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Some Actuarial Formula of Life Insurance for Fuzzy Markets*

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Abstract

This paper presents an actuarial model of life insurance for fuzzy markets based on Liu process. At first, some researches about an actuarial model of life insurance for stochastic market and concepts about fuzzy process have been reviewed. Then, an actuarial model of life insurance for fuzzy process is formulated.

Keywords: Fuzzy Actuarial Model, Fuzzy Process, Liu Process, Geometric Liu Process, Actuarial Formula

1. Introduction

The concept of fuzzy set was initiated by Zadeh [1] via membership function in 1965. In order to measure a fuzzy event, Liu and Liu [2] introduced the concept of credibility measure in 2002. Li and Liu [3] gave a sufficient and necessary condition for credibility measure in 2006 (cf. Liu [4]). Credibility theory was founded by Liu [5] in 2004 and refined by Liu [6] in 2007 as a branch of mathematics for studying the behavior of fuzzy phenomena. Credibility theory is deduced from the normality, monotonicity, self-duality, and maximality axioms. Liu [7] recently introduced the concepts of fuzzy process, Liu process and the geometric Liu process which will be commonly used model in finance for the value of an asset in a fuzzy environment. The two types of fundamental and important fuzzy processes, Liu process and the geometric Liu process, are the counterparts of Brownian motion and the geometric Brownian motion, respectively.

Li [8] presented an alternative assumption that stock price follows geometric Liu process. It is an application of fuzzy process for stock markets. Peng [9] presented A General Stock Model for Fuzzy Markets based on Liu's research in 2008. In 2010, Gao and Zhao [10] first presented fuzzy interests rate for insurance market.

The remainder of this paper is structured as follows. Section 2 is intended to introduce some useful concepts of fuzzy process as they are needed. Section 3 reviews Gao and Zhao's actuarial model for fuzzy market. An actuarial model of life insurance with payouts increased for fuzzy process is formulated in Section 4. Some actuarial formula of pure premium of the n-year continuous life insurance model is discussed in Section 5, as discount factor is a fuzzy process. Finally, some remarks are made in the concluding section.

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2. Preliminaries

In this section, we will introduce some useful definitions and properties about fuzzy process.

2.1. Fuzzy Process

Definition 1. (Liu [7]) Given an index set T and a credibility space (Θ, P, Cr), a fuzzy process is a func-tion from $T \times (\Theta, \mathrm{P}, Cr)$ to the set of real numbers. In other words, a fuzzy process $X(t,\theta)$ is a function of two variables such that the function $X(t^*,\theta)$ is a fuzzy variable for each t^*. For simplicity, sometimes we simply use the symbol X_t instead of longer notation $X(t,\theta)$.

Definition 2. (Liu [7]) A fuzzy process X_t is said to have independent increments if

$$X_{t_1} - X_{t_0}, X_{t_2} - X_{t_1}, \cdots, X_{t_k} - X_{t_{k-1}} \quad (1)$$

are independent fuzzy variables for any times $t_0 < t_1 < \cdots < t_k$. A fuzzy process X_t is said to have stationary increments if, for any given $s > 0$, the $X_{s+t} - X_s$ are identically distributed fuzzy variables for all $t > 0$.

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2.2. Liu Process

Definition 3. (Liu [7]) A fuzzy process C_t is said to be a Liu process if

1) $C_o = 0$
2) C_t has stationary and independent increments
3) every increment $C_{s+t} - C_s$ is a normally distributed fuzzy variable with expected value et and variance $\sigma^2 t^2$, whose membership function is

$$\mu(x) = 2\left(1+\exp\left(\frac{\pi|x-ut|}{\sqrt{6}\sigma t}\right)\right)^{-1}, x \in R \qquad (2)$$

The parameters e and $\sigma(>0)$ are called the drift and diffusion coefficients, respectively. The Liu process is said to be standard if $e = 0$ and $\sigma = 1$. The Liu process plays the role of the counterpart of Brownian motion.

