image11" alt="House" /></td>
<td><img src="image12" alt="Peppers" /></td>
</tr>
<tr>
<td>K-means</td>
<td><img src="image13" alt="IRM" /></td>
<td><img src="image14" alt="M4" /></td>
<td><img src="image15" alt="House" /></td>
<td><img src="image16" alt="Peppers" /></td>
</tr>
</tbody>
</table>

Figure 3. Example of segmented images by HierarchieFuzzy_nD and K-means methods.
real-world objects.

Many segmentation methods exist in literature, but it is difficult to evaluate the efficiency and to make an objective comparison of different segmentation methods. This general problem has been addressed for the evaluation of a segmentation result [20]. There are two main approaches:

- There are supervised evaluation criteria based on the computation of a dissimilarity measure between a segmentation result and ground truth. Baddeley’s distance [30], Vinet’s measure [31], or Hausdorff’s measure [32] are examples of supervised evaluation criteria. In practice the ground truth of natural images is a segmentation results manually made by experts.

- There are unsupervised evaluation criteria that enable the quantification of the quality of a segmentation result without any a priori knowledge. These criteria generally compute statistical measures such as standard deviation or the disparity of each region or class in the segmentation result [33-36].

Currently in practice, no evaluation criterion appears to be satisfactory in all cases [22]. However, we choose here the best criterion for each type of evaluation, i.e. Vinet measure for supervised evaluation and Zeboudj measure in unsupervised case [22].

One potential benefit of supervised methods over unsupervised methods is that the direct comparison between a segmented image and a reference image is believed to provide a finer resolution of evaluation, and as such, discrepancy methods are commonly used for objective evaluation. However, manually generating a reference image is a difficult, subjective, and time-consuming task [20].

5.1. Vinet Measure

The Vinet’s measure [31] that is a supervised criterion which corresponds to the correct classification rate is used as reference for the analysis of the synthetic images. In this case, the ground truth is available. This criterion is often used to compare a segmentation result $I_g$ with a ground truth $I_{gref}$ in the literature. We compute the following superposition table:

$$T(I_g, I_{gref}) = \sum_{i,j} \text{card}\{R_i \cap R_{j}^{\text{ref}}\}$$

(10)

Where $\text{card}\{R_i \cap R_{j}^{\text{ref}}\}$ is the number of pixels resulting from the intersection of regions $R_i$ and $R_j$ in the ground truth. The best match between $R_i$ and $R_j^{\text{ref}}$ is one that maximizes $T(I_g, I_{gref})$. Vinet’s measure gives a dissimilarity measure, it is computed as follows:

$$\text{Vinet}(I_g, I_{gref}) = \frac{\text{Card}(I_g) - \text{Card}(R_i \cap R_{j}^{\text{ref}})}{\text{Card}(I_g)}$$

(11)

This criterion is often used to compute correct classification rate of the segmentation result of a synthetic image.

5.2. Zeboudj Criterion

Zeboudj [35] proposed a measure based on the combined principles of maximum interregions disparity and minimal intraregion disparity measured on a pixel neighborhood. Zeboudj’s criterion is defined by:

$$\text{Zeboudj}(I_g) = \sum_i \frac{\text{Card}(R_i) \times C(R_i)}{\text{Card}(I_g)}$$

(12)

Where $C(R_i) \in [0,1]$ is the disparity of the region $R_i$.

This criterion is suitable for evaluating segmentation of homogeneous and little texture images.

6. Results and Discussion

The tests were performed on many multicomponent images. A synthetic illustration of the evaluation of our method of segmentation is applied to images of Figure 1 with a variation in the number of classes per image to be segmented.

Indexed Table 6(a) to (f) contains the results of the unsupervised evaluation applied to different images of which we doesn’t ground-truth. However Table 7 illustrates the results of the supervised evaluation obtained from 24 images of forsythia and 1 grape image of which we have their ground-truth.

The relevance of our method of segmentation is made with respect to k-means known in the world of image processing because of its simplicity and its performance.

The Table 7 shows that in supervised evaluation, the fuzzy multidimensional hierarchical method of segmentation HierarchieFuzzy_nD outperforms k-means for each image. This is explained by its ability to detect colorimetrically or spectrally similar classes.

