Creative Education, 2015, 6, 961-975
Published Online June 2015 in SciRes. http://www.scirp.org/journal/ce
http://dx.doi.org/10.4236/ce.2015.610098

The Teaching and Learning of Equations: Problems and Possibilities during the Transition from High School to Higher Education

Celia Finck Brandt[1], Ana Lúcia Pereira Baccon[2*]

[1]Department of Methods and Teaching Techniques, State University of Ponta Grossa (UEPG), Parana, Brazil
[2]Department of Mathematics and Statistics, State University of Ponta Grossa (UEPG), Parana, Brazi
Email: [*]ana.baccon@hotmail.com

Received 27 April 2015; accepted 15 June 2015; published 19 June 2015

Abstract

The teaching of equations for elementary school students has been faced with many obstacles, as their difficulties to find the roots of equations of first or second degree. These difficulties are presented by the students in continuing their studies (high school) and also detected on the transition of students to higher education. According to Raymond Duval these difficulties come from the mathematical point of view which is incompatible with the cognitive point of view. In this research we aim to point out to what extent the meanings assigned to algebra by the students of the Bachelor in Mathematics are close among themselves or move away from the cognitive point of view. We developed two instruments to collect qualitative information: a questionnaire with two questions and interviews. The subjects were eighteen students of Degree in Mathematics of the UEPG. The data were analyzed in the light of the discourse functions mentioned by Duval (1995): apophantic, reference and discursive expansion and their cognitive operations: apophantic function and its illocutionary act of predication and operations; referential function and its pure designation of operations, simple categorization and determination; discursive expansion function and its operations narration, description, explanation and reasoning. The speech will be analyzed in terms of semiotic or semantic similarity, internal or external arranged in a double entry table. Empirical data will be entered in cells derived from the crossing of this categorization featuring differentiated discursive expansions: formal expansion, natural, cognitive and lexical. Research is in progress showing no results at this time.

Keywords

Algebra, Equations, Transition from High School to the Top, Functions and Discursive Operations

[*]Corresponding author.

How to cite this paper: Brandt, C. F., & Baccon, A. L. P. (2015). The Teaching and Learning of Equations: Problems and Possibilities during the Transition from High School to Higher Education. *Creative Education, 6,* 961-975.
http://dx.doi.org/10.4236/ce.2015.610098

under Duval, Mathematics Education

1. Introduction

Contemporary society determines some assumptions and expects some attitudes with regard to education and educators. It is often expected that the school, besides providing the formal education, which is its main goal, could also provide the psychological, emotional, ethical and human education in order to fulfill that aspects of education neglected by the parents, which are often so absorbed in their professional works that have less time to properly educate their children. As a result, it is expected from the school a role usually beyond its social function.

We can ask ourselves: what is education? We know that life is an eternal learning (Charlot, 2000). That learning begins from our first step and permeates our whole life. When we are in contact with each other, or handling an object, or reading a book, or talking, or walking, we are always learning.

Learning can occur in several places, and all people can acquire culture, values and skills, often even without going through the school benches. We learn informally in everyday and unexpected situations. On the other hand, what can be said about the formal education, which is done at the school by specialized professionals? That is quite different from informal learning. Formal education has clear objectives, program, schedules, skilled professionals and seek preparation to exercise this function. In this context, education is determined as a job and exercised by professionals duly, and prepared and qualified education is a task that has specific goals, it takes time and occurs at definite places and must to be evaluated..

We also stress that many people see the formal education as the main source—or even the only one—to access knowledge and consequently change their lives after finishing the school. The process of school learning is built gradually and the transition from high school to the top cannot be seen only as training for the labor market and preparation to pass the entrance exam. We highlight that both dimensions are aims and objectives of High School and protected in Article 35 of the LDB (Law of Directives and Bases for the National Education - LDB) (Brasil, 1996), which states very clearly that the process of learning must deepen the knowledge acquired in the previous years in order to study at higher levels. Furthermore, the process of learning should not only abilitate the students to act in the work market, but it should also contribute to the full exercise of their citizenship and critical thinking, thus contributing to develop in them an ethical and humanistic behavior. Summarizing, the learning should allow the student to posses scientific knowledge in order to occupy its place in society, to participate in social relationships in a productive way, with ethical behavior and political commitment, with a critical and creative spirit, exercising its right to the exercise of citizenship.

Within this perspective, Kuenzer (2002: p. 92) points out that we should not give rise to misconceptions that education should be made by qualified teachers, with decent working conditions, including salary, because "education does not support volunteering, improvisation practice missionary or other charity, since it is right, not boon. It therefore requires competent professional treatment, ensured by clearly regulated labor relations".

In most countries, mathematics is presented as an official discipline in the basic curriculum from the early series and permeated all the time at school. It is presented from the beginning of our school life and ends up in the following not only in education, but in our quotidian life and is also very important in other various areas of knowledge. However, we cannot fail to highlight that contact with mathematics has been traumatic for many people. It is very common to find people saying they "hate" math for they think that it is very difficult, what makes it an uninteresting and unexciting discipline. Therefore, discussing and thinking about issues involving mathematics, teaching and learning is always an extremely important action to consolidate more and more what we call mathematics education.

Thinking about the problems of teaching mathematics in Brazil, and trying to analyze some possible causes and consequences on student's learning is what motivates the present investigation, which aims to present some reasonings about the analytical process of teaching and learning equations, under the bias of the difficulties and possibilities during the transition from high school to higher education.