Definition 4. (Liu [7]) Let C_t be a standard Liu process. Then the fuzzy process

$$G_t = \exp(et + \sigma C_t)$$

is called a geometric Liu process, or sometimes exponential Liu process. Li [8] has deduced that G_t is of a lognormal membership function

$$\mu(x) = 2\left(1+\exp\left(\frac{\pi|\ln x-et|}{\sqrt{6}\sigma t}\right)\right)^{-1}, x \ge 0 \qquad (3)$$

and the expected value is

$E[\xi] = e^e \sqrt{6}\sigma \csc\left(\sqrt{6}\sigma\right), \sigma < \pi/\sqrt{6}$ had been proved by Li and Qin [8].

3. Fuzzy Model of Discount Factor

In this section we review a fuzzy model of discount factor which presented by Gao and Zhao, the model is assumed that interests rate is a trapezoidal fuzzy variables.

Accumulation function for force of interest is $y(t)$, as insurer, V_T is present value of payment at the end of m year. As insured, the present value for Survival annuity paid 1 per year is Y_m.

$$y(t) = \delta t, 0 \le t \le n$$

$$V_T = \begin{cases} e^{-y(K-1-m)}, m \le K < n \\ 0 \qquad\qquad , K \ge n \end{cases}$$

where δ is a trapezoidal fuzzy variable (a,b,c,d), $a \le b \le c \le d$.

4. An Actuarial Model for Fuzzy Process

4.1. The Model

The paper hypothesizes that interest rate has two components: risk-free interest and risk interest. δ is used to represent force of interest, $y(t) = \delta_0 t + \beta C_t$ is the accumulation function for force of interest, where δ_0 is a part of interests with no risk, C_t is a standard Liu process, used to describe the interests on risk. β is β-coefficient.

We have the discount factor:

$$V_t = \exp(-y(t))$$

The actuarial model for Liu process is:

$$y(t) = \delta_0 t + \beta C_t$$

$$V_t = \exp(-y(t))$$

4.2. Expected Value for V_t

C_t is a normally distribution fuzzy variable $C_t \sim N(et, \sigma^2 t^2)$. Because C_t is a standard Liu process with $e = 0, \sigma = 1$, $C_t \sim N(0, t^2)$. If $t\beta < \pi/\sqrt{6}$, then it is easy to prove that the expected value is

$$\begin{aligned} E(V_t) &= E\left[\exp(-\delta_0 t - \beta C_t)\right] \\ &= e^{-\delta_0 t} E\left(e^{-\beta C_t}\right) \\ &= e^{-\delta_0 t}\left(\int_0^{+\infty} Cr\left\{e^{-\beta\xi} \ge x\right\}\mathrm{d}x - \int_{-\infty}^0 Cr\left\{e^{-\beta\xi} \le x\right\}\mathrm{d}x\right) \\ &= e^{-\delta_0 t}\int_0^{\infty} Cr\left\{\xi \le \frac{1}{\beta}\ln x\right\}\mathrm{d}x \\ &= e^{-\delta_0 t}\int_0^{\infty} 1 - \left(1+\exp\left(-\frac{\pi\left(\frac{1}{\beta}\ln x\right)}{\sqrt{6}t}\right)\right)^{-1}\mathrm{d}x \\ &= e^{-\delta_0 t}\int_0^{\infty} 1 - \frac{1}{1+x^{\left(-\frac{\pi}{\sqrt{6}t\beta}\right)}}\mathrm{d}x \\ &= e^{-\delta_0 t}\int_0^{\infty} \frac{1}{1+x^{\frac{\pi}{\sqrt{6}t\beta}}}\mathrm{d}x \\ &= e^{-\delta_0 t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right) \end{aligned}$$

Otherwise, we have $E(V_t) = \infty$.

5. Some Actuarial Formula of Pure Premium of the N-Year Continuous Life Insurance

The pure premium of the n-year continuous life insur-

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ance with payment $b(t)$ is

$$\bar{A}^1_{x:\bar{n}} = E[V_T] = \int_0^n b(t)E[V_t]\,{}_t p_x \mu_{x+t}\mathrm{d}t$$

$$= \int_0^n b(t)e^{-\delta_0 t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right){}_t p_x \mu_{x+t}\mathrm{d}t$$

where $\beta t < \pi/\sqrt{6}$

5.1. Pure Premium of the N-Year Continuous Life Insurance under the Assumption of *de Moivre*

Under the assumption of *de Moivre*, we have

$$\mu_{x+t} = \frac{1}{\omega - x - t},\quad {}_t p_x \mu_{x+t} = \frac{1}{\omega - x}$$

