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The Pattern of Prime Numbers

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Abstract

The prime numbers $p \geq 5$ obey a pattern that can be described by two forms or geometric progressions $6m+[5,7] \rightarrow m \in \mathbb{Z}^{+}$ or $6k \pm 1 \rightarrow k \in \mathbb{Z}^{+}$ that facilitates obtaining them sequentially, being possible also to calculate the quantity of primes that are in the geometric progressions $6k \pm 1 \rightarrow k \in \mathbb{Z}^{+}$ as it is described in this document.

Keywords

Prime Numbers and Composite Numbers

1. Introduction

Since ancient times when humans discovered the counting system and natural numbers, prime numbers immediately attracted their attention, were numbers whose only divisors are 1 and the same number. The problem to find them was not to be able to describe by means of an equation. There are countless publications about the properties of prime numbers that can be found in all languages and theorems have been created in different ways, seeking always to find a pattern of ordering [1] [2]. The inability to find an order has been eloquently documented, such as in Havil's book:

> "*The succession of primes is unpredictable. We don't know if they will obey any rule or order that we have not been able to discover still. For centuries, the most illustrious minds tried to put an end to this situation, but without success. Leonhard Euler commented on one occasion: mathematicians have tried in vain to this day to discover some order in the sequence of prime numbers and we have reason to believe that it is a mystery into which the human mind will never penetrate. In a lecture given by D. Zagier in* 1975, *he said:* "*There are two facts about the distribution of prime numbers of*

which I hope to convince you so overwhelmingly that they will be permanently engraved in your hearts. The first is that, [they are] the most arbitrary and ornery objects studied by mathematicians: they grow like weeds among the natural numbers, seeming to obey no other law than that of chance, and nobody can predict where the next one will sprout. The second fact is even more astonishing, for it states just the opposite: that the prime numbers exhibit stunning regularity, that there are laws governing their behavior, and that they obey these laws with almost military precision." (*Havil*, 2003 [3])

To put prime numbers into context, let's begin by saying anecdotally, as late as 20,000 years ago humans marked the bone of Ishango with 19, 17, 13, 11 [4] and 2300 years ago Euclid proved that there are infinitely many prime numbers (e.g. Williamson, 1782) [5]. Later, Euler made another formal proof of it (e.g. Hardy and Woodgold, 2009) [6].

Until now, there is no known efficient formula for primes, nor a recognizable pattern or sequence the primes follow. All recent publications dealing with this issue established that primes are distributed at random and looked more to a white noise distribution [7]. Here will be shown that prime numbers are not random, they obey mathematic rules and can be expressed by equations.

2. Form of Prime Numbers

Porras Ferreira and Andrade (2014) [8] had found that all prime numbers $p_n \geq 7$ have the following form:

$$p_n = [31,7,11,13,17,19,23,29] + 30n, \text{ where } n \in \mathbb{Z}^+ \quad (1)$$

Or all prime numbers $p_m \geq 5$ have the following form:

$$p_m = [5,7] + 6m, \text{ where } m \in \mathbb{Z}_0^+ \quad (2)$$

It can be seen that Equation (2) is a derivation of Equation (1), as demonstrated in [8], although Equation (2) includes, the prime 5 which is not included in Equation (1).

Equation (2) can be transformed to a simpler form where $k = 1 + m$:

$$p_k = 6k \pm 1, \text{ where } k \in \mathbb{Z}^+ \quad (3)$$

It means, the form of all prime numbers only have these three equations: Equation (1) which does not include primes 2, 3 and 5, and Equation (2) and Equation (3) which does not include primes 2 and 3, therefore Equation (2) and Equation (3) are equivalent:

$$\left[p_m = [5,7] + 6m \to m \in \mathbb{Z}_0^+\right] \equiv \left[p_k = 6k \pm 1 \to k \in \mathbb{Z}^+\right] \quad (4)$$

Taking the Equation (3) a table is constructed (**Figure 1**), where all the primes $p_k \geq 5$ are formed only in two columns, the column $6k - 1$ and the column $6k + 1$ fore $k \geq 1$. The cells highlighted in yellow correspond to composite numbers and those that are not highlighted to prime numbers.

