Abstract: In this study, we process the quality of the product characteristics for unilateral specification. Many industrial products can be characterized as bigger-the-best type or smaller-the-best type. Quality characteristics and process yields of unilateral or bilateral specification products can be evaluated by using process capability indices. Process capability indices Cpl or, Cpu are utilized to evaluate quality characteristics and process yields of unilateral and bilateral specification products due to the fact that the formulas for these indices are easy to understand and straightforward to apply. In this paper, a method to incorporate the fuzzy inference with process capability indices in the quality characteristics assessments is presented. Therefore, this paper will be combined with statistical and fuzzy inference square test method to assess whether the process capability requirements to meet product specifications.

Keywords: process capability indices; process yield; fuzzy inference

1 Introduction

In recent years, process capability studies on a lot of discussion. However, the study assumes that most observed and specifications for specific information, but in practice, due to measurement error may result in property value can not be clearly quantified, or products of uncertain quality characteristics of uncertainty, may the traditional assumption measurement data is clear when the process capability assessment methods derived from the uncertainty arising from miscarriage of justice. Therefore, how to consider the ambiguity of the conditions, to more accurately measure the process performance is necessary. In this paper, we have to process the quality characteristics of the product as bigger-the-best type or smaller-the-best type for the research topic, and consider the lower bound of its size and specifications of the corresponding upper bound for the fuzzy specification of conditions, the proposed fuzzy unilateral process capability indicators of the concept. In this study, we unilateral specification of the quality characteristics and specifications of fuzzy observations clearly but the data for the study, the specifications and quality characteristics of other types of bilateral blurred information is not in the scope of this study.

2 Literature review

In recent years, fuzzy theory has been applied in many fields, it is the subject matter hereof, review of fuzzy theory in process capability analysis research. Yongting (1996) have introduced the concept of fuzzy quality and fuzzy process capability indices proposed calculation method. Lee (2001) and Hong(2004) in considering the observed value of ambiguous situations, the proposed fuzzy concept of process capability indices. Prachami et al. (2005) in considering the product specifications for the fuzzy case, the description is related to fuzzy process capability indices, and discuss the relationship between the various indicators. Chen (2003) using fuzzy estimation method, based on process capability indices as selection of suppliers. Tsai, C.C. and Chen, C.C. (2006) to fuzzy process capability indices for process capability of the decision. Wu, C.W. (2009) using fuzzy estimation method to construct fuzzy process capability indices, and p-value for the concept of process capability of the test.

In the related study, many authors have considered data for the fuzzy conditions, fuzzy process capability indices proposed the concept, or information is clearly underway to construct a fuzzy blur estimation process capability indices. However, the main problem in these studies is that for fuzzy process capability of the test and did not propose a feasible method, only the main results are discussed fuzzy process capability indices of building traditional process capability indices and their relationships. Therefore, the content in this study, we propose fuzzy process capability in addition to indicators of view, the most important thing is to complement previous studies did not provide the process capability of the test method.

3 Research Methods

3.1 Traditional unilateral process capability indices

For the analysis of unilateral specifications, Kane
(1986) proposed a bigger-the-best type and smaller-the-best type of the quality characteristic look of process capability indices, these indicators can be defined as follows:

\[ C_{pl} = \frac{\mu - LSL}{3\sigma} \] (bigger-the-best type) \hspace{1cm} (1)

\[ C_{pu} = \frac{USL - \mu}{3\sigma} \] (smaller-the-best type) \hspace{1cm} (2)

However, the parameters of the population is usually unknown, it is necessary to use the sample through the sampling to estimate population parameters estimated value. Sample estimator is defined as follows:

\[ \hat{C}_{pl} = \frac{\bar{X} - LSL}{3S} \] \hspace{1cm} (3)

\[ \hat{C}_{pu} = \frac{USL - \bar{X}}{3S} \] \hspace{1cm} (4)

\( \bar{X} \) is the sample mean, \( S \) is standard deviation for the sample data.

Assuming a normal distribution of the quality characteristics for the hypothesis, unilateral process capability index estimation of the probability distribution of \( \hat{C}_{pu} \) and \( \hat{C}_{pl} \) can be derived, respectively, with non-central \( \delta \) t-distribution parameter. Derived as follows:

\[ \hat{C}_{pl} \sim t_{n-1, \delta} \] , where \( \delta = 3\sqrt{n}C_{pl} \)

\[ \hat{C}_{pu} \sim t_{n-1, \delta} \] , where \( \delta = 3\sqrt{n}C_{pu} \)

\[ \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp[-0.5 \cdot (V - \mu)^2]dv \] \hspace{1cm} (5)

\( C_{pl} \) (\( C_{pu} \)) is strictly increasing process yield function, the difference between the mathematical relationship between one to one, when the larger process capability index value, then the higher process yield, on the contrary, when the process capability indices more hours, then the process good rate is lower..

