A State Variable Method for the Realization of Universal Current-Mode Biquads

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Abstract

A state variable method of converting single-resistance-controlled-oscillators (SRCO), into universal current-mode biquad (offering realizations of all the five standard filter functions namely, low pass, band pass, high pass, notch and all pass) has been highlighted. The workability of the exemplary implementation of the derived current-mode universal biquad has been demonstrated by PSPICE simulation results based upon 0.35 μm technology. It is expected that the proposed method can be applied to other SRCOs to generate other multifunction filter structures.

Keywords: Analog Electronics, Circuit Theory and Design, Current Mode Circuits, Universal Biquads, Current Conveyors

1. Introduction

Current-mode universal filters employing current conveyors (CC) and their many variants have been extensively investigated during the past two decades for instance, see [1-19] and those cited therein. Current-mode universal filters can be broadly classified in two categories namely the single input multi-output (SIMO)-type ([11-18]) and the multiple-input-single-output (MISO)-type ([5,10,14,19]). It may be mentioned that while most of the proposers of current-mode (CM) universal filters have come up with a specific topology rather than disclosing any general method of systematic derivation of such filters. It may also be noted that while a general method for realizing SIMO-type CM universal filters has been known earlier [20], to the best knowledge of the authors, any systematic method of synthesizing MISO-type CM universal biquads has not been reported explicitly in the open literature yet. The purpose of this paper is to fill this void.

The main object of this paper is to present a state variable method by which a given single resistance controlled oscillator (SRCO) can be re-configured as a multiple-input-single-output (MISO)-type current-mode (CM) universal biquad. Although the method to convert a SRCO into universal current-mode biquad proposed here might appear simple but it has not been explicitly published in the open literature earlier.

2. The Proposed Method

Although the proposed method is quite general and can be applied to any given SRCO using any kind of active element, we illustrate the method by choosing an earlier proposed SRCO as an example using fully differential second generation current conveyor (FDCCII) as an active element.

It may be recalled that a FDCCII and its applications for analog VLSI were introduced by El-Adawy, Soliman and Elwan in [1]. FDCCII has since then been used in realizing various signal processing and signal generation circuits for instance, see [1-6]. Simultaneously, improved implementations of FDCCII have also been advanced; see [7] and [8].

Some time back, Chang, Al-Hashimi, Chen, Tu and Wan [2] presented two novel single-resistance-controlled-oscillators (SRCO) using a single FDCCII and all grounded passive elements, which is advantageous for integrated circuit implementation.

Consider now one of the CM SRCOs from [2] (Figure 1 therein). The condition of oscillation (CO) for this circuit is given by \( R_i = R_3 \) and frequency of oscillation...
(FO) is given by \( \omega_b = \sqrt[3]{\frac{R_c R_c C_1 C_2}{R_c}} \).

With all its \( y \)-input terminals unconnected, the circuit can be redrawn as shown in Figure 1(a).

Note that with \( Y_1 \) connected to \( Z^+ \) and \( Y_2 \) connected to \( Z^- \) with \( Y_3 \) and \( Y_4 \) connected to ground, one obtains the SRCO of Figure 1 of [2].

To synthesize a filter providing independent control of \( \omega_b \) (say, by the resistor \( R_c \)) and independent control of bandwidth \( \frac{\omega_b}{Q_0} \) (say, by the resistor \( R_c \)), one must have a transfer function \( \frac{I_{out}}{I_{in}} = \frac{N(s)}{D(s)} \) with its characteristic polynomial \( D(s) \) given by

\[
D(s) = s^2 + \left( \frac{\omega_b}{Q_0} \right) s + \omega_b^2 = s^2 + \frac{s}{R_c C_2} + \frac{1}{C_1 C_2 R_c R_3} \quad (1)
\]

From the above, the characteristic equation of the circuit (assuming zero input) is given by

\[
s^2 + \frac{s}{R_c C_2} + \frac{1}{C_1 C_2 R_c R_3} = 0 \quad (2)
\]

It can be easily worked out that the circuit to be synthesized has to have two capacitors and three resistors only, if the voltages across the assumed capacitors \( C_1 \) and \( C_2 \) are taken as \( x_1 \) and \( x_2 \) respectively such a circuit should be characterized by the following matrix state equation

\[
\begin{bmatrix}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt}
\end{bmatrix} =
\begin{bmatrix}
0 & \frac{1}{C_1 R_c} \\
-\frac{1}{C_2 R_c} & -\frac{1}{C_2 R_c}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} = [A]
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} \quad (3)
\]

so that the consequent characteristic equation

\[
det\{s[I] - [A]\} = 0
\]

would, indeed, result in the characteristic Equation (2).