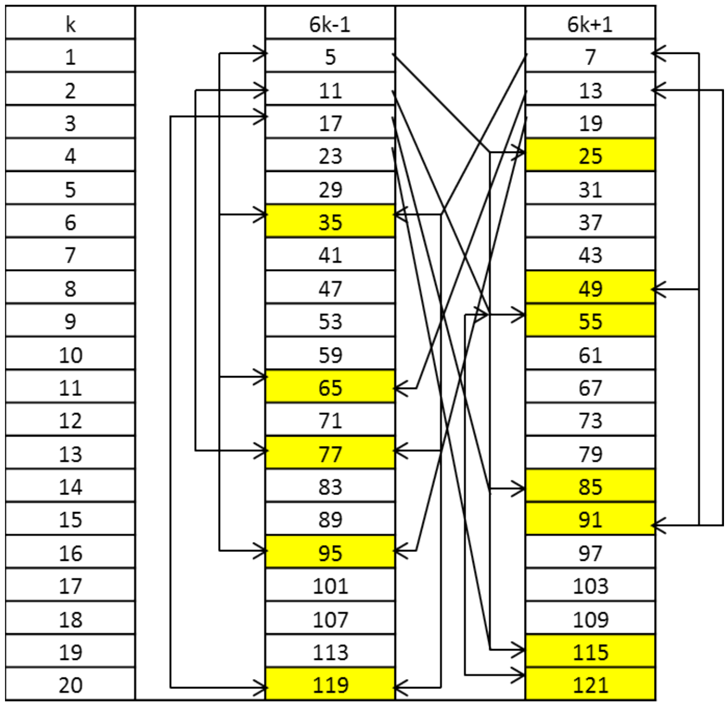

Figure 1. Formation of prime and composite numbers.

3. Analysis of How Composite and Prime Numbers Are Formed in $6k \pm 1$ Columns

In the following analysis, **Figure 1** is used as reference.

3.1. The Composite Numbers Are Formed as Follows

Theorem 1:

If $p = 6k \pm 1$ in $k = k_1$, them all the composite numbers $N_k = 6k \pm 1$ with $k = k_1 + pm$ where $m \geq 1$ will contain p.

Demonstration:

Let $p = 6k \pm 1$ in $k = k_1$, therefore $p = 6k_1 \pm 1$ is a number that can be prime or not and $N_k = 6k \pm 1$ in $k = k_1 + pm$ being $m \geq 1$, therefore:

$$N_k = 6k \pm 1 = 6(k_1 + pm) \pm 1 = 6k_1 \pm 1 + 6pm = p + 6pm$$

Factoring p then:

$$N_k = p(6m + 1)$$

which must be a composite number with two factors p and $6m + 1$.

The theorem is proved.

Corollary 1:

The composite numbers $N_k = 6k \pm 1$ contain two equal or different factors that come from the columns $6k - 1$ and/or $6k + 1$ therefore if $N_k = 6k \pm 1$ $= p(6m + 1)$ is decomposed into their two factors p and $(6m + 1)$; there will be no prime numbers according to Theorem 1, in the rows k of Equation (5) where k_1 is the row where p and $6m + 1$ appears for the first time where $[m, n] \geq 1$.

$$k = \begin{bmatrix} k_1 + pn \\ k_1 + (6m+1)n \end{bmatrix} \quad (5)$$

Corollary 2:

There are not identical composite numbers, one from column $6k-1$ and another from column $6k+1$.

Corollary 3:

All the composite numbers N_k from column $6k-1$ always have a factor $6m-1$ coming from that column and a factor $6n+1$ coming from the other column, which means $N_k = 6k+1 = (6m-1)(6n+1)$ where $[m,n] < k$.

Corollary 4:

All the composite numbers N_k of column $6k+1$ may have two factor from the column $6k-1$, that means $N_k = 6k+1 = (6m-1)(6n-1)$, where $m \le n$ and $[m,n] < k$ or two factors from the same column, that means $N_k = (6m+1)(6n+1)$, where $m < n$ and $[m,n] < k$.

Corollary 5:

Eliminating all k, product of Equation (5), the rest k will only contain primes of the given form of Equation (3) and as it is shown in **Figure 1**. Note that the initial pattern is given by primes 5 and 7, leaving the cells in rows [1] [2] [3] [4] [5] from column $6k-1$ with primes and rows [1] [2] [3] from column $6k+1$ with primes. Subsequently these primes give rise to the composite numbers N_k according to Equation (5), leaving other different rows where there will be prime numbers, this pattern continues until infinity.