Then

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t)e^{\delta_0 t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right){}_t p_x \mu_{x+t}\mathrm{d}t$$

$$= \frac{\sqrt{6}\beta}{\omega - x}\int_0^n t b(t)e^{-\delta_0 t}\csc\left(\sqrt{6}\beta t\right)\mathrm{d}t,\ \beta t < \pi/\sqrt{6}$$

where $\bar{A}^1_{x:\bar{n}}$ respected to pure premium of the n-year continuous life insurance.

If the compensation is a constant, that is to say $b(t) = b$, so we have

$$\bar{A}^1_{x:\bar{n}} = b\int_0^n e^{-\delta_0 t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right){}_t p_x \mu_{x+t}\mathrm{d}t$$

$$= \frac{\sqrt{6}\beta b}{\omega - x}\int_0^n t e^{-\delta_0 t}\csc\left(\sqrt{6}\beta t\right)\mathrm{d}t,\ \beta t < \pi/\sqrt{6}$$

If the payment amount is increasing linearly, $b(t) = b + t$, then

$$I\bar{A}^1_{x:\bar{n}} = \int_0^n (b+t)e^{-\delta_0 t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right){}_t p_x \mu_{x+t}\mathrm{d}t$$

$$= \frac{\sqrt{6}\beta}{\omega - x}\int_0^n (b+t)t e^{-\delta_0 t}\csc\left(\sqrt{6}\beta t\right)\mathrm{d}t$$

$$\beta t < \pi/\sqrt{6}$$

If the payment amount is increasing geometrically, $b(t) = t^k$,

$$\bar{A}^1_{x:\bar{n}} = \int_0^n t^k e^{-\delta_0 t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right){}_t p_x \mu_{x+t}\mathrm{d}t$$

$$= \frac{\sqrt{6}\beta}{\omega - x}\int_0^n t^{k+1} e^{-\delta_0 t}\csc\left(\sqrt{6}\beta t\right)\mathrm{d}t$$

$$\beta t < \pi/\sqrt{6}$$

If the payment amount is increasing exponentially, $b(t) = e^{\alpha t}$,

$$\bar{A}^1_{x:\bar{n}} = \int_0^n e^{\alpha t}e^{-\delta_0 t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right){}_t p_x \mu_{x+t}\mathrm{d}t$$

$$= \frac{\sqrt{6}\beta}{\omega - x}\int_0^n t e^{(\alpha-\delta_0)t}\csc\left(\sqrt{6}\beta t\right)\mathrm{d}t$$

$$\beta t < \pi/\sqrt{6}$$

5.2. Pure Premium of the N-Year Continuous Life Insurance under the Assumption of *Gompertz*

Under the assumption of *Gompertz*, $\mu_{x+t} = BC^{x+t}$, $B > 0$, $C \ge 1$, and

$${}_t p_x \mu_{x+t} = BC^{x+t}e^{\frac{B}{\ln C}C^x\left(1-C^t\right)}$$

If the compensation is a constant, that is to say $b(t) = b$, so we have

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t)e^{-\delta_0 t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right){}_t p_x \mu_{x+t}\mathrm{d}t$$

$$= \sqrt{6}\beta b\int_0^n t e^{-\delta_0 t}\csc\left(\sqrt{6}\beta t\right)BC^{x+t}e^{\frac{B}{\ln C}C^x\left(1-C^t\right)}\mathrm{d}t$$

$$= \sqrt{6}\beta b B\int_0^n tC^{x+t}e^{-\delta_0 t+\frac{B}{\ln C}C^x\left(1-C^t\right)}\csc\left(\sqrt{6}\beta t\right)\mathrm{d}t$$

$$\beta t < \pi/\sqrt{6}$$

If the payment amount is increasing linearly, $b(t) = b + t$, then

$$I\bar{A}^1_{x:\bar{n}} = \int_0^n b(t)e^{-\delta_0 t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right){}_t p_x \mu_{x+t}\mathrm{d}t$$

