Erratum to “Weierstrass’ Elliptic Function Solution to the Autonomous Limit of the String Equation of Type (2,5)” [Advances in Pure Mathematics 4 (2014), 494-497]

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Received 1 June 2014; revised 3 July 2014; accepted 15 July 2014

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The original online version of this article (Sasaki, Y. (2014) Weierstrass’ Elliptic Function Solution to the Autonomous Limit of the String Equation of Type (2,5). Advances in Pure Mathematics, 4, 494-497.
http://dx.doi.org/10.4236/apm.2014.48055) was published in August, 2014. Unfortunately, it contains several mistakes. The author wishes to correct the following errors in [1]:

P. 494, L. 7-: The string equation of type \((q, p)\) should be correctly read as

\[
\begin{align*}
Q, P &= 1, \quad Q := D^q + \sum_{k} w_k D^p, \quad P := D^p + \sum_{k} V_k D^q \\
\end{align*}
\]

P. 496, L. 13 - 14: Theorem B should be correctly read as follows:

**Theorem B.** The autonomous limit Equation (A) has a solution concretely described by the Weierstrass’ elliptic function as

\[
w(z) = \kappa \wp(z),
\]

where \(\kappa = 1\) or 3.

P. 496, L. 17: In Remark, \(g_2\) and \(g_3\) in the elliptic function theory should be correctly read as follows:

\[
\begin{align*}
g_2 &= -4a, \quad g_3 = \frac{4}{3}b, & \text{for } \kappa = 1, \\
g_2 &= -\frac{4}{21}a, \quad g_3 = -\frac{4}{81}b, & \text{for } \kappa = 3.
\end{align*}
\]

P. 496, L. 21: In the r.h.s. of Equation (1), “\(w(4)\)” should be correctly read as “\(-w(4)\).”

P. 496, L. 3 - P. 497, L. 2: These 5 lines should be correctly read as follows:
If both of (2) and (3) are valid, then $a_0$ must vanish and $a_1$ coincides with $4$ or $\frac{4}{3}$.

**Case** $a_0 = 0$ and $a_1 = 4$: In this case, we immediately obtain $a_0^2 = 2(B - 4a)$, $a_3 = B$, $a_4 = -\frac{1}{4}B\sqrt{2(B - 4a)} - \frac{4}{3}b$, where $B$ is a root of $\frac{5}{8}B^2 - 2ab + \frac{4}{3}b\sqrt{2(B - 2a)} = c$. Inversely, if these are satisfied, both of (2) and (3) are valid. $w^2 = 4w^3 + a_2w^2 + a_1w + a_4$ can be reduced to $v^2 = 4v^3 - g_2v - g_3$ by $w = v - a_2/3$. But, for brevity, now we put $a_2 = 0$, and then $a_3 = B = 4a$, $a_4 = -\frac{4}{3}b$, i.e.

$$w^2 = 4w^3 + 4aw - \frac{4}{3}b.$$ Here $g_2 = -4a$ and $g_3 = \frac{4}{3}b$. The irrational equation satisfied by $B$ determines the integral constant $c$ in the r.h.s. of (2) as $c = 2a^2$.

**Case** $a_0 = 0$ and $a_1 = \frac{4}{3}$: In this case, we easily obtain $a_2 = 0$, $a_3 = \frac{4}{7}a$, $a_4 = \frac{4}{9}b$. Only $c = \frac{2}{49}a^2$ is allowed as the integral constant $c$ in the r.h.s. of (2). Inversely, if these are satisfied, both of (2) and (3) are valid.

$$w^2 = \frac{4}{3}w^3 + \frac{4}{7}aw + \frac{4}{9}b$$ is reduced to $v^2 = 4v^3 - g_2v - g_3$ by $w = 3v$, and then $g_2 = -\frac{4}{21}a$ and $g_3 = -\frac{4}{81}b$. □

**References**