2. Mathematics Education and the Teaching of Algebra

At any level of education, it is very common for students to fail in establishing relationship between mathematical knowledge learned in school with things of their daily life and this is one of the greatest difficulties in mathematics learning. Marcozzi (1976: p. 185) notes that “teachers should evaluate their work and conclude that they are the biggest culprits for these difficulties” and emphasizes that this is because “teachers perceive this area as a watertight compartment, as a discipline more to be taught, and do so through formal exercises, where students are trained to react with certain answers to certain questions”. There is another bias related to the teacher’s resistance to change, as stressed by Micotti (1999: p. 153):

“In recent years, curricular changes and new pedagogical proposals are present in the school environment, and those responsible for education have been shown to be sensitive to them. But its implementation is fraught with difficulties, besides the usual resistance to change. In this context is part of the teaching of mathematics.”

The above situation has also been causing concerns to the teachers, who often end up teaching in the same way that they were taught. But, as emphasized by Micotti (1999: p 164), “the renewal of education is not just about teacher attitude change, despite scientific knowledge, but also, and especially, on the student’s knowledge: it is necessary to understand how he understands, builds and organizes knowledge”. We believe that the teaching of mathematics was developed from the needs and realities of the student, would the teaching of mathematics be constructed from the student needs and its reality, by taking into account the historical context and situations from the real world around it, then we could add more significance to its knowledge.

Within the context of teaching and learning of mathematics, we find that some specific contents characterized by a high level of mathematical abstraction become hard to teach and to learn. Among them we can highlight the Algebra education. Traditionally, the Algebra content has been seen as difficult and very abstract. We often hear students saying that working with numbers is difficult, so let us imagine the difficulty of mixing numbers and letters! We believe in fact that students have a great difficulty in the transition from arithmetic to algebra, ranging from: make sense of an algebraic expression; assign specific meaning letters that represent a number; analyze a letter with different status: variable, unknown value or unknown; know represent a relationship between unknowns, unknown values or numbers variables through a sentence in algebraic language, etc.

We highlight that Algebra is one of the great mathematics sub-areas and the study of its historical and epistemological is very important to their teaching. Fiorentini, Miorim and Miguel (1993) propose a teaching methos of algebra in a perspective which students can “think generally perceived regularities and explain this regularity through mathematical structures or expressions, think analytically to establish relationships between variables and quantities” (p. 87).

The learning of algebra, by ira turn, needs to be the focus of investigations in which one way may be the analysis of student speech. This analysis has, in turn, to be supported by a theoretical framework, which in this work we chosen to be the discursive functions of Raymond Duval.

3. Discursive Functions and Operations According to Duval

Duval (1995) states that the discursive functions include cognitive operations featuring a semiotic system must meet to be able to a speech and to be considered as a language. The four discursive functions identified by the author are: reference to designate objects; apophantic to be able to say something about the objects they designate, on the form of a proposition enunciated and also bind the proposition enunciated with others in a coherent whole (description, inference...) and check the value, the mode or the combined status to an expression by the person who proclaims. Discursive expansion, when there is the articulation of complete utterances into a coherent unit; of discursive reflexivity, when there is potentially recurring transformation of a complete statement. These functions and their cognitive operations can best be seen in **Figure 1**.

Discursive functions	Reference	Pure designation Categorization Simples Determination Description					
	Apophantic	Predication Elocution Predication and elocution	Values, logical epistemic social			Forms	
						External similarity	Internal similarity
	Discursive Expansion	Replacement accumulation	Description, explanation, narration, Reasoning	→	Semilitude Semiotics	Lexical expansion	Formal expansion
					Semilitude Semantics	Natural expansion	Cognitive Expansion

Figure 1. Discursive functions.

Regarding the discursive functions of the job of a language, Duval (1995: p. 89) also states that:

> The organization of a speech always depends on discursive functions that meet and discursive operations. The influence of the explanatory functions on the organization of a speech is expressed in the predominance given to one or another of the four discursive functions and selection of some specific operations to this role (our translation).

It is also important to note what Duval (1995) puts in relation to the analysis of a speech, which cannot be done only on its linguistic forms of expression. You need to take into account the discursive functions that the speech meets and operations which it uses power to fulfill them. Recalling that realizing even a single discursive function can mobilize several operations and that the discourse analysis is based on a functional analysis, the use of natural language is inseparable from its social role of communication (Duval, 1995).

Following functions are explained discursive framework, apophantic and discursive expansion and their respective operations that will subsidize the analyzes.

3.1. Referential Function

The "reference" discursive function corresponds to the first function that a language must fulfill, which is to designate objects. This discursive function allows, according to Duval (1995), distinguishing four types of operations: Pure designation, simple categorization, determination and description. Pure designation of operation includes the identification of an object (by a gesture, by a particular brand or a combination of signs), which can be considered sufficient to describe and identify an object in an oral communication, but it depends on other operations in one writing. In the case of triangles, letters characterize pure designation to denote the vertices, sides and angles of the triangle, but they do not work alone. For example, if I say, are AB, BC, AC, A, B, C, α, β and γ, precise categorization operation to designate the sides, corners and angles of a triangle. The simple categorization of operation corresponds to the identification of an object based on one of his qualities. This operation includes the use of nouns, verbs and adjectives that qualify the object. In the example of triangles, needs to identify the object and do it from that categorization operation: let A, B and C vertices of a triangle and AB, BC, and AC sides of this triangle; α is the angle determined by the lines AB, BC, β the angle determined by the segments BC and AC and the included angle γ determined by the sides AB and AC. However, this operation also needs be combined with the determination operation because its job by itself is not sufficient to identify an object. In the case of the determination operation, its job is to determine the field of application of the categorization operation, as for example in a sentence like "the" vertices "of a" triangle, or "the" sides "of a" triangle. The description operations consist in the identification of an object by crossing the results of various categorization operations. In the example of triangles, the determination operations are characterized by definite or indefinite articles (the, a, an) that allow the specification of the field of operation, which in this case is the triangle ABC. Such operation is similar to the categorization, but differs from it by the fact that its use is restricted to the cases in which there is not a specific term to designate a particular object.

