Existence of Periodic Solution for a Non-Autonomous Stage-Structured Predator-Prey System with Impulsive Effects

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Abstract

In this paper, we studied a non-autonomous predator-prey system where the prey dispersal in a two-patch environment. With the help of a continuation theorem based on coincidence degree theory, we establish sufficient conditions for the existence of positive periodic solutions. Finally, we give numerical analysis to show the effectiveness of our theoretical results.

Keywords: Periodic Solution, Coincidence Degree Theory, Stage-Structured, Impulsive

1. Introduction

In recent years, non-autonomous predator-prey systems have been widely studied [1-6]. There has been a growing interest in the study of mathematical models of populations dispersing among patches in the nature world [3,7-9].

In the classical predator-prey models it is usually assumed that each individual predator admits the same ability to feed on prey. However, it is different for some species whose individuals have a life history that takes them through two stages, immature and mature, where immature predators are raised by their parents, so many models with time delays and stage structure for both prey and predator were investigated and rich dynamics have been observed [4,6,10-12].

In this paper, we are considered the effects of prey diffusion in two patches and maturation delay for predator on the dynamics of an impulsive predator-prey model. We discuss the differential equation: (See 1.1)

\[
\begin{cases}
\dot{x}_1(t) = x_1(t) \left( a_1(t) - r_1(t) x_1(t) \right) + d_1(t) x_2(t) - x_1(t), \\
\dot{x}_2(t) = x_2(t) \left( a_2(t) - r_2(t) x_2(t) \right) - k(t) x_2(t) y_2(t) + d_2(t) x_1(t) - x_2(t), \\
\dot{y}_1(t) = c(t) x_2(t) y_2(t) - c(t - \tau) e^{k(t - \tau) x_2(t - \tau)} y_2(t - \tau) - d(t) y_1(t) - q_1(t) y_2(t), \\
\dot{y}_2(t) = c(t - \tau) e^{k(t - \tau) x_2(t - \tau)} y_2(t - \tau) - q_2(t) y_2(t), \\
x_1(t^+_i) = (1 + \theta_1) x_1(t_i), \\
x_2(t^+_i) = (1 + \theta_2) x_2(t_i), \\
y_1(t^+_i) = (1 + \phi_1) y_1(t_i), \\
y_2(t^+_i) = y_2(t_i),
\end{cases}
\]

(1.1)

growth rate; \( \frac{r_i(t)}{a_i(t)} \) \( i = 1, 2 \) is the carrying capacity; 
\( d_i(t) \) \( i = 1, 2 \) is the dispersal rate of prey species;  
k(t) is the capture rate of mature predator.  
\( c(t) \) is a conversion efficiency.  
d(t) is the death rate of the immature predator.  
\( q_i(t) \) \( i = 1, 2 \) is the rate of intra-specific
and \( \theta_k \) and \( \varphi_k \) represent the annual birth pulse of \( x_i(t), y_i(t) (i = 1, 2) \) at \( t_k \ (k \in \mathbb{Z}^+ ) \). We make the following assumptions for our model:

1) \( a_i(t), r_i(t), d_i(t), q_i(t) (i = 1, 2), d(t), k(t), c(t) \) and \( r(t) \) are continuous positive \( \omega \)– periodic functions;

2) \( \theta_{k+q} = \theta_k, \theta_{2k+q} = \theta_{2k}, \varphi_{k+q} = \varphi_k, t_{k+q} = t_k + \omega \).