Equation (3) can now be re-arranged as follows:

\[
C_1 \frac{dx_1}{dt} = -\frac{x_1}{R_3} - \frac{x_2}{R_3} \quad (5)
\]

\[
C_2 \frac{dx_2}{dt} = x_1 - x_2 \quad (6)
\]

The above equations can be considered to be the node equations (NE) of the circuit to be synthesized. In view of the FDCCII characterization, which is given by the equations:

\[
\begin{align*}
Y_{ki} & = k, \quad k = 1 - 4; \\
Y_{ii} & = Y_{v1} - Y_{v2} + Y_{v3}, \\
Y_{ii} & = (-Y_{v1} + Y_{v2} + Y_{v4}), \\
Y_{i2} & = -Y_{v1}, \\
Y_{i3} & = i_{v1}, \\
Y_{i4} & = i_{v4}
\end{align*}
\]

the circuit shown in Figure 1(a) (where terminal \( Z^+ \) is replaced by \( -Z^- \)) is characterized by the following equation

\[
\begin{bmatrix}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 \\
-\frac{1}{C_2 R_c} & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
-\frac{1}{C_1 R_c} & 0 & 1 & 0 \\
0 & -\frac{1}{C_2 R_c} & 1 & 0 \\
-\frac{1}{C_2 R_c} & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
v_{v1} \\
v_{v2} \\
v_{v3} \\
v_{v4}
\end{bmatrix}
\quad (7)
\]

where voltages across capacitors \( C_1 \) and \( C_2 \) are chosen as state variables \( x_1 \) and \( x_2 \) respectively. If the various \( Y \)-terminal voltages of FDCCII are chosen as:

\[
v_{v1} = 0, \quad v_{v2} = 0, \quad v_{v3} = x_1, \quad v_{v4} = x_2
\]

then it can be verified that resulting state equations will be same as in Equations (5) and (6).

After having made the above state variable assignment and by appropriately connecting the required external terminals of the FDCCII in accordance with the requirements in Equation (8), a non-autonomous circuit with multiple-inputs and single-output (MISO) is subsequently created by augmenting the circuit with four external current input signals \( i_{v1}, i_{v2}, i_{v3}, \) and \( i_{v4} \) and extending the FDCCII to have one additional \( Z^+ \) output terminal (henceforth to be referred as multiple output FDCCII (MO-FDCCII)). The output current of the resulting cir-
circuit (shown in Figure 1(b)) is found to be:

\[
-i_0 \left( s^2 + \frac{s}{C_2 R_2} + \left( i_2 + i_4 \right) \frac{s}{C_2 R_2} - i_1 \frac{1}{C_1 C_2 R_2 R_3} \right)
\]

\[
= \left( s^2 + \frac{s}{C_2 R_2} + \frac{1}{C_1 C_2 R_2 R_3} \right)
\]

(9)

Note that the D(s) of (9) is exactly same as (1).

The five filter responses can be realized from the circuit of Figure 1(b) as follows: Low pass (LP): making \( i_2 = i_4 = i_0 = 0 \) and taking \( i_1 = i_0 \). Band pass (BP): making \( i_1 = i_4 = 0 \) and taking one of \( i_2 \) or \( i_3 \) as \( i_0 \).

High pass (HP): making \( i_1 = 0 \) and taking \( i_2 \) or \( i_3 \) as \( i_0 \).

Notch: making \( i_2 \) or \( i_4 \) as \( i_0 \) along with \( R_2 = R_1 \). All pass (AP): making \( i_1 = i_2 = i_3 = i_4 = i_0 \) along with \( R_2 = R_1 \).

The various parameters of the realized filters are given by

\[
\omega_o = \sqrt{\frac{C_1^2 R_2 R_3}{R_1}} \quad \text{BW} = \frac{1}{C_2 R_1} \quad Q_o = R_1 \sqrt{\frac{C_2}{C_1 R_2 R_3}}
\]

\( H_o = \begin{cases} \frac{R_1}{R_2} & \text{for BP} \\ -1 & \text{for LP / HP / AP / Notch} \end{cases} \) (10)

where \( \omega_o \) is cut-off frequency in radian/sec, BW = bandwidth, \( Q_o \) = quality factor and \( H_o \) = gain. In the last three cases, having fixed the bandwidth (BW) by \( R_1 \), \( \omega_o \) can be independently controlled by \( R_1 \) while in the first two cases \( \omega_o \) (with \( R_1 \) and/or \( R_2 \)) and BW (by \( R_1 \)) are independently adjustable.