To better express the particularities of the discursive function "referential" Duval (1995) also points out some considerations about the lexicons for assignment operations. However, the author states that not all lexicons permit compliance with the four designated operations and to understand them better, he distinguishes two types of lexicons: the systematic and associative. The systematic lexicon only allows pure designation operation and not the categorization or determination. Such lexicons have the restriction of not allowing designating more than the objects belonging to a particular domain, for example, the numerals. A lexicon is associative when linking a diversity of objects and phenomena of the physical environment and the socio-cultural environment and not just a set of theoretically elementary objects. For example, A, B and C can denote vertices and midpoints of segments.

Description and characterization operations proper to the referential function allow the identification number represented by one of its qualities, e.g. the meaning or the prefix represents a group of 10, of 100, of 1000 and likewise in Arabic numerals. This ID needs the addition of addition and multiplication operations to follow this structure in Decimal Numbering System (SND). The appointment of a triangle ABC can be represented in **Table 1**.

These operations are needed because the lexicons are the associative type. These lexicons are not familiar to students and should constitute learning object. As an example, in the case of trigonometric relationships we have the word "side". It can be associated to the inside or outside, right side or left side of geometric figures. The side will be associated with the thread that separates the inner and outer region enclosed by the line. It is a seman-

Table 1. Appointment of a triangle ABC.

Pure designation	A, B, C	AB, BC, AC	α, β e γ	ABC
Simple categorization	vertex	Sides	Angles Determined by the sides AB and AC, AB and BC, BC and CA respectively triangle	Triangle
Determination	The	The	The	Of one
Description The description of operation will allow the reference to a triangle without nominates each of them separately and to lead the individual to become aware of the form of representation of this object that have no physical or actual existence.	The sides… of a triangle.			

Source: The authors.

tic extension procedure. Existing words for the study of trigonometry need to be explained or explained: ... side is half, is ... midpoint, is ..., etc.

That is what is important in the case of trigonometry. For example, the sides' nouns, vertices can be assigned by the students as tips, inside or outside the defined area by the plan. In this case it takes patience, precision and awareness by the teacher at the difficulty of the students and this is only possible if it is taken into account in the teaching and evaluation functions of speech.

Also in relation to the referential function, Duval (1995) highlights the forms associated with this function. The author states that there are many forms of expression that may mark the fulfillment of the reference function of discourse objects and to distinguish them and analyze them, one must consider the operations which they mark linguistically. To illustrate, Duval presents a table, which is reproduced here as **Table 2**, with the operations of the referential functions with some expressions related to language records.

3.2. Apophantic Function

According to Duval (1995) the possibility of designating objects is not sufficient to allow a discursive activity, because the language or the system of signs that fulfill this function is only reduced to a code. Thus, a language must also allow that one can say something about the objects is called, which characterizes the call apophantic function, according to the author. The expressions that perform this function in some instances are referred to as complete utterances. But Duval (1995) states that in order to make the difference between a full statement and referring expressions, it is necessary to consider that

> A statement has a "complete sense" because the act of expression that produces it is complete. An act of expression is a complete act of speech when the output expression takes a certain value in the cognitive universe, representational or relational interlocutors. In other words, the specificity of the sense of a "full statement" in relation to a reference expression, must be sought in its value (Duval, 1995: p.105—our translation).

About this value the author points out that this may be a logical value of truth or falsehood, an epistemic value of certainty, necessity, probability, possibility or absurd, a social value of question that requires an answer in order to run, of desire, of promise, among others. Duval (1995: p. 105) also points out that a full statement can have:

- ✓ only a social value, "come quick", "call later";
- ✓ an epistemic value and social value, for example, when you make a promise whose fulfillment seems unlikely or absurd;
- ✓ an epistemic value and a logical value if the act of speech is located in a theoretical context, "the sum of the angles of a triangle is greater than 180".

The value that this announced full may depend on the context of the act of speech and cognitive universe, representational and relational interlocutors.

Table 2. Referential function of designating objects and their cognitive operations.

Referential function of operations	Elementary operations of "natural logic" according Grize	Corresponding expression Forms of operations in natural languages	Corresponding expression Forms of operations on formal languages
Pure designation	Operations of ... and the objects operations	Deictic[3], proper nouns	Constant symbols
Categorization		Nouns, adjectives adjectives, verbs	Predicate symbols (or propositional function)
Description		Genitive constructions, proposals with regard	Conjunction or disjunction of propositions formed by saturation propositional functions
Determination		Articles, adjectives and indefinite pronouns	Symbols quantifying

Source: Duval (1995: p. 100).

3.3. Discursive Functions

Among the four discursive functions Duval (1995: p. 94) states that the discursive expansion is considered as the most important because it is possible to "articulate a range of complete utterances in coherent unity of a narrative, a description, an explanation or a raciocinamento". He states that a language should not only express complete utterances, but should also link them into a drive "discursive thematically continuous and semantically not tautological: report, description, explanation, commentary, argument, deduction, calculation, etc." (Duval, 1995: p. 113). That is, a language needs to link different statements on the same theme in order to better explain it, but without falling into redundancy.

Duval (1995) also points out that a major problem that arises in understanding a speech is related to what he makes explicit or implicit. To exemplify how two successive sentences with no apparent connections can be related, he points out the following "burst a gas canister, the house burned" (Duval, 1995: p. 113). These phrases have no explicit link, but it can be inferred that this is cause and effect, that because this inference is supported by two treatments. The first treatment refers to place in correspondence two words that match the first and the second sentence, if "gas cylinder" and "burning" which are associated with the word "fire", which mobilize the same semantic relationships for two sentences (Duval, 1995). The second treatment, for the author refers to the mobilization of knowledge, as you can only infer that the first sentence is related to the fire, if you have the knowledge, for example, to blow up a gas cylinder may cause fire.