2. Preliminaries

Denote by \( PC(J, R) \ (J \subset R) \) the set of functions \( \psi : J \to R \), which are piecewise continuous in \( [0, \omega] \), and have points of discontinuity \( t_k \in [0, \omega] \). Let \( PC^c(J, R) \) denote the set of functions \( \psi \) with derivative \( \dot{\psi}(t) \in PC(J, R) \). We define the Banach space of \( \omega \)– periodic functions \( PC_\omega = \{ \psi \in PC ([0, \omega], R) \mid \psi(0) = \psi(\omega) \} \) with \( \| \psi \|_{PC_\omega} = \sup_{t \in [0, \omega]} \| \psi(t) \| \) and \( PC_{\omega}^c \) with \( \| \psi \|_{PC_{\omega}^c} = \max \{ \| \psi(t) \|_{C_\omega}, \| \dot{\psi}(t) \|_{C_\omega} \} \), we will considered the \( PC_\omega \times PC_{\omega}^c \) with the norm

\[
\| \psi_1, \psi_2 \|_{PC_\omega} = \| \psi_1 \|_{PC_\omega} + \| \psi_2 \|_{PC_{\omega}^c}.
\]

We define:

\[
\bar{f} = \frac{1}{\omega} \int_0^\omega f(t) dt, \quad f^L = \min_{t \in [0, \omega]} f(t), \quad f^U = \max_{t \in [0, \omega]} f(t).
\]

3. Existence of Positive Periodic Solutions

In this section, we study the existence of positive periodic solutions of system (1.1).

Before stating our result on positive \( \omega \)– periodic solutions of system (1.1), we need the following lemma:

**Lemma 3.1** ([13]). Let \( \Omega \in X \) be an open bounded set. Let \( L \) be a Fredholm mapping of index zero and \( N \) be \( L \)– compact on \( \overline{\Omega} \). Assume

1) for each \( \lambda \in (0, 1) \), \( x \) is any solution of \( Lx = \lambda Nx \) such that \( x \in \partial \Omega \);

2) for each \( QNx \neq 0 \) for each \( x \in \partial \Omega \cap \ker L \);

3) \( \deg \{ QNx, \partial \Omega \cap \ker L, 0 \} \neq 0 \).

Then the equation \( Lx = Nx \) has at least one solution in \( \overline{\Omega} \cap \text{Dom} L \).

**Theorem 3.1** If the system (1.1) satisfies

\[
(H1) \quad a_\omega + \ln \prod_{k=1}^q (1 + \theta_k) > 0,
\]

\[
(H2) \quad a_i - d_i + a_\omega + \ln \prod_{k=1}^q (1 + \theta_k) > 0,
\]

\[
(H3) \quad c \ e^{\omega_2 t} - c \ e^{\omega_3 t} > 0,
\]

the system (1.1) has at least one \( \omega \)– periodic positive solution.

**Proof.** Let \( x_1(t) = e^{\omega_1 t}, x_2(t) = e^{\omega_2 t}, y_1(t) = e^{\omega_1 t}, y_2(t) = e^{\omega_2 t} \), then

\[
\begin{align*}
\dot{u}_i(t) &= a_i(t) - r_i(t) e^{\omega_1 t} + d_i(t) e^{\omega_2 t} - a_\omega e^{\omega_1 t} - d_\omega e^{\omega_2 t}, \\
\dot{u}_2(t) &= a_2(t) - r_2(t) e^{\omega_2 t} - k(t) e^{\omega_1 t} + d_2(t) e^{\omega_2 t} - d_\omega e^{\omega_2 t}, \\
\dot{u}_3(t) &= c(t) e^{\omega_2 t} - q_1(t) e^{\omega_1 t} - d(t) - q_1(t) e^{\omega_1 t} \\
&\quad - c(t - \tau) e^{\omega_2 t} - q_2(t) e^{\omega_1 t}, \\
\dot{u}_4(t) &= c(t - \tau) e^{\omega_2 t} - q_2(t) e^{\omega_1 t} - q_2(t) e^{\omega_1 t} - q_1(t) e^{\omega_1 t}.
\end{align*}
\]

One can easily see that if system (3.1) has one \( \omega \)– periodic solution \( (u_1(t), u_2(t), u_3(t), u_4(t)) \), then

\[
\begin{pmatrix}
\dot{x}_1(t), \dot{y}_1(t), \dot{x}_2(t), \dot{y}_2(t)
\end{pmatrix} = \begin{pmatrix}
x_1(t), y_1(t), x_2(t), y_2(t)
\end{pmatrix},
\]

is a positive \( \omega \)– periodic solution of system (1.1). Thus, in what follows our goal is to show that system (3.1) has at least one \( \omega \)– periodic solution.