### 3. Analysis Incorporating Nonideal Parameters

Considering the non-ideal MO-FDCCIIs sources, two parameters, \( \alpha \) and \( \beta \) (where \( \alpha = (1 - \varepsilon_\alpha) \) and \( \beta = (1 - \varepsilon_\beta) \), with \( \varepsilon_\alpha (\varepsilon_\beta) \langle 1 \rangle \) and \( \varepsilon_\alpha (\varepsilon_\beta) \langle 1 \rangle \) denote the current and voltage tracking errors respectively) need to be considered. Incorporating these sources of error, we have the following non-ideal characterization of the MO-FDCCI:

\[
i_0 = -\alpha_{02} i_2 \left( s^2 + \frac{s}{C_2 R_2} + \left( \alpha_{02} i_2 + i_4 \right) \frac{s}{C_2 R_2} - i_1 \frac{1}{C_1 C_2 R_2 R_3} \right)
\]

\[
= \left( s^2 + \frac{s}{C_2 R_2} + \frac{1}{C_1 C_2 R_2 R_3} \right)
\]

\( \alpha \) (13)

Taking Equation (12) into consideration, the non-ideal expression for the output current is given by (13)

\[
\omega_o = \sqrt{\frac{C_1^2 R_2 R_3}{R_1}} \quad Q_o = R_1 \sqrt{\frac{C_2}{C_1 R_2 R_3}}
\]

\( H_{oLP} = \alpha_{01} \alpha_{02} \beta_{13} \beta_{24} \) (14)

The non-ideal gains and realization conditions (whenever applicable) are modified as follows:

\( H_{oBP} = -1 \) (remains unaffected by non-ideal voltage/current gains)

\( H_{oAP} = -1 \) (remains unaffected by non-ideal voltage/current gains)

\( H_{oBP} = -1 \) (remains unaffected by non-ideal voltage/current gains)

\( H_{oAP} = -1 \) (remains unaffected by non-ideal voltage/current gains)

\( H_{oNotch} = -1, \) if \( \alpha_{02} = 1 \); realization condition being same as in HP.

\( H_{oBP} = -1, \) if \( \alpha_{02} = \alpha_{01} = \beta_{24} = 1 \) and \( R_2 = R_1 \).

From the above, the active and passive sensitivities of the non-ideal \( \omega_o \) and \( Q_o \) are given by

\[
S_{\omega_o}^{\alpha} = S_{\omega_o}^{\beta}, \quad S_{\omega_o}^{\alpha} = S_{\omega_o}^{\beta} = S_{\omega_o}^{\alpha} = S_{\omega_o}^{\beta} = -1/2
\]

\( S_{\omega_o}^{\alpha} = S_{\omega_o}^{\beta}, \quad S_{\omega_o}^{\alpha} = S_{\omega_o}^{\beta} = S_{\omega_o}^{\alpha} = S_{\omega_o}^{\beta} = 1/2
\]

\( S_{\omega_o}^{\alpha} = S_{\omega_o}^{\beta}, \quad S_{\omega_o}^{\alpha} = S_{\omega_o}^{\beta} = -1/2, \quad S_{\omega_o}^{\alpha} = 1
\]

From Equation (15) the active and passive sensitivities of \( \omega_o \) and \( Q_o \) are found to be in the range \( -1/2 \leq S_{\omega_o} \leq 1 \), and the circuit, thus, enjoys low sensitivities.

\( \alpha_{02} \) (15)

Although additional circuitry e.g. a multiple-output current follower will be needed at the front end of the proposed universal CM filter circuits to realize the conditions of the kind \( i_4 = i_3 = i_0 \), the total amount of the hardware required, even after including such additional circuitry, will be lesser than the three-FDCCII-based universal filter structures of [1].
4. Simulation Results

To verify the validity of the proposed configuration, current mode filters have been simulated in SPICE by making a CMOS MO-FDCCI based upon the FDCCI from [3] (Figure 3 therein) which is shown here in Figure 2.

PSPICE simulation implementation was based upon a CMOS MO-FDCCI in 0.35 μm technology where the aspect ratios of the MOSFETs are shown in Table 1.