3.1.1. Examples of Composite and Prime Numbers from $6k-1$ Column

1) The prime $p=5$ appears for the first time in the row $k_1=1$, applying Equation (5) there are no primes but composite numbers that have the 5 as one of its factors in the rows $k=1+5n$ for $n \ge 1$. These compound numbers have the form $N_k = 6k-1 = 6(1+5n)-1 = 5(6n+1)$. The second factor of the composite number N_k having as the first factor, the prime 5 always has the form $6n+1$ (Corollary 3).

2) The prime $p=7$ from column $6k+1$ appears for the first time in the row $k_1=6$, applying Equation (5) there are no primes but composite numbers that have the 7 as one of its factors in the rows $k=6+7n$ for $n \ge 1$. These compound numbers have the form $N_k = 6k-1 = 6(6+7n)-1 = 7(6n-1)$. The second factor of the composite number N_k having as the first factor, the prime 7 always has the form $6n-1$ (Corollary 3).

3) The prime $p=11$ appears for the first time in the row $k_1=2$, applying Equation (5) there are no primes but composite numbers that have the 11 as one of its factors in the rows $k=2+11n$ for $n \ge 1$. These compound numbers have the form $N_k = 6k-1 = 6(2+11n)-1 = 11(6n+1)$. The second factor of the composite number N_k having as the first factor, the prime 5 always has the form $6n+1$ (Corollary 3).

4) The prime $p=13$ from column $6k+1$ appears for the first time in the row $k_1=11$, applying Equation (5) there are no primes but composite numbers

that have the 13 as one of its factors in the rows $k = 11 + 13n$ for $n \geq 1$. These compound numbers have the form $N_k = 6k - 1 = 6(11 + 13n) - 1 = 13(6n - 1)$. The second factor of the composite number N_k having as the first factor the prime 13, always has the form $6n - 1$ (Corollary 3).

5) From the analysis of the previous 4 points and applying Theorem 1 we can conclude the following with respect to Equation (5) being the geometric progression $p = 6m - 1$:

With $k_1 = m$, $p = 6m - 1$, we have:

$$k_{m,n} = m + (6m - 1)n \to [m, n] \geq 1 \tag{6}$$

They contain composite numbers of the form $6k_{m.n} - 1$

And

$$k \neq k_{m,n} = m + (6m - 1)n \to [m, n] \geq 1 \tag{7}$$

They contain prime numbers of the form $6k - 1$.

In Equation (6) there exist symmetry of cells where $k_{1+5m,n} = k_{1,1+7n}$, $k_{11,n} = k_{1,1+14n}$, $k_{m,4} = k_{1,4+5(n-1)}$ and so on, this is important to take into account in order to calculate the number of primes in this column as will be seen later. **Table 1** gives an example of the above.

Table 1. Examples where there are symmetry of cells (in yellow) $k_{1+5m,n} = k_{1,1+7n}$ (in blue) $k_{11,n} = k_{1,1+14n}$ and (in green) $k_{m,4} = k_{1,4+5(n-1)}$, in Equation (6).

	m														
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
	$k_{m,n} = m + (6m - 1)n$														
1	6	13	20	27	34	41	48	55	62	69	76	83	90	97	104
2	11	24	37	50	63	76	89	102	115	128	141	154	167	180	193
3	16	35	54	73	92	111	130	149	168	187	206	225	244	263	282
4	21	46	71	96	121	146	171	196	221	246	271	296	321	346	371
5	26	57	88	119	150	181	212	243	274	305	336	367	398	429	460
6	31	68	105	142	179	216	253	290	327	364	401	438	475	512	549
7	36	79	122	165	208	251	294	337	380	423	466	509	552	595	638
8	41	90	139	188	237	286	335	384	433	482	531	580	629	678	727
9	46	101	156	211	266	321	376	431	486	541	596	651	706	761	816
10	51	112	173	234	295	356	417	478	539	600	661	722	783	844	905
11	56	123	190	257	324	391	458	525	592	659	726	793	860	927	994
12	61	134	207	280	353	426	499	572	645	718	791	864	937	1010	1083
13	66	145	224	303	382	461	540	619	698	777	856	935	1014	1093	1172
14	71	156	241	326	411	496	581	666	751	836	921	1006	1091	1176	1261
15	76	167	258	349	440	531	622	713	804	895	986	1077	1168	1259	1350

Table 2 shows examples where there are composite and prime numbers of the form $6k-1$ in the rows $1 \le k \le 51$ applying the Equation (6) and Equation (7).