$$= \sqrt{6}\beta\int_0^n (b+t)t e^{-\delta_0 t}\csc\left(\sqrt{6}\beta t\right)BC^{x+t}e^{\frac{B}{\ln C}C^x\left(1-C^t\right)}\mathrm{d}t$$

$$= \sqrt{6}\beta B\int_0^n (b+t)tC^{x+t}e^{-\delta_0 t+\frac{B}{\ln C}C^x\left(1-C^t\right)}\csc\left(\sqrt{6}\beta t\right)\mathrm{d}t$$

$$\beta t < \pi/\sqrt{6}$$

If the payment amount is increasing geometrically, $b(t) = t^k$,

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t)e^{-\delta t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right){}_t p_x \mu_{x+t}\mathrm{d}t$$

$$= \sqrt{6}\beta\int_0^n t^k t e^{-\delta t}\csc\left(\sqrt{6}\beta t\right)BC^{x+t}e^{\frac{B}{\ln C}C^x\left(1-C^t\right)}\mathrm{d}t$$

$$= \sqrt{6}\beta B\int_0^n t^{k+1}C^{x+t}e^{-\delta t+\frac{B}{\ln C}C^x\left(1-C^t\right)}\csc\left(\sqrt{6}\beta t\right)\mathrm{d}t$$

$$\beta t < \pi/\sqrt{6}$$

If the payment amount is increasing exponentially, $b(t) = e^{\alpha t}$,

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t)e^{-\delta_0 t}\sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right){}_t p_x \mu_{x+t}\mathrm{d}t$$

$$= \sqrt{6}\beta\int_0^n e^{\alpha t}t e^{-\delta_0 t}\csc\left(\sqrt{6}\beta t\right)BC^{x+t}e^{\frac{B}{\ln C}C^x\left(1-C^t\right)}\mathrm{d}t$$

$$= \sqrt{6}\beta B\int_0^n C^{x+t}e^{(\alpha-\delta_0)t+\frac{B}{\ln C}C^x\left(1-C^t\right)}\csc\left(\sqrt{6}\beta t\right)\mathrm{d}t$$

$$\beta t < \pi/\sqrt{6}$$

5.3. Pure Premium of the N-Year Continuous Life Insurance under the Assumption of *Makeham*

Under the assumption of *Makeham*,

$$\mu_{x+t} = A + BC^{x+t},\ B > 0,\ C \geq 1,\ A \geq -B$$

$$_t p_x \mu_{x+t} = \left(A + BC^{x+t}\right) e^{-At + \frac{B}{\ln C} C^x \left(1 - C^t\right)}$$

If the compensation is a constant, that is to say $b(t) = b$, so we have

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t) e^{-\delta_0 t} \sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right)\, {}_t p_x \mu_{x+t} \mathrm{d}t$$
$$= \sqrt{6}\beta b \int_0^n t e^{-\delta_0 t} \csc\left(\sqrt{6}\beta t\right)\left(A + BC^{x+t}\right)$$
$$\cdot e^{-At + \frac{B}{\ln C} C^x \left(1 - C^t\right)} \mathrm{d}t = \sqrt{6}\beta b \int_0^n t\left(A + BC^{x+t}\right)$$
$$\cdot e^{-(\delta_0 + A)t + \frac{B}{\ln C} C^x \left(1 - C^t\right)} \csc\left(\sqrt{6}\beta t\right) \mathrm{d}t$$
$$\beta t < \pi / \sqrt{6}$$

If the payment amount is increasing linearly, $b(t) = b + t$, then

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t) e^{-\delta_0 t} \sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right)\, {}_t p_x \mu_{x+t} \mathrm{d}t$$
$$= \sqrt{6}\beta \int_0^n (b + t) t e^{-\delta_0 t} \csc\left(\sqrt{6}\beta t\right)\left(A + BC^{x+t}\right)$$
$$\cdot e^{-At + \frac{B}{\ln C} C^x \left(1 - C^t\right)} \mathrm{d}t = \sqrt{6}\beta \int_0^n t(b + t)\left(A + BC^{x+t}\right)$$
$$\cdot e^{-(\delta_0 + A)t + \frac{B}{\ln C} C^x \left(1 - C^t\right)} \csc\left(\sqrt{6}\beta t\right) \mathrm{d}t$$
$$\beta t < \pi / \sqrt{6}$$

