Developing Critical Thinking Skills from Dispositions to Abilities: Mathematics Education from Early Childhood to High School

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Abstract

In light of the importance of developing critical thinking, and given the scarcity of research on critical thinking in mathematics education in the broader context of higher order thinking skills, we have carried out a research that examined how teaching strategies oriented towards developing higher-order thinking skills influenced students’ critical thinking abilities. The guiding rationale of the work was that such teaching can foster the students’ skills of and dispositions towards critical thinking. In this article, we discuss ways in which critical thinking can be incorporated in mathematics instruction. In particular, we highlight how content taught in the probability strand can intentionally be focused on the development of students’ critical thinking. We report results of a study demonstrating improvement in secondary mathematics students’ dispositions towards critical thinking and abilities to think critically.

Keywords

Critical Thinking, Probability, Mathematics, High School, Early Childhood

1. Introduction

Critical thinking (CT) is a capability essential to contemporary life. Furthermore, the benefits of critical thinking are lifelong, supporting students in the regulation of their study skills, and subsequently empowering individuals to contribute creatively to their chosen profession. In this paper, we argue that critical thinking is constituted
through both dispositions and abilities. While the abilities may be developed through direct instruction, the dispositions are better thought of as “habits of mind” and their development requires long-term participation in learning environments conducive to reflection and argumentation. Our research into the promotion of critical thinking has been undertaken in high school mathematics classrooms through the teaching of probability, but we will argue that all mathematics topics can be taught with the underlying goal of helping students learn to think critically and become mathematically proficient citizens of the world. We provide examples of activities that can help foster students’ critical thinking, from early childhood through high school. Critical thinking can be thought of as both a capability and a “habit of mind” by which individuals interact with the world of experience by questioning it, rather than just accepting it. This is more than a taught skill to be employed selectively; it is an orientation to the world. To encounter the world critically, ideally an individual must be initiated into the practices of critical thinking from an early age. Routines such as “I see, I think, and I wonder” can be used with very young children to develop the three skills of careful observation, thoughtful interpretation, and imaginative speculation. The development of such questioning habits of mind in young people may be culturally confronting to some communities. In this chapter, we explore approaches to the lifelong development of critical thinking as an essential process by which we can regulate our experience of a world in which few things are constant, where reliance on routines will not maintain our quality of life, and emergent problems will be overcome with imagination rather than just persistence. Central to our argument is the distinction between abilities and dispositions. It is our contention that critical thinking must become a pervasive element of the educational experiences of all students from pre-school to upper high school and university settings and that a structured program in critical thinking should start with the promotion of appropriate dispositions and progressively move to the development of critical thinking abilities.

2. Critical Thinking

2.1. Critical Thinking: An Overview

Critical thinking was originally developed during the ancient times when the sophists (740-399 BC) and Socrates (5th century BC) pondered theories of ethics and societal governance. In order for Socrates to understand an opinion on a certain issue, he first clarified the definition of that issue and then evaluated whether that definition was true. He was a person who thought independently, and taught others to think for themselves. He also believed that thinking independently was a learned trait; that is, people could be educated in thinking critically, rather than being innately born with critical thinking capabilities (Bryan, 1987; Regev, 1997).

Critical thinking is reflective and reasonable thinking that focuses on deciding what to do or believe (Ennis, 1985). Critical thinking abilities include such things as: applying available information to new situations, analyzing causes or motives for situations, and evaluating opinions on subjects.

The consensus emerging amongst science and mathematics educators worldwide is that the acquisition of higher-order cognitive skills (HOCS) by our students should be a major instructional goal in education (Zohar & Dori, 2003). HOCS include, among other skills, the ability to ask questions, solve problems, make decisions, and engage in critical thinking (Ennis, 1989; Innabi & Sheikh, 2007; Perkins, 1992). Ideally, students will gradually build upon their learning of critical thinking as they progress through their schooling; as opposed to being asked to think critically one year but not the next. National curriculum initiatives around the world (Australia, China, Israel, Japan and Singapore, for example) all advocate the structured promotion of thinking skills that can be variously labeled “higher order”, “heuristic”, “metacognitive”, “imaginative”, “creative” or “critical”. We address some of the strategies by which such structured promotion of critical thinking might be effected at different stages of schooling, from pre-school onwards.