In Table 2, two cells $k_{m,n}$ which are the same, but with different $[m,n]$, are highlighted in yellow and green. This occurs when the compound number in $k_{m,n}$ contains primes with powers greater than 1, Example

$$k_{1,8} = k_{6,1} = 41, \quad N_{k=41} = 6 \times 41 - 1 = 245 = 7^2 \times 5$$

and

$$k_{1,9} = k_{2,4} = 46, \quad N_{k=46} = 6 \times 46 - 1 = 275 = 5^2 \times 11.$$

The number of times the above occurs in $N_k = 6k-1$, where $p_m = 6m-1$; $p_n = 6n+1$; $p_{m1} = 6m_1 - 1$; $p_{m2} = 6m_2 - 1$; $p_{n1} = 6n_1 + 1$; $p_{n2} = 6n_2 + 1$ with $[m,n,m_1,m_2,n_1,n_2] < k \to$ ($m_1 \ne m_2$ n_1 and n_2 can be the same or different), can be calculated by

$$\left(\sum \left| \frac{N_k}{p_{n1} p_{n2} p_m} \right| \ge 1 + \sum \left| \frac{N_k}{p_{m1} p_{m2} p_m} \right| \ge 1 \right).$$

This is important for calculating the number of primes smaller or equal to N_k, as will be seen later.

3.1.2. Examples of Composite and Prime Numbers from $6k+1$ Column

1) The prime $p = 7$ appears for the first time in the row $k_1 = 1$, applying Equation (5) there are no primes but composite numbers that have the 7 as one of its factors in the rows $k = 1 + 7n$ for $n \ge 1$. These compound numbers have the form $N_k = 6k + 1 = 6(1+7n) - 1 = 7(6n+1)$. The second factor of the composite number N_k having the first factor, the prime 5 always has the form $6n+1$ (Corollary 4).

Table 2. Examples where there are composite and prime numbers of the form $6k-1$ in the rows $1 \le k \le 51$ applying the Equation (6) and Equation (7).

	m							$1 \le k \le 51$	
n	1	2	3	4	5	6	7	$k_{m,n}$ with composite numbers	$k \ne k_{m,n}$ with primes
	$k_{m,n} = m + (6m-1)n$								
1	6	13	20	27	34	41	48	6, 11, 13	1, 2, 3, 4, 5
2	11	24	37	50				16, 20, 21	7, 8, 9, 10, 12
3	16	35						24, 26, 27	14, 15, 17, 18
4	21	46						31, 34, 35	19, 22, 23, 25
5	26							36, 37, 41	28, 29, 30, 32
6	31							46, 48, 50	33, 38, 39, 40
7	36							51	42, 43, 44, 45
8	41								47, 49
9	46								
10	51								

2) The prime $p=5$ from column $6k-1$ appears for the first time in the row $k_1=4$, applying Equation (5) there are no primes but composite numbers that have the 5 as one of its factors in the rows $k=4+5(n-1)$ for $n\geq 1$. These compound numbers have the form $N_k=6k+1=6(4+5(n-1))+1=5(6n-1)$. The second factor of the composite number N_k having as the first factor, the prime 5 always has the form $6n-1$ (Corollary 4).

3) The prime $p=13$ appears for the first time in the row $k_1=2$, applying Equation (5) there are no primes but composite numbers that have the 13 as one of its factors in the rows $k=2+13n$ for $n\geq 1$. These compound numbers have the form $N_k=6k+1=6(2+13n)+1=13(6n+1)$. The second factor of the composite number N_k having as the first factor, the prime 13 always has the form $6n+1$ (Corollary 4).

4) The prime $p=11$ from column $6k-1$ appears for the first time in the row $k_1=9$, applying Equation (5) there are no primes but composite numbers that have the 11 as one of its factors in the rows $k=9+11(n-1)$ for $n\geq 1$. These compound numbers have the form $N_k=6k+1=6(9+11(n-1)+1=11(6n-1)$. The second factor of the composite number N_k having as the first factor, the prime 11 always has the form $6n-1$ (Corollary 4).

5) From the analysis of the previous 4 points and applying Theorem 1 we can conclude the following with respect to Equation (5) being the geometric progression $p=6m-1$ or $p=6m+1$:

With $k_1=m+pn$, $p=6m+1$, them $K_{m,n}=m+(6m+1)n$, where $[m,n]\geq 1$, and $N_k=6k_{m,n}+1$ contains composite numbers of the form $(6m+1)(6n+1)$.