### 3.2 Fuzzy unilateral process capability indices

Specifications or specifications unilateral lower bound on the size of the fuzzy boundary conditions, the traditional process capability indices will no longer apply. Observations obey the normal distribution assumption, if the specifications are the lower bound and upper bound
specifications from the triangular fuzzy number, that is, $\text{LSL} \sim Tr(a, b, c)$ and $\text{USL} \sim Tr(d, e, f)$ is based on the nature of fuzzy operation, unilateral specification of the fuzzy process capability index is derived as follows

$$\overline{C}_{pl} = \frac{\text{Tr}(\mu, \mu, \mu) - \text{Tr}(a, b, c)}{3\sigma}$$

$$= \frac{\text{Tr}(\mu - c, \mu - b, \mu - a)}{3\sigma}$$

$$= \frac{\text{Tr}(\mu - c, \mu - b, \mu - a) - \text{Tr}(\mu, \mu, \mu)}{3\sigma}$$

$$\overline{C}_{pu} = \frac{\text{Tr}(d, e, f) - \text{Tr}(\mu, \mu, \mu)}{3\sigma}$$

$$= \frac{\text{Tr}(d - \mu, e - \mu, f - \mu)}{3\sigma}$$

$$= \frac{\text{Tr}(d - \mu, e - \mu, f - \mu) - \text{Tr}(\mu, \mu, \mu)}{3\sigma}$$

Obviously, when the specification is ambiguous situation, unilateral specification of the fuzzy process capability indices for the triangular fuzzy numbers. However, when $a = b = c, d = e = f$, and the specification is clear that the data is fuzzy process capability index is the tradition of process capability indices. In other words, the traditional process capability indices for fuzzy process capability index of a special case.

### 3.3 Fuzzy process capability index of the test

In the implementation of fuzzy process capability indices before the test, how to compare the size of fuzzy numbers is necessary. an effective and simple sorting idea is as follows

**Ranking function**

Ranking function $r : F(R) \rightarrow R$

Sort function $r$ maps each fuzzy number to a real value on the relationship between the size of two fuzzy numbers can be expressed as follows:

- $\overline{A} \geq \overline{B}$ if and only if $r(\overline{A}) \geq r(\overline{B})$
- $\overline{A} \leq \overline{B}$ if and only if $r(\overline{A}) \leq r(\overline{B})$
- $\overline{A} = \overline{B}$ if and only if $r(\overline{A}) = r(\overline{B})$

Based Roubens (1990) proposed the method of sorting function, $r(\overline{A})$ defined as

$$r(\overline{A}) = \frac{1}{2} \int_{-\infty}^{\infty} \inf \overline{A}[a] + \sup \overline{A}[a] da$$

When the fuzzy number is triangular fuzzy number $Tr(a, b, c)$, then the equation (3) simplifies to

$$r(\text{Tr}(a, b, c)) = \frac{2b + a + c}{4}$$

**Process capability indices of the test**

In the actual situation, to fully understand the parent information is more difficult, requiring the use of the mother out of the sample to determine the statistical assumptions.

Of the fuzzy process capability assessment, the sample data must be collected to evaluate the sample of the fuzzy process capability indices are defined as follows:

$$\overline{C}_{pl} = \frac{\text{Tr}(x - c, x - b, x - a)}{3\sigma}$$

$$\overline{C}_{pu} = \frac{\text{Tr}(d - x, e - x, f - x)}{3\sigma}$$

As mentioned above, when $a = b = c, d = e = f$ and the specification is clear that the data is fuzzy process capability index is the traditional style of the estimated process capability index estimation.

For fuzzy process capability indices $\overline{C}_{pl}$ and $\overline{C}_{pu}$ of the test, can be expressed as

$$H_{0} : \overline{C}_{pl} \leq \overline{C}_{0} \text{ vs. } H_{1} : \overline{C}_{pl} > \overline{C}_{0}$$

$$H_{0} : \overline{C}_{pu} \leq \overline{C}_{0} \text{ vs. } H_{1} : \overline{C}_{pu} > \overline{C}_{0}$$

Which $\overline{C}_{0}$ means that for fuzzy process capability index the minimum requirements of the fuzzy numbers.

Here, we take $\overline{C}_{pl}$ test of an example as follows:

Given significance level $\alpha$, the fuzzy concept of the critical value can be expressed

$$a = P\left(\overline{C}_{pl} > \overline{C} | \overline{C}_{pl} = \overline{C}_{0}\right)$$

as

$$= P\left(\text{Tr}\left(\frac{x - c}{3\sigma}, \frac{x - b}{3\sigma}, \frac{x - a}{3\sigma}\right) > \overline{C} | \overline{C}_{pl} = \overline{C}_{0}\right)$$

Final solution of the equation, and try to solve the fuzzy critical value $\overline{C}$, in order to determine the process capability as a standard

### 4 Conclusion

In the real industrial environment, some specifica-tions are not clear digital product characteristics, or cause property values due to measurement error can not be clearly quantified, based on the traditional assumption measurement of these data is clear when the process capability assessment of uncertainty derived produce false positives, in this study fuzzy process capability indices proposed method, the ability to find process to determine
the threshold standard for the relevant product characteristics consistent with the industry reference.

References