If the payment amount is increasing geometrically, $b(t) = t^k$

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t) e^{\delta_0 t} \sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right)\, {}_t p_x \mu_{x+t} \mathrm{d}t$$
$$= \sqrt{6}\beta \int_0^n t^k t e^{\delta_0 t} \csc\left(\sqrt{6}\beta t\right)\left(A + BC^{x+t}\right)$$
$$\cdot e^{-At + \frac{B}{\ln C} C^x \left(1 - C^t\right)} \mathrm{d}t = \sqrt{6}\beta \int_0^n t^{k+1}\left(A + BC^{x+t}\right)$$
$$\cdot e^{-(\delta_0 + A)t + \frac{B}{\ln C} C^x \left(1 - C^t\right)} \csc\left(\sqrt{6}\beta t\right) \mathrm{d}t$$
$$\beta t < \pi / \sqrt{6}$$

If the payment amount is increasing exponentially, $b(t) = e^{\alpha t}$

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t) e^{\delta_0 t} \sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right)\, {}_t p_x \mu_{x+t} \mathrm{d}t$$
$$= \sqrt{6}\beta \int_0^n e^{\alpha t} t e^{-\delta_0 t} \csc\left(\sqrt{6}\beta t\right)\left(A + BC^{x+t}\right)$$
$$\cdot e^{-At + \frac{B}{\ln C} C^x \left(1 - C^t\right)} \mathrm{d}t \sqrt{6}\beta \int_0^n t\left(A + BC^{x+t}\right)$$
$$\cdot e^{(\alpha - \delta_0 - A)t + \frac{B}{\ln C} C^x \left(1 - C^t\right)} \csc\left(\sqrt{6}\beta t\right) \mathrm{d}t$$
$$\beta t < \pi / \sqrt{6}$$

5.4. Pure Premium of the N-Year Continuous Life Insurance under the Assumption of *Weibull*

Under the assumption of *Weibull*,

$$\mu_{x+t} = k(x + t)^n, k > 0, n > 0$$

$$_t p_x \mu_{x+t} = k(x + t)^n e^{k(n+1)\left[x^{n+1} - (x+t)^{n+1}\right]}$$

If the compensation is a constant, that is to say $b(t) = b$, so we have

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t) e^{-\delta_0 t} \sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right)\, {}_t p_x \mu_{x+t} \mathrm{d}t$$
$$= \sqrt{6}\beta b \int_0^n t e^{-\delta_0 t} \csc\left(\sqrt{6}\beta t\right) k(x + t)^n$$
$$\cdot e^{k(n+1)\left[x^{n+1} - (x+t)^{n+1}\right]} \mathrm{d}t = \sqrt{6}\beta b k e^{k(n+1)x^{n+1}}$$
$$\cdot \int_0^n t(x + t)^n e^{-\delta_0 t - k(n+1)(x+t)^{n+1}} \csc\left(\sqrt{6}\beta t\right) \mathrm{d}t$$
$$\beta t < \pi / \sqrt{6}$$

If the payment amount is increasing linearly, $b(t) = b + t$, then

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t) e^{-\delta_0 t} \sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right)\, {}_t p_x \mu_{x+t} \mathrm{d}t$$
$$= \sqrt{6}\beta \int_0^n t(b + t) e^{-\delta_0 t} \csc\left(\sqrt{6}\beta t\right) k(x + t)^n$$
$$\cdot e^{k(n+1)\left[x^{n+1} - (x+t)^{n+1}\right]} \mathrm{d}t = \sqrt{6}\beta k e^{k(n+1)x^{n+1}}$$
$$\cdot \int_0^n t(b + t)(x + t)^n e^{-\delta_0 t - k(n+1)(x+t)^{n+1}} \csc\left(\sqrt{6}\beta t\right) \mathrm{d}t$$
$$\beta t < \pi / \sqrt{6}$$