With $k_1=4+5(m-1)+p(n-1)$, $p=6m-1$, then

$$k_{m,n}=4+5(m-1)+(6m-1)(n-1),$$

where $[m,n]\geq 1$, and $N_k=6k_{m,n}+1$ contains composite numbers of the form $(6m-1)(6n-1)$.

Therefore:

$$k_{m,n}=\begin{bmatrix} m+(6m+1)n\to[m,n]\geq 1 \\ 4+5(m-1)+(6m-1)(n-1)\to[m,n]\geq 1\end{bmatrix} \quad (8)$$

They contain composite numbers of the form $6k_{m.n}+1$

$$k\neq k_{m,n}=\begin{bmatrix} m+(6m+1)n\to[m,n]\geq 1 \\ 4+5(m-1)+(6m-1)(n-1)\to[m,n]\geq 1\end{bmatrix} \quad (9)$$

They contain prime numbers of the form $6k+1$.

In $k_{m,n}=m+(6m+1)n$, there exists symmetry of equal cells in $k_{1,n}=k_{m,1}$, $k_{2,(n+1)}=k_{(m+1),2}$ and so on, *i.e.* $k_{a,(n+a-1)}=k_{(m+a-1),a}$, where $[a,m,n]\geq 1$.

Also in $k_{m,n}=4+5(m-1)+(6m-1)(n-1)$, there exist symmetries of equal cells in $k_{1,n}=k_{m,1}$, $k_{2,(n+1)}=k_{(m+1),2}$ and so on, *i.e.* $k_{a,(n+a-1)}=k_{(m+a-1),a}$, where $[a,m,n]\geq 1$.

Table 3 shows examples of the above, where the horizontal and vertical cells having the same value have been colored with the same color.

Table 3. Examples of symmetry in Equation (8), where horizontal and vertical cells having the same value have been colored with the same color.

	m									
n	1	2	3	4	5	6	7	8	9	10
	$k_{m,n}=m+(6m+1)n$									
1	8	15	22	29	36	43	50	57	64	71
2	15	28	41	54	67	80	93	106	119	132
3	22	41	60	79	98	117	136	155	174	193
4	29	54	79	104	129	154	179	204	229	254
5	36	67	98	129	160	191	222	253	284	315
6	43	80	117	154	191	228	265	302	339	376
7	50	93	136	179	222	265	308	351	394	437
8	57	106	155	204	253	302	351	400	449	498
9	64	119	174	229	284	339	394	449	504	559
10	71	132	193	254	315	376	437	498	559	620
	m									
n	1	2	3	4	5	6	7	8	9	10
	$k_{m,n}=4+5(m-1)+(6m-1)(n-1)$									
1	4	9	14	19	24	29	34	39	44	49
2	9	20	31	42	53	64	75	86	97	108
3	14	31	48	65	82	99	116	133	150	167
4	19	42	65	88	111	134	157	180	203	226
5	24	53	82	111	140	169	198	227	256	285
6	29	64	99	134	169	204	239	274	309	344
7	34	75	116	157	198	239	280	321	362	403
8	39	86	133	180	227	274	321	368	415	462
9	44	97	150	203	256	309	362	415	468	521
10	49	108	167	226	285	344	403	462	521	580

The number of cells repeated in each column m is $m-1$ and the sum until m of all columns of a matrix $m \times n$, being $m=n$ would be $\frac{m^2-m}{2}$ both in $k_{m,n}=m+(6m+1)n$, as in $k_{m,n}=4+5(m-1)+(6m-1)(n-1)$. This is important to find the number of primes less than or equal to k in column $6k+1$, as will be seen later.

The cell symmetry in Equation (8) can be eliminated by taking only the values of $n \geq m$, so Equation (8) and Equation (9) would be:

$$k_{m,n}=\begin{bmatrix} m+(6m+1)n \to [n \geq m] \geq 1 \\ 4+5(m-1)+(6m-1)(n-1) \to [n \geq m] \geq 1 \end{bmatrix} \tag{10}$$

They contain composite numbers of the form $6k_{m,n}+1$

$$k \neq k_{m,n} = \begin{bmatrix} m+(6m+1)n \to [n \geq m] \geq 1 \\ 4+5(m-1)+(6m-1)(n-1) \to [n \geq m] \geq 1 \end{bmatrix} \tag{11}$$

They contain prime numbers of the form $6k+1$.

Table 4 shows examples where there are composite and prime numbers in rows $1 \leq k \leq 51$ applying Equation (10) and Equation (11).