Here, we rewrite

\[
\begin{align*}
f_1(t) &= \dot{u}_1(t), f_2(t) = \dot{u}_2(t), \\
f_3(t) &= \dot{u}_3(t), f_4(t) = \dot{u}_4(t).
\end{align*}
\]

Let

\[
\begin{align*}
 DomL &= PC_\omega^c \times PC_\omega^c \times PC_\omega^c, \\
 N : PC_\omega^c \times PC_\omega^c \times PC_\omega^c &\to R.
\end{align*}
\]
\[
\begin{pmatrix}
u_1 \\ u_2 \\ u_3 \\ u_4
\end{pmatrix} = \begin{pmatrix}
 f_1(t) \\ f_2(t) \\ f_3(t) \\ f_4(t)
\end{pmatrix},
\begin{pmatrix}
 \ln(1 + \theta_{ik}) \\ \ln(1 + \theta_{ik}) \\ \ln(1 + \varphi_i) \\ 0
\end{pmatrix} \quad \text{for} \quad i = 1, \ldots, q \quad \text{and} \quad k = 1, \ldots, 4,
\]

\[
\text{Ker} L = \left\{ \begin{pmatrix}
u_1 \\ u_2 \\ u_3 \\ u_4
\end{pmatrix} \in R^4, t \in [0, \omega] \right\}.
\]

Where \( Q \) is defined by
\[
QZ = \begin{pmatrix}
\int_0^\omega f(t) \, dt + \sum_{0 < k < \omega} a_k - \frac{1}{\omega} \int_0^\omega \int_0^t f(s) \, ds \, dt - \frac{\omega}{q} \sum_{k=1}^q a_k \\
\int_0^\omega g(t) \, dt + \sum_{0 < k < \omega} b_k - \frac{1}{\omega} \int_0^\omega \int_0^t g(s) \, ds \, dt - \frac{\omega}{q} \sum_{k=1}^q b_k \\
\int_0^\omega h(t) \, dt + \sum_{0 < k < \omega} c_k - \frac{1}{\omega} \int_0^\omega \int_0^t h(s) \, ds \, dt - \frac{\omega}{q} \sum_{k=1}^q c_k \\
\int_0^\omega j(t) \, dt + \sum_{0 < k < \omega} d_k - \frac{1}{\omega} \int_0^\omega \int_0^t j(s) \, ds \, dt - \frac{\omega}{q} \sum_{k=1}^q d_k
\end{pmatrix}
\]

Furthermore, \( K_p : \text{Im} L \rightarrow \text{Ker} P \cap \text{Dom} L \) is given by
\[
K_p Z = \begin{pmatrix}
\int_0^\omega f(t) \, dt + \sum_{0 < k < \omega} a_k - \frac{1}{\omega} \int_0^\omega \int_0^t f(s) \, ds \, dt - \frac{\omega}{q} \sum_{k=1}^q a_k \\
\int_0^\omega g(t) \, dt + \sum_{0 < k < \omega} b_k - \frac{1}{\omega} \int_0^\omega \int_0^t g(s) \, ds \, dt - \frac{\omega}{q} \sum_{k=1}^q b_k \\
\int_0^\omega h(t) \, dt + \sum_{0 < k < \omega} c_k - \frac{1}{\omega} \int_0^\omega \int_0^t h(s) \, ds \, dt - \frac{\omega}{q} \sum_{k=1}^q c_k \\
\int_0^\omega j(t) \, dt + \sum_{0 < k < \omega} d_k - \frac{1}{\omega} \int_0^\omega \int_0^t j(s) \, ds \, dt - \frac{\omega}{q} \sum_{k=1}^q d_k
\end{pmatrix}
\]