From these considerations, Duval (1995: p. 113) states that "considering the importance of the omissions, reductions and assumptions of spontaneous speech in natural language, the first models of text comprehension focus on the kind of discursive expansion in which the link is an obvious knowledge that is implicit". This means that in order to have a discourse expansion, among other things, the previously required implicit knowledge must be made explicit, is necessary in explanation of implicit knowledge, which constitutes as a basic pre-requisite expertise.

The inference to Duval (1995) is only a particular form of discursive expansion. He considers that it cannot realize what may have different discursive expansion in a description of a report or even a raciocinamento. It also states that the problems in understanding the discursive expansion cannot be reduced to an explanation of the implicit knowledge, and the texts that are distant from a speech of oral practice and in which the inference rules are explicitly given "often turn out to be the that whose apprehension and understanding manifest as more difficult" (Duval, 1995: p. 114). Regarding this aspect, the author shows that it is more difficult to understand a discursive expansion about a situation that is not used in oral communication.

With regard to the operations of discourse expansion function, Duval (1995) states that to determine them is necessary as the difference between the progress mode in which a speech is characterized as logical and other characterized as natural, because it is more spontaneous.

The speech is restricted to produce inferences according to Duval (1995) shows the progression of the propositions by replacement by new inferences from previous inferences. He states that a discursive expansion by inference works by replacing as in a calculation and the understanding of speech developed according to this expansion requires that "every time they realize the application of the rule used, and know what is explicitly stated or what remains implicit" (Duval, 1995: p. 114).

But Duval (1995) points out that it is not that way that makes the progression in a narration, description, or even in an explanation. In such cases the sentences are added each other because speech progressively determines the objects. The author further states that understanding the discourse developed by this expansion mode requires "a synoptic seizure of all sentences and all the relationships that exist between them" (Duval, 1995: p. 114).

With regard to the forms associated with the discursive expansion function, Duval (1995: p. 116) states that they are for the recognition of a series of phrases unity of purpose "a step of a raciocinamento, an episode of a story, the description of an object, the justification of a statement, not a disjointed succession of statements jumping from topic to another" [emphasis added]. He also states that the subject of the discursive expansion is related to the different ways an apophantic unit can be produced in discursive continuity with another apophantic unit, which underlies in a similarity between both. This similarity can be determined by two dimensions: a) the presence or absence of significant common to the two units; and b) mediating or not through a third apophantic unit (Duval, 1995).

Through these aspects, Duval (1995: p. 116) states that it is possible to get "four possible basic forms of discursive expansion on record of a language" (with the presence of common and significant mediation through a third unit, with no common and significant mediation by a third unit, the presence of significant common and not through the mediation of a third unit, with no significant common and not through the mediation of a third unit).

We can illustrate with apophantic units "the gas canister exploded" and "the house burned" that do not have significant in common, but that allow a discursive expansion through the mediation of a third apophantic unit, "the house burned because the bottled gas exploded". This expansion is possible through a mediator apophantic site, since the explosion of a gas canister causes fire.

Another example is presented by similarity Duval (1995) for pointing out the possibility of discourse because of expansion dual sense/reference. Two apophantic units can be semantically similar because there continuity between two statements in the absence of common significant: 50 + 1 and 18/9. In this case despite the different direction from each of these expressions refers to the same number. These two units apophantic expand the speech can be replaced by the number 2.

The texts may combine various forms of expansion, but the author states that all use at least one of four ways. In connection with these forms, he states that "one cannot assume a learning written production and comprehension of texts without taking into account the development of the discrimination capabilities of these four forms of discursive expansion" (Duval, 1995: p. 117—our translation). These four forms are better observed in **Figure 2**.

The semiotic similarity corresponds to the continuation of statements by repeating the same signs or the same words. And the semantic similarity happens when there is reference invariance between statements, which means that there is a thematic continuity between statements, allowing continuous progress (Duval, 1995). The author also warns that the semiotic and semantic similarity does not assure the continuity of the discourse, and hence a second dimension is needed, which is the necessity of resorting to a third point.

Mechanisms	**Internal Similarity Similarity external**	
Expansion	(continued without a third point) (continuity with a third statement)	
Similarity Semiotics (are recovered some significant)	LEXICAL expansion (recovery of the sense of the same unit of vocabulary in a phonetic-auditory mode or graphic-visual) Verbal associations, events Unconscious language	FORMAL expansion (exclusive use of symbols: ratings, algebraic writing, ...) Raciocinamento deductive (functional structure of propositions) Propositional calculus, calculations predicates
Similaridade semântica Law of Frege: Significant different and the same object. (Strict or global reference invariance)	NATURAL expansion (only knowledge of everyday language is sufficient) Description, Narration arguments rhetoric Aristotelian syllogism (predicative thematic proposition) Raciocinamento the absurd	Cognitive expansion (requires knowledge of definitions, rules and laws for domain objects) explanation Raciocinamento deductive (conditional thematic proposition) Raciocinamento the absurd

Figure 2. The four forms of discursive expansion of a second expression Duval (1995). Source: Duval (1995) cited by Tozetto (2010).

The most common case, according to Duval (1995), is when the passage of a statement for its expansion happens directly, without the need for a third point. This happens when “only the recognition of the basic vocabulary of the language used is sufficient to recognize the semiotic similarity or semantic similarity between statements” (Duval, 1995: p. 118). This case is called the internal similarity of two statements.

When the change is indirect, it requires the explicit or implicit mediation of a third point, Duval (1995) calls this case of external similarity. He also states that “there is no discursive expansion of a statement that is not based on the combination of a semiotic or semantic similarity and an internal or external similarity” (Duval, 1995: p. 119).

These questions are important as they allow inferences about the meanings of discourses. Not always the semantic similarity or semiotic ensures this expansion, but that does not prevent us from resorting to a third point.