If the payment amount is increasing geometrically, $b(t) = t^a$

$$\bar{A}^1_{x:\bar{n}} = \int_0^n b(t) e^{-\delta_0 t} \sqrt{6}\beta t \csc\left(\sqrt{6}\beta t\right)\, {}_t p_x \mu_{x+t} \mathrm{d}t$$
$$= \sqrt{6}\beta \int_0^n t t^a e^{-\delta_0 t} \csc\left(\sqrt{6}\beta t\right) k(x + t)^n$$
$$\cdot e^{k(n+1)\left[x^{n+1} - (x+t)^{n+1}\right]} \mathrm{d}t = \sqrt{6}\beta k e^{k(n+1)x^{n+1}}$$
$$\cdot \int_0^n t^{a+1}(x + t)^n e^{-\delta_0 t - k(n+1)(x+t)^{n+1}} \csc\left(\sqrt{6}\beta t\right) \mathrm{d}t$$
$$\beta t < \pi / \sqrt{6}$$

If the payment amount is increasing exponentially, $b(t)=e^{\alpha t}$

$$\bar{A}^{1}_{x:\bar{n}}=\int_0^n b(t)e^{-\delta_0 t}\sqrt{6}\beta t\csc\left(\sqrt{6}\beta t\right)\,{}_t p_x\mu_{x+t}\mathrm{d}t$$

$$=\sqrt{6}\beta\int_0^n te^{\alpha t}e^{-\delta_0 t}\csc\left(\sqrt{6}\beta t\right)k\left(x+t\right)^n$$

$$\cdot e^{k(n+1)\left[x^{n+1}-(x+t)^{n+1}\right]}\mathrm{d}t=\sqrt{6}\beta ke^{k(n+1)x^{n+1}}$$

$$\cdot\int_0^n t\left(x+t\right)^n e^{(\alpha-\delta_0)t-k(n+1)(x+t)^{n+1}}\csc\left(\sqrt{6}\beta t\right)\mathrm{d}t$$

$$\beta t<\pi/\sqrt{6}$$

6. Conclusions

The main contribution of this paper is to suggest a new actuarial for fuzzy markets by means of Liu process. Some actuarial formulas of pure premium of the n-year continuous life insurance on the proposed fuzzy model are investigated.

7. References

[1] L. A. Zadeh, "Fuzzy Sets," *Information and Control*, Vol. 8, No. 3, 1965, pp. 338-353. doi:10.1016/S0019-9958(65)90241-X

[2] B. D. Liu and Y. K. Liu, "Expected Value of Fuzzy Variable and Fuzzy Expected Value Models," *IEEE Transactions on Fuzzy Systems*, Vol. 10, No. 4, 2002, pp. 445-450. doi:10.1109/TFUZZ.2002.800692

[3] X. Li and B. D. Liu, "A Sufficient and Necessary Condition for Credibility Measures," *International Journal of Uncertainty, Fuzziness & Knowledge-Based Systems*, Vol. 14, No. 5, 2006, pp. 527-535. doi:10.1142/S0218488506004175

[4] B. D. Liu, "Uncertainty Theory," Springer-Verlag, Berlin, 2004.

[5] B. D. Liu, "Uncertainty Theory," 2nd Edition, Springer-Verlag, Berlin, 2007.

[6] B. D. Liu, "A Survey of Credibility Theory," *Fuzzy Optimization and Decision Making*, Vol. 5, No. 4, 2006, pp. 387-408. doi:10.1007/s10700-006-0016-x

[7] B. D. Liu, "Fuzzy Process, Hybrid Process and Uncertain Process," *Journal of Uncertain Systems*, Vol. 2, No. 1, 2008, pp. 3-16.

[8] X. Li and F. Q. Zhong, "Eepected Value and Variance of Geometric Liu Process," *Far East Journal of Experimental and Theoretical Artificial Intelligence*, Vol. 1, No. 2, 2008, pp. 127-135.

[9] J. Peng, "A General Stock Model for Fuzzy Markets," *Journal of Uncertain Systems*, Vol. 2, No. 4, 2008, pp. 248-254.

[10] J. G. Gao and M. Q. Zhao, "The Life Insurance Model for Fuzzy Interest Rate (Chinese)," *Journal of Systems Engineering*, Vol. 25, 2010, pp. 603-608.

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