Thus,
\[
K_p (I - Q) N = \begin{pmatrix}
\int_0^\omega f_1(t) \, dt + \sum_{0 < k < \omega} \ln(1 + \theta_{ik}) \\
\int_0^\omega f_2(t) \, dt + \sum_{0 < k < \omega} \ln(1 + \theta_{ik}) \\
\int_0^\omega f_3(t) \, dt + \sum_{0 < k < \omega} \ln(1 + \varphi_i) \\
\int_0^\omega f_4(t) \, dt
\end{pmatrix}
\]

In order to apply the Lemma 3.1, we also need to find an appropriate open and bounded subset \( \Omega \). Corresponding to the operator equation \( Lu = \lambda Nu \), here, \( \lambda \in (0, 1) \), \( u = (u_1, u_2, u_3, u_4)^T \), we can get
\[
\begin{align*}
\dot{u}_1(t) &= \lambda f_1(t), \quad \dot{u}_2(t) = \lambda f_2(t), \\
\dot{u}_3(t) &= \lambda f_3(t), \quad \dot{u}_4(t) = \lambda f_4(t),
\end{align*}
\]

Suppose \( u = (u_1, u_2, u_3, u_4)^T \) is a \( \omega \)– periodic solution to (3.2). By integrating over \([0, \omega]\),
\[
\begin{align*}
\frac{a_1 - d_1}{\omega} + \frac{1}{\omega} \ln \left( \prod_{k=1}^q (1 + \theta_{ik}) \right) &= \frac{1}{\omega} \int_0^\omega \left( r_1(t) e^{\alpha(t)} - d_1(t) e^{\alpha(t) - \alpha(t)} \right) \, dt, \\
\frac{a_2 - d_2}{\omega} + \frac{1}{\omega} \ln \left( \prod_{k=1}^q (1 + \theta_{ik}) \right) &= \frac{1}{\omega} \int_0^\omega \left( r_2(t) e^{\alpha(t)} + k(t) e^{\alpha(t)} - d_2(t) e^{\alpha(t) - \alpha(t)} \right) \, dt, \\
\frac{a_3 - d_3}{\omega} + \frac{1}{\omega} \ln \left( \prod_{k=1}^q (1 + \varphi_i) \right) &= \frac{1}{\omega} \int_0^\omega \left( c(t) e^{\beta(t) + u(t) - u(t)} - q_3(t) e^{\alpha(t)} \right) \, dt \\
&\quad - \frac{1}{\omega} \int_0^\omega c(t) e^{\beta(t) - \alpha(t)} e^{\alpha(t)} \, dt, \\
\frac{a_4 - d_4}{\omega} + \frac{1}{\omega} \ln \left( \prod_{k=1}^q (1 + \varphi_i) \right) &= \frac{1}{\omega} \int_0^\omega \left( q_2(t) e^{\alpha(t)} \right) \, dt \\
&\quad - \frac{1}{\omega} \int_0^\omega c(t) e^{\beta(t) - \alpha(t)} e^{\alpha(t)} \, dt.
\end{align*}
\]