In some cases $k_{m,n} = m+(6m+1)n$ can be equal to $k_{m,n} = 4+5(m-1)+(6m-1)n$, example $k_{1,4} = k_{1,6} = 29$, value highlighted in the yellow cells in Table 4. These cases correspond to cells containing a composite number $N_k = 6k+1 = p_{m1}p_{m2}p_n$, where $p_{m1} = 6m_1 - 1$ and $p_{m2} = 6m_2 - 1$ for $[m_1, m_2] \geq 1$. Example cells 29, 64, 141 are repeated in $k_{m,n} = m+(6m+1)n$ and en $k_{m,n} = 4+5(m-1)+(6m-1)(n-1)$. This is important in order to calculate the number of primes in $6k+1$ column, as will be seen later.

4. How to Recognize the Prime Numbers of $6k \pm 1$ Columns

Regardless of how to find primes with Equation (7) and Equation (11), there is the traditional test for whether or not a number is prime, using Theorem 2.

1) For $6k-1$ column

Theorem 2:

If $N_k > 5$ is a composite integer of the form $N_k = 6k-1$, then N_k has a prime factor p, with $p \leq \sqrt{N_k}$.

Demonstration:

Let $N_k > 5$ be a composite integer of the form $N_k = 6k-1$. Then $N_k = a \cdot b$, with $5 \leq a < N_k$ and $5 \leq b < N_k$. We affirm that one of them a or b is less than or equal to $\sqrt{N_k}$. As $a > 5$, from the previous lemma, there exists a prime p such that $p | a$.

Table 4. Examples where there are composite and prime numbers in rows $1 \leq k \leq 51$ applying Equation (10) and Equation (11).

	m					$1 \leq k \leq 51$	
n	1		2		3	$k_{m,n}$ with composite numbers	$k \neq k_{m,n}$ with prime numbers
	$k_{m,n} = m+(6m+1)n$		$k_{m,n} = 4+5(m-1)+(6m-1)(n-1)$				
1	8		4			4, 8, 9	1, 2, 3, 5, 6
2	15	28	9	20		14, 15, 19	7, 10, 11, 12
3	22	41	14	31	48	20, 22, 24	13, 16, 17, 18
4	29		19	42		28, 29, 31	21, 23, 25, 26
5	36		24			34, 36, 39	27, 30, 32, 33
6	43		29			41, 42, 43	35, 37, 38, 40
7	50		34			44, 48, 49	45, 46, 47, 51
8			39			50	
9			44				
10			49				

As $a|N_k$, it follows that $p|N_k$ and as $p|a$, we also have $p \le a \le \sqrt{N_k}$. The theorem has being proved.

As it is known that for $k = [1,2,3,4,5]$, $p = 6k-1$ are primes, then for $k \ge 6$ and $p = 6k-1$ is a prime greater than 5 according to Theorem 2, as long as:

$$(6m-1) \nmid p \text{ Or } (6m+1) \nmid p \text{ where } m \le \left|\sqrt{k/6}\right| \text{ and } k \ge 6 \tag{12}$$

Example: primes (11, 17, 23, 29, and 41). The problem with Equation (12) is that its application is more difficult for very large primes; one would have to know all primes smaller than $\sqrt{p}$, and the application of Equation (8) and Equation (9), is simpler; all $6k-1$ primes can be sequentially obtained.

2) For the $6k+1$ column

Applying the same theorem shown for the $6k-1$ column, we have that $p = 6k+1$ is a prime greater than 7 as long as:

$$(6m-1) \nmid p \text{ Or } (6m+1) \nmid p \text{ where } m \le 1 + \left|\sqrt{k/6}\right| \text{ and } k \ge 4 \tag{13}$$

Example: primes (13, 19, 31, and 37). Similarly, the application of Equation (13), is more difficult for very large primes, while the application of Equation (10) and Equation (11), is simpler to obtain all $6k+1$ primes sequentially.

5. Numbers of Primes in $6k \pm 1$ Columns

1) The total of primes Π including primes 2 and 3 will be $\Pi = 2 + \Pi_1 + \Pi_2$, where Π_1 is the total of primes in $6k-1$ column and Π_2 the total of primes in $6k+1$ column for the same k in both columns. The remarkable thing about this is that $\Pi_1 \ge \Pi_2$, by the way the composite numbers that have 5 and 7 appear as one of their prime factors in each column, therefore $2\Pi_2 \le \Pi - 2 \le 2\Pi_1$.