The indexed Table 6 by (a) to (e) evaluates in a synthetic way HierarchieFuzzy_nD comparing it systematically to k-means and by putting forward the evaluation of HierarchieFuzzy_nD in the case of the classical connectivity between speks i.e for $\alpha = 0.5$. Thus can be drawn the following conclusions:

- The segmentation method HierarchieFuzzy_nD proposed gives better results overall compared to k-means as well for color images as multicomponent images in unsupervised evaluation. This is justified by taking into account the uncertainty in
the spectral data by solving the problem of colorimetric or spectral similarity between spels of the multidimensional histogram.

- While limiting oneself to alpha = 0.5 corresponding to the traditional connexity, we note that for the images M4 and Orge of which the number of spectral plans is high, that HierarchieFuzzy\_nD is systematically considered to be less powerful than k-means. This lower performance is a consequence of the phenomenon of over-segmentation justified by the Table 4 which shows the widespread problem of the spels of the nD histograms in a multidimensional space when n is large, hence the interest of the introduction of fuzzy connectedness between spels characterized by the parameter $\alpha$, highlighting the relevance of HierarchieFuzzy\_nD. The major drawback of this last is to be costly in computing time.

Table 6. Unsupervised evaluation of multicomponent images segmentation; in red, segmentations considered to be the best.

(a)

<table>
<thead>
<tr>
<th></th>
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<th>8 classes</th>
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(b)

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(c)

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(d)

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(e)

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Table 7. Supervised evaluation of real images segmentation; in red, segmentations considered to be the best.

| Forsythia | \begin{tabular}{l|c|c} 
  & \text{Hierarchie Fuzzy} & \text{nD K-means} \\
  & \text{Vinet (%)} & \text{Vinet (%)} \\
  \hline
Seg_Img01 & 5.78 & 8.43 \\
Seg_Img02 & 6.11 & 10.22 \\
Seg_Img03 & 6.04 & 8.98 \\
Seg_Img04 & 5.34 & 11.06 \\
Seg_Img05 & 5.46 & 9.84 \\
Seg_Img06 & 4.34 & 9.36 \\
Seg_Img07 & 8.26 & 14.06 \\
Seg_Img08 & 5.46 & 10.43 \\
Seg_Img09 & 4.69 & 10.66 \\
Seg_Img10 & 4.85 & 10.35 \\
Seg_Img11 & 5.58 & 12.21 \\
Seg_Img12 & 3.65 & 6.93 \\
Seg_Img13 & 3.57 & 6.88 \\
Seg_Img14 & 4.00 & 7.07 \\
Seg_Img15 & 5.27 & 12.08 \\
Seg_Img16 & 5.58 & 11.71 \\
Seg_Img17 & 4.52 & 8.05 \\
Seg_Img18 & 4.21 & 13.22 \\
Seg_Img19 & 4.18 & 8.57 \\
Seg_Img20 & 4.88 & 8.44 \\
Seg_Img21 & 6.87 & 18.99 \\
Seg_Img22 & 5.04 & 9.42 \\
Seg_Img23 & 5.53 & 8.52 \\
Seg_Img24 & 4.19 & 9.44 \\
Seg_Raisin & \textbf{5.76} & \textbf{5.96} \\
  \hline
\end{tabular} |

7. Conclusion

This work is a contribution to the classification by the analysis of multidimensional histograms. It proposes a vectorial strategy to segment color or multicomponent images and provides the tools necessary for its implementation. This work is accompanied by a more general reflection on the principle of classification in a multidimensional space in connection with the evaluation methods of segmentation.

We have liberated the large volume of multidimensional histograms using the nD compact histogram for which we have proposed a fuzzy algorithm for labeling connected components. This algorithm allowed us to develop an unsupervised and nonparametric method by vectorial classification of multidimensional histograms. We made a solution to the problem of over-segmentation generated by the appearance diffuse of multidimensional histograms and evaluated our segmentation results.

We chose to approach the Classification by analyzing histograms nonparametrically with emphasis on algorithmic and geometric approaches comparatively to statistical approaches. We justify this choice by the need first there was to provide solutions to the problem of dealing vectorially histograms of multicomponent images because of their size. The tools that we have developed, allowed us to better understand the mechanism of classification in a multidimensional space and to open channels to predict outcomes.

In Perspective, this work can be directly exploited to define new vectorial methods or strategies for the analysis of multidimensional histograms and their classification. The algorithms developed here can be used to serve statistical or spatio-colorimetric approaches for the segmentation of color and multicomponent images.

8. References