According to (3.2) and (3.3), we have...
\[ 0 \leq u_i (\xi_i) + \int_0^\omega |L_i (t)| dt + \ln \left[ \prod_{k=1}^q (1 + \theta_{ik}) \right] \]
\[ u_i (t) \leq u_i (\xi_i) + \int_0^\omega |L_i (t)| dt + \ln \left[ \prod_{k=1}^q (1 + \theta_{ik}) \right] \]
\[ a \omega + \ln \left[ \prod_{k=1}^q (1 + \theta_{ik}) \right] \]
\[ u_i (\xi_i) \leq \ln \frac{a \omega + \ln \left[ \prod_{k=1}^q (1 + \theta_{ik}) \right]}{p \omega} \quad (i = 1, 2), \]
\[ \int_0^\omega e^{\phi(t)} dt \leq \int_0^\omega e^{\omega(t)} dt \]
According to the fourth equation of (3.3), we have
\[ \int_0^\omega q_2 (t) e^{2\omega(t)} dt = \int_0^\omega c(t) e^{e^{-t} + M_2 + u_2(t)} dt \]
Due to
\[ \phi(t) \leq \omega(t) \]
From (3.11) and (3.12), we have
\[ \int_0^\omega e^{\phi(t)} dt \leq \frac{a \omega}{q_2} \]
According to (3.7) and (3.14), we get
\[ \int_0^\omega |L_i (t)| dt < 2 \int_0^\omega q_2 (t) e^{\omega(t)} dt \]
\[ u_4 (t) \leq u_4 (\xi_4) + \int_0^\omega |L_i (t)| dt \]
\[ \leq \ln \frac{c^M e^{e^{-t} + M_2 + u_4(t)}}{q_2^2} \]
According to the third equation of (3.3), we have
\[ \int_0^\omega c(t) e^{e(t) + u_1(t) - u_1(t)} dt \geq \omega - \ln \left[ \prod_{k=1}^q (1 + \phi_{ik}) \right] \]
Due to \( u_4 (t) < M_2, u_4 (t) < M_4 \), we have
\[ c^M e^{M_2 + M_4} \int_0^\omega e^{\omega(t)} dt \geq \omega - \ln \left[ \prod_{k=1}^q (1 + \phi_{ik}) \right] \]
and
\[ u_i (\xi_i) \leq \ln \frac{c^M e^{M_2 + M_4} \omega}{\omega - \ln \left[ \prod_{k=1}^q (1 + \phi_{ik}) \right]}, \]
From (3.11) we have
\[
2^2 \leq \sum_{i=1}^{q} (1 + \theta_i) n_i \leq \Delta M_3, \tag{3.16}
\]
From the first equation of (3.3), we have
\[
\int_0^\omega r_i(t) e^{\mu_i(t)} dt \geq \int_0^\omega r_i(t) e^{\eta_i(t)} dt \geq a_i - d_i \omega + \ln \left[ \sum_{i=1}^{q} (1 + \theta_i) \right],
\]
So,
\[
u_i(\eta_i) \geq \ln \frac{a_i - d_i \omega + \ln \left[ \sum_{i=1}^{q} (1 + \theta_i) \right]}{r_i \omega}, \tag{3.17}
\]
From the second equation of (3.3), we have
\[