McPeck defined critical thinking as “skills and dispositions to appropriately use reflective skepticism” (McPeck, 1981). Lipman claimed that critical thinking is “thinking which enables judgment, is based on criteria, corrects itself, and is context sensitive” (Lipman, 1991). A third definition, by Ennis (1987a), is the one on which we have based our research. Ennis (1985) defined critical thinking as “reasonable reflective thinking focused on deciding what to believe or do.”

In light of his definition, Ennis developed a critical thinking (CT) taxonomy that relates to skills that include an intellectual aspect as well as a behavioral aspect. In addition to skills, Ennis’ (1987a) taxonomy includes dispositions and abilities. Ennis claims that CT is a reflective and practical activity the purpose of which is to mod-
erate action or belief. In the study that is the principal focus of this chapter, we address the challenge of promoting both students’ abilities and their dispositions.

2.2. Critical Thinking Activity Examples in Mathematics

In order to give readers a more concrete view of the kinds of grade activities suggested by researchers internationally, we focus on the mathematical strand of probability, which can be taught starting from preschool children through high school and beyond. We provide examples from only this one significant mathematical topic, in order to provide a coherent picture of the progression of critical thinking with respect to mathematical content introduced.

In early childhood, students are often asked to sort objects by shapes and characteristics. For instance, they could have a bucket of red cars, red trucks, blue cars, and blue trucks; and be asked to sort out all of the cars from all of the trucks, or all of the red vehicles from the blue vehicles. This kind of activity can help students think about the definition of each class of objects and the application of this definition amongst the current sample space available to them. Lead by instructors who have critical thinking in mind, students can build fundamentals of inductive reasoning by comparing newly introduced objects with characteristics of the existing objects. Should a yellow car belong in the group of cars? Students must consider whether the definition of car is dependent upon color. Students can also be asked to develop categories of their own and to articulate the defining characteristics of each category.

Another kind of activity involves presenting students with dilemmas, or situations in which two solutions can be equally valid. For instance, elementary school students can explore the dilemma, “is it okay for someone to take another person’s pencil without asking?” and discuss whether there are conditions where it is acceptable (e.g., a doctor needing to write out a prescription for another classmate) and not acceptable (e.g., greed). By evaluating moral situations relevant to students in their daily lives, students are led to recognize and challenge their existing beliefs and employ such reasoning strategies as considering exceptions to a rule.

When students are sufficiently cognitively advanced as to be able to consider at least three choices within a context, they can be asked questions requiring them to evaluate whether a situation is “always, sometimes, or never” true. For instance, “if a person has a dream to become a sports star, does this person always, sometimes, or never accomplish this dream?” or “when you add an odd number and an even number, is the result always, sometimes, or never an odd number?” The obligation to justify an assertion with evidence should be emphasized as soon as students can make claims. This would lead to the development of principles of reasoning, such as, when a statement has a counterexample, it can never be always true; however, if a statement is supported by only one example, the statement may not necessarily be always true. This is an important logical distinction to make and can be introduced early in a child’s school experience.

Some simple situations and related propositions can be evaluated and even proposed by students from the early years of schooling through high school. Here are three simple examples:

1) Did you notice that Dani is very clever at maths and he has really big feet? Do you suppose that all students with big feet are good at maths? (Grade 4)

2) The population of a country town fell by 50% over five years. Suggest why this might have happened and some methods that you could use to test your hypothesis. (Grade 6)

3) A research study showed a negative correlation between disciplinary statements and maths achievement. That is, classes where the teacher made many disciplinary statements were low achieving and classes where the teacher made few disciplinary statements were high achieving. It is clear that students would be much more successful if teachers would just stop disciplining them. Please comment. (Grade 9)

In upper elementary school, students could learn about fractions simultaneously with their meanings from a probabilistic point of view. Fractions are one representation of the notion of chance. When there is a 7/8 chance that you will pull out a blue marble out of a bag of blue and red marbles, it is still possible 1/8 of the time to pull out a red marble. Students should become aware of the notion of taking calculated risks and the implications of making decisions based upon chances; often seen in the contexts of sports, financial decisions, etc. There are many ways in which decision-making based on likelihood can be simulated in the classroom. Games of chance can be adapted to simulate such situations. Once statistical situations require decision-making, we have conditions conducive to the development of critical thinking.