4. Methodology and Data Presentation

This research is a qualitative approach (Bogdan & Biklen, 1994) carried out with eighteen (18) students of the third year of the Bachelor’s Degree in Mathematics from the State University of Ponta Grossa. Data were collected in two stages:

- The First time-students were asked to submit five words that related to the equation. This activity had as aim to identify which design students had about what’s equation. They were then asked to answer two questions: What is an equation? For you, equation and its teaching are related to what?
- Second time-an activity involving the resolution of equations with support concrete material (golden stuff) was performed. Then the students were asked to answer two questions: What activities with golden stuff added in its equation of conception? What do you find most interesting or most you called attention?

To preserve the identity of the subjects, attach a letter (L-licensing), followed by a number. Thus, L1 represents the first licensing, L2, the second, and so on. **Table 3** below features the first moment of gathering research data:

This first data organization became possible reorganization for further analysis. The first column was worked separately at first, aiming at a reduction of evoked words. This reduction can be observed in **Table 4**.

When dealing with the designation of operations is possible to see the words that are associated immediately to the equations of the first and second degrees. These words are associative lexical and meet the simple categorization of operation, since they have a property or quality of the represented object identified in used nouns. In the case of discursive expansion function words used expand the speech by one of its forms of expansion. In this particular case, the words evoked narrated allow a cognitive expansion, they require the knowledge of definitions, rules and laws for a domain objects.

When it comes to learning the equations of first and second degree is important to identify what was retained for that object of knowledge. In fact many of these words say a lot about this object of knowledge in the case of the conceptual field, the meaning attributed by Gérard Vergnaud, associated with it.

Teachers need to be aware of the students’ understanding of both to organize and reorganize its educational practice in different educational levels. If this is not done teaching can be seated on quicksand.

The apophantic function is also evident since this function is characterized by two operations: preaching and speech. The speech happens as there is an order given by the teacher to be met by students. This order has social value, it is not likely to be true or false, nor has epistemic value.

However it addresses a student returns with words. These rather may have epistemic or logical value. In the case of answers presented it appears that all of them are associated to the first or second degree equation as indicated in **Table 4**.

The same analysis was performed with the question 2 answers: “For you equation is...”. Responses were grouped and presented in **Table 5**.

The analysis of their responses inferred from the social, logical or epistemic of their responses, made possible by one of the forms of expansion of speech: cognitive expansion.

Interestingly, first, that the letters are now associated to the unknowns, now the variables and now the unknown value (L6, L7, L8, L9, L10, L14). These distinct statutes assigned to the letters give the logical value to the response of the students and the teacher reveals how students over their education attach significance to this significant expressed in natural language (equation). In the case letters representing the variables, an equation features not equal but rather a function and unknown value in the case of the expression characterizes a formula (for example, base x height = area or $A = B \times H$).

Table 3. Empirical data from the first activity.

Licensing	Words-Equation 1 and 2nd degree	What is an equation	For you, equation and its teaching is related to:
L1	Equal-degree (1st and 2nd)-Modular-irrational-logarithmic-Exponential-systems	It is an algebraic expression of equality	From the sixth grade is related to an unknown value to be searched.
L2	Side-part number-operations-addition-subtraction-multiplication-division-remarkable product-factoring-equivalnce-MMC-rationalization	It is the formation of numerical and literal parts where it employs mathematical tools (operations) to find a solution.	Is related to mathematical interpretation and its application, through operations to achieve a solution and validate your study.
L3	Numbers-Graphical incógnitas-solution-adding-multiplicação-division-Bhaskara formula-complete square-power-roots-coefficient	Algebraic expression formed by an equality.	-Know define what is a function (introduction). -Build graphics. -Equality. -Master the concepts and development of sum and product enhancement, roots. problem-analysis using equation as a solution.
L4	Equality-variables-independent coefficients-terms-inequação fraction-exponent-problem-solving data-graph method of re-ta-formula Bhaskara square-graphic-Complete a parable	Defined by an algebraic expression which has equal x1, x2, x3, xn variables which together with the coefficients a1, a2, a3, in that the sum results in an independent term logo is formed equation ... a1x1 + a2x2 + a3x3 + ... = b anxn	-by troubleshooting. -analysis of the construction of graphics. Introduction to function.
L5	Equality-coeficientes un-known-mathematical equation -Operations-frações-numberpad -raízes-resolution methods-inverse operations-representation graphically systems of equations-types of equations 2 exponent of the unknown and non-zero coefficient-formula Bháskara-complete square-sum and product of roots-numberpad.	Algebraic expression defined by an equality.	-questioning -técnica troubleshooting. -introduction to the study of functions. Construction, analysis and interpretation of graphs
L6	Unknown-coefficients-equality-sum-subtraction-multiplication-division-operations-School-exponent.	It is an expression used to find the value of an unknown that is something.	Equality-algebra-system balance-application.
L7	Find the value of the unknown -equation 1 and 2 degree-roots-power -subtraction-multiplication-addition-function-graphic--coefficient angular equation of the line-equality-algebra-balance-Bhaskara formula-polynomials-graphic algebraic expressions-complete square.	It is an expression, which has unknown and the equal sign.-balance, which I put on one side, have to have the other.	-Complete square. -Find the unknown value. -Graphics. -Polynomials.
L8	Unknowns-numbers-letters-addition-subtraction-division-multiplication-equality–algebra	Equality where there are unknowns representing values.	Algebra-primary education-equality-systems-unknown-function
L9	Equality-unknown-of the equation-algebraic term-value of x-equation system-2 roots-Baskara formula -complete Square-coefficients-complete and incomplete equation.	Every equation has an equal and unknown, aims to discover the unknown variable.	Concept of equality.
L10	Numbers-unknown-variable-activity-root-results-values for unknown-various terms-Baskara-sum and product-real numbers-complex numbers	It is a pair with unknowns	Mathematics, contents of the seventh year of elementary school, literal expression and algebraic expression
L11	Degree -Numbers-problems-polynomial-roots–values	It is an algebraic expression, equality	From the sixth year, and is related to unknown values found from the equation