\int_0^\omega r_i(t) e^{\nu_i(t)} dt \geq \int_0^\omega r_i(t) e^{\eta_i(t)} dt \geq a_i - d_i \omega + \ln \left[ \sum_{i=1}^{q} (1 + \theta_i) \right] - k^M e^{M \omega} \omega_i
\]
\[
u_i(\eta_i) \geq \ln \frac{a_i - d_i \omega + \ln \left[ \sum_{i=1}^{q} (1 + \theta_i) \right] - k^M e^{M \omega} \omega_i}{r_i \omega}, \tag{3.18}
\]
From (3.11) we have
\[
q_d e^{\eta_i(\omega)} \int_0^\omega q_d(t) e^{\eta_i(t)} dt \geq \int_0^\omega q_d(t) e^{\eta_i(t)} dt \geq c_l e^{c_l M_{t+M_2+M_1}} \int_0^\omega e^{\eta_i(t)} dt,
\]
\[
u_i(\eta_i) \geq \ln \frac{c_l e^{c_l M_{t+M_2+M_1}}}{q_d}, \tag{3.19}
\]
According to the third equation of (3.3), we have
\[
\left( c^M e^{m_1 + m_2} - c^M e^{-r^M_{t+M_2+M_1}} \right) \int_0^\omega e^{\eta_i(t)} dt \leq d + q^M \omega^2 e^{M_2 + M_2} + \ln \left[ \sum_{i=1}^{q} (1 + \theta_i) \right],
\]
\[
u_i(\eta_i) \geq \ln \frac{c^M e^{m_1 + m_2} - c^M e^{-r^M_{t+M_2+M_1}}}{q_d}, \tag{3.20}
\]
Thus, we have
\[
\sup_{i \in \{1,3,4\}} \nu_i(\omega) \leq \max \left\{ \left| M_1 \right|, \left| M_2 \right|, \left| M_3 \right|, \left| M_4 \right|, \left| m_1 \right|, \left| m_2 \right|, \left| m_3 \right|, \left| m_4 \right| \right\} \Delta D_2 \quad (i = 1,2,3,4),
\]
Denote \( M = \max \{D_1, D_2, D_3, D_4, D_0 \} \), where \( D_0 \) may be taken sufficiently large such that each solution to Equations (3.21)
\[
\left\{ \begin{array}{l}
\hat{\nu}_1(\omega) - \hat{a}_1 e^{\eta_1} + \hat{d}_1 e^{\eta_1} = \ln \left[ \sum_{i=1}^{q} (1 + \theta_i) \right], \\
\hat{a}_1 - \hat{a}_1 e^{\eta_1} - \hat{k}_1 e^{\eta_1} = \ln \left[ \sum_{i=1}^{q} (1 + \theta_i) \right], \\
\hat{a}_2 - \hat{a}_2 e^{\eta_2} - \hat{k}_2 e^{\eta_2} = \ln \left[ \sum_{i=1}^{q} (1 + \theta_i) \right], \\
\hat{a}_3 - \hat{a}_3 e^{\eta_3} - \hat{k}_3 e^{\eta_3} = \ln \left[ \sum_{i=1}^{q} (1 + \theta_i) \right], \\
\hat{a}_4 - \hat{a}_4 e^{\eta_4} - \hat{k}_4 e^{\eta_4} = \ln \left[ \sum_{i=1}^{q} (1 + \theta_i) \right], \\
\end{array} \right. \tag{3.21}
\]
satisfies \[ \left( u_1, u_2, u_3, u_4 \right)^T < D_0, \text{ then } \|u\| < M. \]