Students generally learn about measures of central tendency during their years in upper elementary and mid-
There are many opportunities for students to think critically about mean, median, and mode. Students can begin to calculate, “if I want to earn an A in this class, what grade do I need to earn on the final exam?” or “if the majority of my classmates did not enjoy visiting a national monument, what are the chances that I will enjoy visiting it?” Reasoning on the basis of measures of central tendency simultaneously invokes statistical and critical thinking. Situations that highlight the interplay between mean, median, and mode (e.g., a mode that is low but mean is high, a median that is low but mode is high, etc.) can also help students to re-evaluate their first reactions to a situation and understand the underlying mathematics before making an informed decision. In such situations, critical reflection on the relevance and relationships of statistical concepts can enhance both the students’ understanding of statistics and their capacity for critical thought. That is, such classroom activities bring the multiple benefits of simultaneously developing cognitive or argumentative skills at the same time as statistical knowledge, and the pragmatic utility of developing effective strategies for evidence-based decision-making.

In middle school, students are ready to learn how to use argumentation, that is, using warrants and backings to support any claims that they may make (Forman et al., 1998). Skills of argumentation can be applied to any discussion of a mathematical or non-mathematical nature, and can be especially used in mathematics classes when students solve cognitively challenging tasks (Aizikovitsh-Udi, 2012). An example of a cognitively demanding task is to evaluate the claim, “in larger and better equipped hospitals, the rates of mortality are higher than in smaller and less well equipped hospitals”. Students would need to compare and contrast the two different kinds of hospitals, and coordinate their comparison with the mortality rates in those hospitals. They would need to identify relevant variables and consider alternative scenarios, leading to the development and investigation of rival hypotheses. Such tasks can be expressed simply and create the conditions to optimize student reflection and critical thinking in instructional settings.

3. Method: A Study Supporting the Early Introduction of CT

There are two approaches to teaching critical thinking using content disciplines: a) The “infusion” approach, where critical thinking skills are taught explicitly using the discipline’s content; b) The “embedded” or general approach, where critical thinking skills are taught in indirect ways without spelling it out to students (Swartz & Parks, 1994). The study by Aizikovitsh-Udi (2012) used the “infusion” approach that had also been used elsewhere (Ennis, 1987b; Swartz, 1992; Swartz & Parks, 1994). In the infusion approach, the skills are taught in the framework of a particular subject or topic, and thinking becomes an integral part of teaching specific subject content, while general principles and terminology of thinking are explicitly emphasized.

In the course of the study, students were challenged to consciously examine data supporting claims and determine the validity of any data to be interpreted. The course consisted of fifteen sessions of ninety minutes each. During this course, students were given recent news from the newspaper or from survey results and were asked to judge how true these claims could be. In order to properly justify students’ claims, they needed to learn conditional probability, Bayes Theorem, and additional probabilistic and statistical tools.

3.1. Research Questions

In this study, Aizikovitsh-Udi (2012) examined the following two research questions:

1) To what extent do students learn critical thinking skills (abilities and disposition) via a CT-infused presentation of probability?

2) What critical thinking processes are invoked as students learn probability via an infusion approach?

3.2. Tools

Data collection was purposefully triangulated: 1) Five students were interviewed at the end of a class period as well as one week after. These personal interviews were conducted in order to reveal a change in the students’ attitudes throughout the academic year; 2) Students’ exams, in-class handouts, and homework were collected; 3) All class sessions were documented and analyzed—the sessions were recorded and transcribed; 4) The participant group was tested twice, both Pre and Post the instructional program, using the two questionnaires: Dispositions of Critical Thinking (CCDTI) and Abilities of Critical Thinking (Cornell test). Each of the questionnaires (both the CCTDI and Cornell test) generates sub-scores and a total-score.

CCTDI was used in order to evaluate the students’ dispositions toward critical thinking. This tool was de-
signed to measure the general dispositions profiles of the students. CCTDI is divided into seven sub-tests: truth-seeking (sub-test T), intellectual openness (sub-test O), analyticity (sub-test A), systematicity (sub-test S), self-confidence in critical thinking (sub-test C), inquisitiveness (sub-test I) and maturity (sub-test M), although thinking dispositions are far less numerous than thinking skills (Facione & Facione, 1992; Facione, Facione, & Giancarlo, 1996).