Continued

L12	Degree-Function-problem-Báskhara	It is an algebraic expression of equality	From the sixth year is related to a value to be searched
L13	Unknown–Degree-Báskhara-soma and product-multiplication by 1-isolation of the terms-system-replacement-definition-find value-methodology.	It is an algebraic expression of equality	In the sixth year is made unknown value of the approach, which gives an introduction of the equation. Being related to missing data
L14	Variable-independent term-root-equality-unknowns-term x squared-a guess-solution set-real solution-complex solution-solving methods	Having an equal or more unknown, or not having solution	Mathematics-proportion-area-perimeter-measures-comparison values
L15	Equal-find the value of x-problem solving-balance-only solution-in a chart describing the form of a parable-Baskara-delta root–quadratic	Equality, as if a balance which has to balance the sides.	Like a balance
L16	Variable-constant-complete and incomplete equations--incógnita equality-only a root-Báskhara-two roots-there are real solution or not-incomplete full equation-delta-parable-roots–x		Mathematics, algebriza situations of everyday life
L17	Letters-numbers-problem-reasoning-Signs of operations-system power-Báskhara formula	One way of solving exercises that have lyrics	Solve problems that you do not know one of the elements
L18	Numbers-Mathematics-sum of variables-mixture of numbers and letters.	One way to express a mathematical problem. It is a statement that establishes an equality between two mathematical expressions.	Algebraic expressions, interpretation.

Source: The authors.

Table 4. Words evoked related to the word equation.

Undergraduates	WORDS-EQUATION the 1st and 2nd grade
L1, L4, L5, L6, L8, L9, L14, L15, L16	Equality
L3, L5, L6, L7, L8, L9, L10, L13, L14, L16	Unknown
L1, L6, L7, L11, L12, L13	Degree
L3, L5, L8, L10,L17, L18	Numbers
L8, L17, L18	Letters
L3, L4, L5, L7, L9, L10, L12, L13, L15, L16, L17	Bhaskara
L4, L10, L14L, L16, L18	Variable
L3, L10, L14, L15, L16	Solution
L3, L5, L7, L9, L11, L14, L16	Root
L15, L16	Delta
L3, L4, L5, L7, l15	Graphic
L7. L12	Function
L3, L5, L7, L9	Complete square
L4, L5, L14, L15	Method, methodology

Continued

L2, L3, L5, L6, L7, L8, L10, L13, L17	Operations
L2, L3, L4, L5, L6, L7, L9	Coefficients
L4, L9, L16	Complete and incomplete equations
L9, L14, L15, L16	X
L4, L9, L10, L13, L14, L16	Terms
L2	Literal part and numeric part
L2, L4, L5, L7, L9, L15	Equation and inequality
L4, L11, L12, L17	Problem
L4, L5, L6	Exponent
L1	Modular, irrational, logarithmic, exponential and parabola
L7, L8	Algebra
L1, L3, L17	System
L4, L10	Numerical Sets
L2	Algebraic operations
L7, L11	Polynomials
L15, L7	Balance
L2, L7	Equality and equivalence

Source: The authors.

Table 5. Students' responses to the question.

Licensing	What is an equation?
L1, L3, L4, L5, L11, L12, L13	It is an algebraic expression of equality.
L2	It is the formation of numerical and literal parts where it employs mathematical tools (operations) to find a solution.
L4	Defined by an algebraic expression which has equal ×1, ×2, ×3, xn variables which together with the coefficients a1, a2, a3, in that the sum results in an independent term logo is formed equation a1x1 + a2x2 + a3x3 + ... in the b = xn.
L6, L7, L8, L9, L10, L14	It is an expression (Equality that has something unknown that is, values) used to find (aims) find the value of an unknown that is something, having solution or not.
L15	Equality, like a balance where you have to balance the sides.
L17	One way of solving exercises that have lyrics.
L18	One way to express a mathematical problem. It is a statement that establishes an equality between two mathematical expressions.

Source: The authors.

Considering the event's theme this is an issue that shows what is important in the transition of the average student to the top, is, their understandings and meanings on different objects of knowledge.

The apophantic function and discursive expansion function (cognitive expansion based on definitions, rules properties) provide logical and epistemic value to the students' responses. In the case of L4 response is possible to identify that he associates letter to the variable, but the way this variable is designed featuring an unknown whose value to be found can become true or not equal (if any or no solution).

Another response that draws attention is the L15 which combines the equation "equality like a balance where you have to balance the sides." This meaning suffers the consequences of educational obstacles coming from experienced empirical practices in education during their schooling process. From a logical point of view of value is false and the epistemic value does not relate to the concept equation.

Also it would be natural expansion L2 speech "is the formation of numeric literals and parts which make use of mathematical tools (operations) to find a solution" that is not based on properties, laws, rules and admits to being an equation one formation of these numerical and literal parts. This training (of what?) Is not explained and not related to mathematical operations that interconnect (multiplication and addition), but only the mathematical operations known as mathematical tools that are used to find a non explicit solution (solution that?). We can form expressions to combine numbers and letters which do not necessarily characterized as an equation, for example, in the case of formula ($2a + 2b = P$, where P is the name of the perimeter of a rectangle and the letters a and b designating the measures of the sides this rectangle).

This demonstrates that the designation of an equation by means of a mathematical sentence that expresses equality, in which letters (with unknown status) and numbers are related by means of addition and multiplication operations, do not mention these relationships and that status of letter. Therefore all the other mathematical expressions involving letters and numbers are associated with an equation such as, for example, the analytical expression for a function $y = 2x + 3$, or a mathematical formula that letters designate relations between the unknown status value, numbers and mathematical operations.