The CMOS MO-FDCCI was biased with DC power supply voltages \( V_{DD} = +1.5 \) V, \( V_{SS} = -1.5 \) V, \( V_{DP} = 0.2 \) V, and \( V_{BM} = -0.66 \) V. To achieve the filters with \( f_0 = 1 \) MHz, the component values chosen were \( R_1 = R_2 = 0.71 \) kΩ, \( R_3 = 1.39 \) kΩ, and \( C_1 = C_2 = 0.16 \) nF. The frequency responses of LPF, BPF, HPF, Notch and APF are shown in Figure 3. Thus, a very good correspondence between theoretical values and PSPICE simulations is observed.

To test the input dynamic range of the proposed filters, the simulation of the band-pass filter as an example has been done for a sinusoidal input signal of \( f_0 = 1 \) MHz. Figure 4 shows that the input dynamic range of the filter extends up to amplitude of 300 μA without significant distortion. The dependence of the output harmonic distortion on the input signal amplitude is illustrated in Figure 5.

Although FDCCI-based filters have been proposed by many authors as [1,3-6], with the exception of [1] (Figure 11 therein), [6] (Figure 2 therein), all others deal with voltage-mode filters.

In view of this, a comparison with MISO-type CM universal biquads using FDCCIIIs presented recently in [1] (Figure 11 there in) and [6] (Figure 2 therein) is now in order. When compared with the circuit of [1], the circuit of Figure 2 has the advantage of using only one active building block (one FDCCI) as against three FDCCII’s in biquads proposed in [1] and use of all grounded passive elements (AGPE) which is an attractive feature for IC implementation. On the other hand, when compared with the circuit of [6] (Figure 2 there in) our circuit has

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**Figure 2. CMOS realization of the FDCCI.**

**Table 1. Aspect ratios of MOSFETs.**

<table>
<thead>
<tr>
<th>MOS transistors</th>
<th>W/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 - M6</td>
<td>0.7/0.35</td>
</tr>
<tr>
<td>M7, M8, M10, M13</td>
<td>1/1.2</td>
</tr>
<tr>
<td>M16, M11, M12, M24</td>
<td>0.7/0.35</td>
</tr>
<tr>
<td>M14, M19, M20, M21, M22, M23, M25, M30, M31, M32, M33, M34, M35, M36, M40</td>
<td>20/0.35</td>
</tr>
<tr>
<td>M15, M17, M29, M31, M32, M33, M35, M36, M41, M42, M43, M44</td>
<td>25/0.35</td>
</tr>
<tr>
<td>M22, M23, M27, M28</td>
<td>0.35/0.35</td>
</tr>
</tbody>
</table>
Figure 3. PSPICE simulation results. (a) Gain response of LPF, BPF, HPF and Notch. (b) Gain and phase response of APF.

Figure 4. Input and output waveforms of the band-pass filter of the proposed circuit for 1 MHz sinusoidal input current of 300 μA.

Figure 5. Dependence of output current total harmonic distortion on input current amplitude for the band-pass filter realized from the proposed configuration.
the advantages of independent tunability of BW or $Q_0$, which is not feasible in the quoted circuit of [6] which also needs two outputs to implement APF. Our FDCCII has nine terminals in contrast to the FDCCII in [6] which has eleven terminals to implement the biquad filter.

The comparison with [9] and [10] is now in order. The circuit of [9] (Figure 8 there in) is also current-mode MISO type and uses five grounded passive elements but used two FDCCII (the first has ten terminals and the other has nine terminals to implement the biquad filter) and not independent tunability of BW or $Q_0$.

The circuit of [10] although uses one FDCCII (eleven terminals to implement the biquad filter in current-mode and voltage mode) but has one floating resistance and needs two outputs to implement LPF and APF.

5. Concluding Remarks

A method has been presented by which the FDCCII-based CM SRCOs of [2] can be reconfigured as MISO-type universal biquads offering realizations of all the five standard filter functions also, thereby enhancing their capabilities. One exemplary biquad resulting from the application of the proposed method was presented and its workability was demonstrated by SPICE simulation using an FDCCII implementation in 0.35 μm CMOS technology.

The methodology presented here could also be applied to all other SRCOs published earlier using other kinds of active building blocks thereby giving rise to a large number of new MISO-type CM universal biquads, some of which may possess some interesting features. This, however, is left for further investigations.

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7. References