2) The number of primes Π_1 in $6k-1$ column, smaller than or equal to $N_k = 6k-1$; where $p_m = 6m-1$; $p_n = 6n+1$; $p_{m1} = 6m_1 - 1$; $p_{m2} = 6m_2 - 1$; $p_{n_1} = 6n_1 + 1$; $p_2 = 6n_2 + 1$ with $[m, n, m_1, m_2, n_1, n_2] < k \rightarrow$ ($m_1 \neq m_2$, n_1 and n_2 can be the same or different), can be calculated by Equation (14):

$$\Pi_1 = k - \sum\left|\frac{k-m}{p_m}\right| \ge 1 + \left(\sum\left|\frac{N_k}{p_{n1}p_{n2}p_m}\right| \ge 1 + \sum\left|\frac{N_k}{p_{m1}p_{m2}p_m}\right| \ge 1\right) \tag{14}$$

The term $\sum\left|\frac{k-m}{p_m}\right| \ge 1$ corresponds to the number of composite numbers in N_k, including cells that have equal numbers in different cells as shown in Table 1 and Table 2 and the term $\left(\sum\left|\frac{N_k}{p_{n1}p_{n2}p_m}\right| \ge 1 + \sum\left|\frac{N_k}{p_{m1}p_{m2}p_m}\right| \ge 1\right)$ corresponds to the number of cells that are repeated with the same value in $\sum\left|\frac{k-m}{p_m}\right| \ge 1$.

Example of Π_1 for $k = 16, N_k = 95$, with $p_1 = 5$ and $p_2 = 11$, therefore $\sum\left|\frac{k-m}{p_m}\right| \ge 1 = 3 + 1 = 4$ and

$$\left(\sum\left|\frac{N_k}{p_{n1}p_{n2}p_m}\right|\geq 1+\sum\left|\frac{N_k}{p_{m1}p_{m2}p_m}\right|\geq 1\right)=\left[\left(\sum\left|\frac{95}{7\times 7\times 5}\right|\geq 1\right)=0\right]$$
$$+\left[\left(\sum\left|\frac{95}{5\times 11\times 5}\right|\geq 1\right)=0\right]=0,$$

then

$$\Pi_1=16-4=12.$$

Example of Π_1 for $k=51$, $N_k=305$, with $p_1=5$, $p_2=11$, $p_3=17$, $p_4=23$, $p_5=29$, $p_6=35$ *y*

$$p_7=41\sum\left|\frac{k-m}{p_m}\right|\geq 1=10+4+2+2+1+1+1=21$$

and

$$\left(\sum\left|\frac{N_k}{p_{n1}p_{n2}p_m}\right|\geq 1+\sum\left|\frac{N_k}{p_{m1}p_{m2}p_m}\right|\geq 1\right)=\left(\left|\frac{305}{7\times 7\times 5}\right|=1\right)+\left(\left|\frac{305}{5\times 11\times 5}\right|=1\right)=2$$

therefore $\Pi_1=51-21+2=32$ (see Table 2).

3) The number of primes Π_2 in $6k+1$ column, less than or equal to $N_k=6k+1$, where $p_m=6m-1$ and $p_n=6n+1$ with $[m,n]<k$, is a little more complicated than Equation (14) because of the large number of cells that are repeated as shown in Table 3 and Table 4 and can be calculated by Equation (15):

$$\Pi_2=k-\left|\frac{k+1}{5}\right|-\sum a_m-\sum a_n+\sum\left|\frac{N_k}{p_{m1}p_{m2}p_n}\right|\geq 1$$
$$a_m=\left|\frac{k-(4+5(m-1))}{p_{m\geq 2}}\right|-(m-2)\rightarrow\left|\frac{k-(4+5(m-1))}{p_{m\geq 2}}\right|\geq(m-2) \tag{15}$$
$$a_n=\left|\frac{k-n}{p_n}\right|-(n-1)\rightarrow\left|\frac{k-n}{p_n}\right|\geq(n-1)$$

The term $\left|\frac{k+1}{5}\right|$ contains the total number of composite having 5 as one of its factors, the term $\sum a_m$ contains the total number of composite numbers having $p_{m\geq 2}$ as one of its Factors after eliminating the repeated cells in $k_{m,n}=4+5(m-1)+(6m-1)(n-1)$, the term $\sum a_n$ contains the total number of composite numbers having p_n as one of its factors after eliminating the repeated cells in $k_{m,n}=m+(6m+1)n$ and the term $\sum\left|\frac{N_k}{p_{m1}p_{m2}p_n}\right|\geq 1$ corresponds to the number of repeated cells where $k_{m,n}=m+(6m+1)n$ is equal to $k_{m,n}=4+5(m-1)+(6m-1)(n-1)$ where its factors are $p_{m1}p_{m2}p_n$ with $p_{m1}=6m_1-1$, $p_{m2}=6m_2-1$ and $p_n=6n+1\rightarrow[m_1,m_2,n]\geq 1$.