This identification by the teacher in that moment of transition from high school students for higher education is essential, mainly because they are students of a Bachelor's Degree in Mathematics that will be future teachers who teach mathematics.

The responses of L17 and L18 "One way of solving exercises that have letters" or "[...] to express a mathematical problem. It is a statement that establishes an equality between two mathematical expressions" not conceptualize an equation. This discourse expands the meaning given by the students to the use of an equation and from the point of view of the illocutionary act, means they did not answer the question asked by the teacher (which is an equation). This is because a mathematical problem can be related to a function and not necessarily an equation (eg. A liter of gasoline costs 2.89 reais. What is the mathematical statement that expresses the price spent on a subject that fills your tank with a number any liters of gasoline? In that case the price to pay must be designated by a letter and the number of liters of gasoline by another letter. The formed expression is an equality involving numbers and letters related to each other by a multiplication operation, but will not be an equation, since the letters have the variable status).This same analysis can be extended to L1 response, L3, L4, L5, L11, L12, L13 "is an algebraic expression of equality".

The answer came closest to the concept of equation and its definition was presented by students L6, L7, L8, L9, L10, L14 when defining equation as: "[...] an expression (Equality that has unknown that is something , values), used to find (aims) find the value of an unknown that is something, having solution or not".

To the statement "for you and teaching equation is related to": grouped the responses given to allow complete the sentence and can be viewed in **Table 6**.

The above question fulfilled the cognitive operation of predication of apophantic function with social status of an order to be long. The illocutionary act is present in the answers given by the students who can take a logical or epistemic value. The responses of (L1) unknown value to be searched, (L11) is related to the unknown and found values from the equation, (L13) being related to missing data. It made the approach of unknown value and (L3) know how to define what is a function, (L4) and (L5) Introduction to the study of function and (L8) function, are not true and not epistemically valid. In the case of discursive expansion function allowed by the speech we can infer that the letter is identified the unknown value and variable. A seemingly simple question gives opportunity to the teacher the discovery of erroneous associations made by the students with the letters designating an unknown a representative mathematical statement of an equation.

In this high school transition time for higher education this identification is not only important but fundamental as it is related to the teacher's knowledge of the type of content knowledge, pointed out by Lee Shulmman, to be included in the initial training of future teachers teach mathematics.

Through these answers we can infer that the association of the equations to functions occurs in the process of obtaining the roots of an equation when those roots are considered as the abcissae corresponding to the zeroes of a function. The mathematical statement obtained is actually an equation and necessary to obtain the roots that represent the point (s) of intersection of the curve with the x-axis. This analysis is corroborated by the answers

Table 6. Total answers to the question.

For you and your education equation are related to:		
Relations	Licensing	Answer
Unknown	(L7)	Find the unknown value
	(L8)	Unknowns
	(L12)	Is related to a value to be searched
Equality	(L3) (L9)	Concept of equality
	(L6) (L8)	Equality
Balance	(L6)	Balance
	(L15)	Like a balance
	(L7)	Balance, which I put on one side, have to have the other
System	(L6) (L8)	System
Mathematics	(L10) (L16) (L14)	Mathematics
Application	(L2)	Is related to mathematical interpretation of implementation, through operations to achieve a solution and validate their study in question.
	(L6)	Application
Concepts	(L3)	Mastery of the concepts and development of sum and product enhancement, roots.
Complete Square	(L7)	Complete Square
Expressions Algebraic	(L10) (L18)	Expressions Algebraic
	(L7)	Polynomials
	(L10)	Literal expression
Comparison of values	(L14)	Ratio, area, perimeter, measures, Comparison of values
Interpretation	(L18)	Interpretation

Source: The authors.

of (L3) (L4) and (L5) construction; analysis interpretation of graphics (L7) with graphics.

We also see the association of the algebra equations: "(L6) algebra, (L8) algebra, (L16) algebriza situations of everyday life" and problem solving as claimed "(L3) problem analysis, using equation as a solution, (L4) by solving problems, (L5) technical problem-solving problems, (L17) solve problems which you do not know one of the elements". Such statements have a logical true value, but not epistemic, they are not capable of substantiation. The equations can relate to functions (functions without being considered).

Regarding the teaching degree equations are associated only to the series they are introduced as assertion "(L1) from the sixth grade, (L8) elementary school (L10) content of the seventh year of elementary school (L11) from the sixth year (L12) from the sixth year (L13) in the sixth year he gives an introduction of the equation". Students forget that the equations are also required for calculating the roots of a polynomial or to obtain the point of intersection of the curves.

5. Conclusions

The above discussion and analysis allow us to infer about the social, logical or epistemic values that emerge from the teaching of Algebra, specifically concerning the content of equations. Such inferences are made possible through a form of discourse expansion, namely the cognitive expansion.

From the answers presented, we can observe, first, that the letters are associated with the unknowns, now the variables and the value unknown to the research subjects. This association reveals how students throughout their schooling attach significance to this significant expression in natural language (equation). In the case of letters representing variables, instead of being related to equations the letters are associated to functions, and in the case of an unknown value they are associated to formulas.

The apophantic function and discursive expansion function (cognitive expansion based on definitions, rules, properties) provide logical and epistemic value to student responses, they attach several meanings for the term equation. This meaning suffers the consequences of educational obstacles coming from experienced empirical practices in education during their schooling, which appears justified by the research subjects and sometimes not, and it is not even related to the mathematical operations related among themselves (multiplication and addition), but instead this meaning is related only to the mathematical operations viewed as tools useful to find an unknown solution.

This demonstrates that the designation of an equation by means of a mathematical sentence that expresses equality, in which letters (with unknown status) and numbers are related by means of addition and multiplication operations, does not mention these relationships and that status of letter. For this reason, all other mathematical expressions involving letters and numbers are associated with an equation or a mathematical formula that designates relationships between letters with unknown value status, numbers and mathematical operations.