The Cornell Test was used to check the development of the students’ critical thinking abilities according to Ennis’ taxonomy. This test was developed by Ennis and his colleagues (e.g., Ennis & Millman, 2005b). The Cornell Critical Thinking Test, Level Z and X were chosen by the researchers as it was more suited to the advanced high-fool students in the group. The test includes general content with which most of the students would be familiar and it assesses various aspects of critical thinking. It is a multiple-choice test with three choices and one correct answer. Although the test is meant to be taken within a fifty-minute period, it was thought that the students in the group would be unable to complete it within that time limit. For this reason it was decided to give them eighty minutes in which to take the test. The test includes five sub-tests and evaluates different aspects of critical thinking, including induction, deduction, value judging, observation, credibility, assumptions, and meaning (Ennis & Millman, 2005a).

In order to answer the research questions and to evaluate the change in the students’ knowledge, pre and post questionnaires were used according to the Counter Balance method. To achieve these goals, we have passed the following questionnaires: 1) the CCTDI (Facione & Facione, 1992): a 75-item, Likert test of critical thinking; 2) the Cornell Critical Thinking Test version “Z” (Ennis & Millman, 2005b), a 52-item, multiple choice test of critical thinking abilities.

The teacher kept a reflection journal after every session, as well. Data was processed using qualitative (interpretive) methods, which allowed the researcher to follow the students’ patterns of thinking and interpretation with regards to the learned materiel in different contexts.

3.3. Population

Fifty-five children between the ages of fifteen and sixteen participated in an extra curriculum program aimed at students from different cultural backgrounds and socio-economic levels. The students all attended the tenth-grade and recently finished learning from the textbook, “Probability in daily life” (Lieberman & Tversky, 1996; 2002).

4. Result Related to the Disposition of Critical Thinking

In order to develop and foster critical thinking, we need first to define it and to understand the mental processes it involves. Previous research has investigated the ways in which students acquire technical tools such as evaluation, verifying results, assessing problems, making comparisons and conclusions, choosing solution strategies etc. This study, by contrast, focused on the more general and universal aspects of critical thinking, investigating the ways in which students develop abilities, such as induction, deduction, value judging, observation, checking the sources’ reliability, identifying assumptions, and extracting meaning, and dispositions, such as truth-seeking, open-mindedness and inquisitiveness, according to the taxonomy of Ennis (1989). The overarching purpose of this study was to determine whether critical thinking abilities and dispositions could be developed through the study of probability.

Two pre-facing remarks summarize the key findings:
- Learning did occur but gains in specific CT areas could have been higher;
- Students in interview asserted that CT was important and valued, and suggested that there were multiple uses of CT in their lives; however they thought CT should appear earlier in the educational process.

The details behind these general findings fall into two specific categories: those results concerned with student disposition towards critical thinking and those concerned with student critical thinking abilities. Results related to both are reported below.

4.1. Result Related to the Disposition of Critical Thinking

The distinction between Abilities and Dispositions is a vital one, because the individual who possesses the appropriate thinking abilities may still lack the drive, the will or inclination to act upon them. According to Stern-
berg, there are “close to a thousand” thinking skills, and according to Lipmann, “the list is infinite” (Sternberg, 1984; Lipman, 1991). This distinction is by no means clear-cut—thinking dispositions include and encourage disposition to thinking—but it has theoretical and practical justifications. A thinking disposition, as we define it, is a rational impulse toward a particular thinking pattern or thinking quality, which encourages becoming actively involved in the process of thinking, investing oneself in thinking. Unfortunately, school does not traditionally provide room for the type of thinking that involves intellectual awakening; it can even appear to oppose it. Baber describes the following dialogue between a teacher and his student: “Teacher: What are you doing? Student: I’m thinking. Teacher: Then stop thinking and start working!” (Beyer, 1987). We suggest, as a contemporary imperative, that schools initiate a structured program to develop students’ critical thinking and purposefully promote students’ disposition to thinking. We further suggest that the promotion of critical thinking must start in the early years of schooling if we are to maximize the educational benefits of such a program.