Example of Π_2 where $k=16$ and $N_k=97$:

With $p_m=[5,11]$ and $p_n=[7,13]$ then

$$\left|\frac{k+1}{5}\right|=\left|\frac{16+1}{5}\right|=3,$$

$$\sum a_m = \left|\frac{k-(4+5(m-1))}{p_{m\geq 2}}\right| - (m-2) = \left[\left(\left|\frac{16-9}{11}\right| \geq 0\right) = 0\right] - 0 = 0,$$

$$\sum a_n = \sum\left(\left|\frac{k-n}{p_n}\right| \geq (n-1)\right) - (n-1)$$

$$= \left[\left\{\left(\left|\frac{16-1}{7}\right| \geq 0\right) = 2\right\} - 0 = 2\right] + \left[\left\{\left(\left|\frac{16-2}{13}\right| \geq 1\right) = 1\right\} - 1 = 0\right] = 2,$$

$\left(\sum\left|\frac{N_k}{p_{m1}p_{m2}p_n}\right| \geq 1\right) = \left(\left|\frac{97}{5\times5\times7}\right| \geq 1\right) = 0.$ Therefore $\Pi_2 = 16-3-0-2+0 = 11$.

Example of Π_2 where $k = 51$ and $N_k = 307$:

With $p_m = [5,11,17,23]$ and $p_n = [7,13,19]$ then

$$\left|\frac{k+1}{5}\right| = \left|\frac{51+1}{5}\right| = 10,$$

$$\sum a_m = \left|\frac{k-(4+5(m-1))}{p_{m\geq 2}}\right| - (m-2)$$

$$= \left[\left\{\left(\left|\frac{51-9}{11}\right| \geq 0\right) = 3\right\} - 0 = 3\right] + \left[\left\{\left(\left|\frac{51-14}{17}\right| \geq 3\right) = 2\right\} - 1 = 1\right]$$

$$+ \left[\left\{\left(\left|\frac{51-19}{23}\right| \geq 4\right) = 2\right\} - 2 = 0\right] = 4,$$

$$\sum a_n = \sum\left(\left|\frac{k-n}{p_n}\right| \geq (n-1)\right) - (n-1)$$

$$= \left[\left\{\left(\left|\frac{51-1}{7}\right| \geq 0\right) = 7\right\} - 0 = 7\right] + \left[\left\{\left(\left|\frac{51-2}{13}\right| \geq 1\right) = 3\right\} - 1 = 2\right]$$

$$+ \left[\left\{\left(\left|\frac{51-3}{19}\right| \geq 2\right) = 2\right\} - 2 = 0\right] = 9,$$

$\left(\sum\left|\frac{N_k}{p_{m1}p_{m2}p_n}\right| \geq 1\right) = \left(\left|\frac{307}{5\times5\times7}\right| \geq 1\right) = 1.$ Therefore $\Pi_2 = 51-10-4-9+1 = 29$ (see Table 4).

4) With $k = 16$ then $\Pi = 2+\Pi_1+\Pi_2 = 2+12+11 = 25$ and with $k = 51$ then $\Pi = 2+\Pi_1+\Pi_2 = 2+32+29 = 63$. In both cases, $2\Pi_2 \leq \Pi - 2 \leq 2\Pi_1$.

6. Conclusions

All prime numbers $p \geq 5$ have a pattern or form described by Equation (4).

Verification of whether a number is prime can be done in $6k-1$ column by means of Equation (6) and Equation (7) or Equation (12) and in $6k+1$ column by means of Equation (10) and Equation (11) or Equation (13).

The primes do not appear in random form, their sequence is determined by the Equation (6) and Equation (7) in $6k-1$ column and Equation (10) and Equation (11) for $6k+1$ column.

The number of primes Π_1 less than or equal to $N_k = 6k-1$ can be calculated exactly by Equation (14) without needing to know all primes having that form equally for Π_2 with Equation (15).

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