In the case of discursive expansion function allowed by the speech we can infer that the letter is identified the unknown value and variable. A seemingly simple question gives opportunity to the teacher to discover the erroneous associations made by the students with the letters designating an unknown but a representative mathematical statement of an equation.

This identification, by the teacher, in the moment of transition from high school students for higher education, is essential, mainly because they are students of a Bachelor's Degree in Mathematics and will be future teachers who teach mathematics.

In this time of transition such identification is not only important but fundamental as it is related to the teacher's knowledge of the type of content knowledge which, as pointed out by Lee Shulmman, should to be included in the process of initial training of the future mathematics teachers. Therefore, the algebraic approach, which fosters the abstract thinking and the construction of a symbolic language which could be meaningful to the student is important in this process.

The organization of algebra for its teaching to the students can be considered from two opposite and incompatible points of view: the mathematical point of view and the cognitive point of view. What should be considered, whatever the option is, is whether there will be consequences for teacher training and for researchers' formation.

From the cognitive point of view the teaching:

• There should not be focused on the knowledge to be acquired nor on the cognitive functioning needed by students to comprehend and to do the required work, but instead it should be focused on the intellectual activity and on the act of thinking from which the knowledge emerges.

• It should focus on the intellectual activity which is needed to the act of thinking, without which the knowledge cannot emerge.

These cognitive operations are of a discursive nature and are specific to the function description. The main focus in what regards such operations must be the decomposition of these in terms of cognitive operations according to the semiotic systems considered. Different discursive operations emerge depending on the semiotic system (natural language, numerical language, algebraic language). The types of signs may be words, numbers or letters and symbols. The discursive operation designation will designate every type of object in words, describe the numbers and operations and assign to point calculations and algorithms to use.

Duval (2011) shows us that there is a cognitive complexity to appoint or designate anything because: there are no words that enable sometimes to designate some objects; all the objects have not the same nature to be nominated (individuals, classes, unknowns...); operations or ways of naming are different according to the nature of the designated objects; the fact that each system tends to favor an operation on the other.

Duval (2011) shows us the ways for assignment operations. They can be direct with words from one language to proper names, nouns, time, among others, or with letters or symbols for variables, missing data, parameters, unknowns, etc. They can be built by combining many terms, for example, d/v to designate the time spent by a car at a given speed and at a specific time, or by comparison with a docking object, for example, a designated object according to another "y = x + 8".

According to Duval (2011) the recognition and production of two names of the same object is a strange practice to spontaneous practices of the language around the math.

Depending on these issues, Duval (2011) shows us what to do to take into account, in education, the importance of the appointment of operation. First, it is necessary that teachers are aware of the problems arising from the designation in natural language and the need for designations built by condensation and combination (descriptive).

It allows teachers to take necessary awareness of the designations built by condensation on the numerical data (in the case of open lists and the appointment of pattern regularity linking the numbers from a list in another function).

Another issue concerns the introduction of the letters is made to denote not only an object, but many different objects. This question concerns the different own cognitive operations of the designation function: Pure designation (through systematic lexicons), simple categorization (through associative lexical), determination and description. To exemplify with: "Let A, B and C of a triangle vertices or let A, B and C the midpoints of three segments". The introduction of a letter is not to describe an object, but to provide the meanings to be able to designate many different objects.

Another issue pointed out by Duval (2011) for the designation operation is not proposing situations instantiation, but open lists of numbers that will eventually require a letter to the designation of the regular pattern with use of a mathematical sentence. Teachers in the teaching of algebra should propose situations that lead students to identify the reference to the same object after changes or conversions among different semiotic systems. For example: "a number added to three results in eight" is to reference the same object of the sentence "$x + 3 = 8$". This reference equivalence must be addressed in education. It concerns the hidden face of mathematics and is essential to the learning of algebra.

Another issue pointed out by Duval concerns the equation notion with its own possibility of processing by changing side to solve this equation. This action has to be perceived by students as a way of expressing the same amount in two ways and this is what is named equation. Again, it can be observed the referential equivalence and the use of the addition and multiplication principles to support this transformation. In the following we shall consider the teaching proposals which will consider the notions of differences among referential equivalence, equality and identity.

References

Bogdan, R , & Biklen, S. (1994). *Investigação qualitativa em educação: uma introdução à teoria e aos métodos*. Porto: Porto Editora.

Brasil (1996). *Lei 9394/96. Lei de Diretrizes e Bases da Educação Nacional.*

Charlot, B. (2000). *Da relação com o saber: elementos para uma teoria.* São Paulo: Artmed.

Duval, R. (1995). *Semiosis et pensée humaine: sémiotiques registres et apprentissages intellectuels*. Berna: Peter Lang.

Duval, R. (2011). Deux regards sur les opposés points critiques sur l'enseignement de l'algèbre au collège (11-15 ans). Lecture given at the Graduate Program in Mathematics Education of Mato Grosso Federal University.

Fiorentini, D., Miorim, M. A., & Miguel, A. (1993). Contribuição para um repensar... A educação algébrica elementar. *Pro-Posições, 4,* 78-91.

Kuenzer, A. (2002). *Ensino Médio: construindo uma proposta para os que vivem do trabalho*. São Paulo: Cortez.

Marcozzi, A. M., & Dornelles, L. W., & Rêgo, M. V. B. S. (1976). *Ensinando a criança: um guia para o professor* (3rd ed.). Rio de Janeiro: Ao Livro Técnico.

Micotti, M. C. O. (1999). O ensino e as propostas pedagógicas. In: M. A. V. Bicudo (Ed.), *Pesquisa em Educação Matemática: concepções & Perspectivas* (pp. 153-167). São Paulo: Editora UNESP.