Denote \( \phi : Dom L \times [0, 1] \rightarrow X \) as the form
\[
\phi(u_1, u_2, u_3, u_4, \mu) = \begin{cases}
\frac{a_i - d_i - q_i e^{\alpha_i}}{\omega} + \frac{1}{\omega} \ln \left( \prod_{k=1}^{q} \left( 1 + \theta_{ik} \right) \right) \\
\frac{a_j - d_j - q_j e^{\alpha_j}}{\omega} + \frac{1}{\omega} \ln \left( \prod_{k=1}^{q} \left( 1 + \theta_{jk} \right) \right) \\
- \frac{c(t - \tau)}{e^{\alpha + q_i}} - q_i e^{\alpha} \\
0
\end{cases},
\]

Where \( \mu \in [0, 1] \) is a parameter. With the mapping \( \phi \), we have \( \phi(u_1, u_2, u_3, u_4, \mu) \neq 0 \) for \( (u_1, u_2, u_3, u_4)^T \in \mathcal{C} \Omega \cap Ker L \). So we know that \( \|u\| < M \).

Obviously, the algebraic Equation (3.22) has a unique solution \((u_1, u_2, u_3, u_4)^T\).

\[
\begin{aligned}
\frac{a_i - d_i - q_i e^{\alpha_i}}{\omega} + \frac{1}{\omega} \ln \left( \prod_{k=1}^{q} \left( 1 + \theta_{ik} \right) \right) &= 0, \\
\frac{a_j - d_j - q_j e^{\alpha_j}}{\omega} + \frac{1}{\omega} \ln \left( \prod_{k=1}^{q} \left( 1 + \theta_{jk} \right) \right) &= 0, \\
- \frac{c(t - \tau)}{e^{\alpha + q_i}} - q_i e^{\alpha} &= 0, \\
0 &= 0.
\end{aligned}
\]

From the coincidence degree theory, we can obtain
\[
\text{deg} (\mathcal{J}QNu, \Omega \cap Ker L, 0) = \text{deg} (\phi(u_1, u_2, u_3, u_4, \mu), \Omega \cap Ker L, 0) = 1.
\]

\section*{4. Numerical Analysis}

In this paper, we have focused on the dynamics complexity of a stage-structured system with diffusion and impulsive effects. By using the method of coincidence degree, we obtain the sufficient condition for the existence of at least one positive \( \omega \) - periodic solution. In this section, we give the numerical results.

\[
\begin{aligned}
\dot{x}_1(t) &= x_1(t)[3 - 1.6 \cos(\omega t) - 1.5 x_1(t)] \\
&\quad + (2 - \cos(\omega t)) x_2(t) - x_1(t), \\
\dot{x}_2(t) &= x_2(t)[5.2 - 3.2 \sin(\omega t) - 2.4 x_2(t)] \\
&\quad - (3 - 2.5 \sin(\omega t)) x_3(t) - x_2(t), \\
\dot{y}_1(t) &= x_1(t) - x_2(t) \\
&\quad - (1.2 - \sin(\omega t)) x_2(t) y_2(t) - \\
&\quad - 0.2 x_2(t) - (1 - 0.5 \cos(\omega t)) y_2(t), \\
\dot{y}_2(t) &= (1 - 0.75 \cos(\omega t)) y_2(t) + \\
&\quad (1 - 1.2 \sin(\omega t)) y_2(t) x_3(t) - x_2(t) y_2(t), \\
x_1(t') &= (1 + \theta_1) x_1(t), \\
x_2(t') &= (1 + \theta_2) x_2(t), \\
y_1(t') &= (1 + \varphi_1) y_1(t), \\
y_2(t') &= y_2(t).
\end{aligned}
\]

(4.1)

Numerical analysis indicates that the complex dynamic behavior of system (1.1) depends on the values of impulsive perturbations \( \theta_i, \theta_j \) \((i = 1, 2) \) in model (1.1). Our theoretical results are confirmed by numerical simulations. We can see that the dynamic behavior of the system (4.1) has obviously varied as the impulse value changing. Let \( \theta_1 = 0.001, \theta_2 = 0.002, \varphi = 0.003 \), it is easily proved that the system (4.1) satisfies all the conditions of Theorem 3.1, that mean the system (4.1) has at least one positive periodic solution (Figure 1). As impulses increase, the periodic oscillation of system (4.1) will be destroyed (Figure 2).

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\section*{6. Conclusions}

There is much previous work reported on non-autonomous stage-structured system or diffusive system. This motivates us to study a non-autonomous stage-structured predator-prey system with impulsive effects. As pointed out in Section 1, we built system (1.1). In Section 2, we give some preliminaries. In Section 3, by using the method of coincidence degree, we obtain the sufficient condition for the existence of at least one positive periodic solution. In Section 4, we give the numerical simulations on the dynamic behaviors of the system through two examples. But we did not discuss the global stability of the periodic solutions periodic solution of system (1.1). We
Figure 1. Dynamic behavior of the system (4.1) with initial values $(1.2, 1.2, 0.8, 0.6), \tau = 0.1$ and impulsive perturbations $\theta_1 = 0.001, \theta_2 = 0.002, \varphi = 0.003$.

Figure 2. Dynamic behavior of the system (4.1) with initial values $(1.2, 1.2, 0.8, 0.6), \tau = 0.1$ and impulsive perturbations $\theta_1 = 0.1, \theta_2 = 0.2, \varphi = 0.3$. 

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leave these aspects for future research.

7. References