A consistent effort to encourage higher-order thinking skills not only promotes student critical thinking during the current instructional period, but also has a long-term effect, by becoming an integral part of these students’ thinking habits. Statistical comparison of averages in CCTDI tests administered in this study focused on the relative rate of improvement. A series of t-tests showed that the students of the Experimental group considerably improved their critical thinking dispositions relative to the Control group. Thinking dispositions in a specific area do not necessarily extend to thinking patterns in general. For example, one may tend to deep and nuanced thinking in one’s area of specialization but exhibit superficial thinking on political matters. That is, thinking dispositions are context-dependent. Aizikovitsh-Udi & Amit (2008) demonstrated that teaching the “Probability in Daily Life” unit in the infusion approach greatly developed the students’ critical thinking. It is clear from this research that one of the fundamental elements of good critical thinking is the development of the dispositions already discussed. Specifically, with respect to the major focus of this paper, the idea of dispositions as an inclination to perceive and interact with experience in a certain way suggests that the initiation of individuals (children) into this way of thinking (that is, the development of these dispositions) should begin at the earliest age possible.

4.2. Results Related to Abilities of Critical Thinking

Both training and knowledge are necessary to promote critical thinking abilities in students. Ennis (1962; 1963; 2002) believes that critical thinking involves inquiry, asking questions, offering alternative answers, and questioning traditional and accepted beliefs. He suggests that because society does not welcome people who challenge authority, critical thinking is not encouraged. “Old” education, as Dewey referred to it, focused on passing on knowledge, where knowledge was conceived to be a body of facts and propositions that need only be memorized and understood.

As in the case of critical thinking dispositions, consistent effort to encourage high order thinking skills contributed to the development of the students’ ability to think critically (not only during the experimental learning period but also in the longer term, as these skills became an integral part of the students’ thinking habits). To conclude, studying the learning unit “Probability in Daily Life” in the infusion approach contributed to the development of the students’ thinking skills. It is our opinion that instruction in such high order thinking skills requires that the teacher be personally expert in the various aspects of critical thinking. Further, the relative sophistication of the abilities being developed suggests that programmatic development of critical thinking abilities should be structured to provide a well-established dispositional foundation before attempting to develop the more sophisticated critical thinking abilities.

5. Conclusion

The results of this research demonstrate that if teachers consistently and systematically encourage CT in their classes by applying mathematics to real-life problems, encouraging debate, and planning investigative lessons, students are likely to practice CT skills and develop the language of critical thinking. The research cited in this paper also shows how the infusion approach can work in the specific case of the teaching of probability related to real life problems. Students practiced critical thinking using probability, while the stimulus material presented constituted the basis for practicing critical thinking skills together with the subject of probability.

It was important to note that in addition to the CT skills mentioned above, in the course of the reported study, students also gained intellectual skills such as conceptual thinking and benefited from a classroom culture that
fostered CT. But to practice CT skills under teachers’ guide and develop the language of critical thinking does not mean that the internalization of critical thinking skills has been achieved by students. Such internalization is essential if our ultimate goal is that students should be able to apply critical thinking in any and all settings and situations in which they find themselves. This internalization is deeply connected to the establishment of the relevant dispositions and will only come over time, and hence we strongly advocate the use of critical thinking instruction for students beginning in early childhood education. The general educational implications derived from this research can and should be used to encourage the intellectual development of students beyond the technical content of conventional mathematics courses, by creating learning environments that foster CT. This approach will, in turn, encourage students to interrogate the issue at hand, evaluate relevant information, and respond as critical thinkers.

From this research’s findings and discussion, there arise the following research recommendations. A more comprehensive examination of the processes of critical thinking: to what extent could the students describe, orally and in writing, the processes of thinking, activate them and apply the thinking skills they studied at the procedural and metacognitive level? Did they make an informed use of terms and strategies of higher order thinking, including critical thinking? To examine the former question, researchers should examine to what extent participants use the “language of thinking” (Ben-Chaim et al., 2000; Costa, 1991; Costa & Marzano, 1991; Harpaz, 1996; Zohar, 1996). Developing such a language involves, on the part of teachers, such skills as using precise vocabulary, presenting critical questions, presenting data rather than answers, aspiring for exactness, giving directions, and developing metacognition. As has already been discussed, students can be assisted to talk about their thinking, even at a very young age, through routines such as “I see, I think, and I wonder”. Through such relatively simple approaches, we feel that an appropriate classroom climate can be developed for the promotion of students’ critical thinking dispositions. Once these dispositions are established in the early years of schooling, the instructional program can progressively increase the sophistication of the critical thinking skills being developed. The research cited here suggests that instructional approaches such as the infusion approach are suitable for the development of both critical thinking dispositions and abilities throughout all years of schooling. It is hoped that the findings of this study will contribute to our understanding of the nature of critical thinking and to the further development of instructional approaches relevant to its promotion.

References


