

Some Properties of the g -Good-Neighbor (g -Extra) Diagnosability of a Multiprocessor System

Yunxia Ren, Shiyong Wang

School of Mathematics and Information Science, Henan Normal University, Xinxiang, China
Email: wangshiyong@htu.edu.cn, shiyong@sxu.edu.cn

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Abstract

Diagnosability of a multiprocessor system is one important study topic. In 2012, Peng *et al.* proposed a measure for fault tolerance of the system, which is called the g -good-neighbor diagnosability that restrains every fault-free node containing at least g fault-free neighbors. In 2015, Zhang *et al.* proposed a measure for fault diagnosis of the system, namely, g -extra diagnosability, which restrains that every fault-free component has at least $(g + 1)$ fault-free nodes. In this paper, we obtain some properties of the g -good-neighbor (g -extra) diagnosability of the system and give the g -good-neighbor (g -extra) diagnosability of some graphs under the PMC model and MM^* model.

Keywords

Interconnection Network, Combinatorics, Diagnosability

1. Introduction

Many multiprocessor systems take interconnection networks (networks for short) as underlying topologies and a network is usually represented by a graph where nodes represent processors and links represent communication links between processors. We use graphs and networks interchangeably. For a multiprocessor system, study on the topological properties of its network is important. Furthermore, some processors may fail in the system, so processor fault identification plays an important role for reliable computing. The first step to deal with faults is to identify the faulty processors from the fault-free ones. The identification process is called the diagnosis of the system. A system is said to be t -diagnosable if all faulty processors can be identified without replacement, provided that the number of faults presented does not exceed t . The diagnosability $t(G)$ of a system G is the

maximum value of t such that G is t -diagnosable [1]-[3]. For a t -diagnosable system, Dahbura and Masson [1] proposed an algorithm with time complex $O(n^{2.5})$, which can effectively identify the set of faulty processors.

Several diagnosis models were proposed to identify the faulty processors. One major approach is the Preparata, Metze, and Chien's (PMC) diagnosis model introduced by Preparata *et al.* [4]. The diagnosis of the system is achieved through two linked processors testing each other. Another major approach, namely the comparison diagnosis model (MM model), was proposed by Maeng and Malek [5]. In the MM model, to diagnose a system, a node sends the same task to two of its neighbors, and then compares their responses. In 2005, Lai *et al.* [3] introduced a restricted diagnosability of multiprocessor systems called conditional diagnosability. They consider the situation that any fault set cannot contain all the neighbors of any vertex in a system. In 2012, Peng *et al.* [6] proposed a measure for fault diagnosis of the system, namely, g -good-neighbor diagnosability (which is also called g -good-neighbor conditional diagnosability), which requires that every fault-free node has at least g fault-free neighbors. In [6], they studied the g -good-neighbor diagnosability of the n -dimensional hypercube under the PMC model. In [7], Wang and Han studied the g -good-neighbor diagnosability of the n -dimensional hypercube under MM* model. Yuan *et al.* [8] and [9] studied the g -good-neighbor diagnosability of the k -ary n -cube ($k \geq 3$) under the PMC model and MM* model. The Cayley graph CT_n generated by the transposition tree Γ_n has recently received considerable attention. In [10] [11], Wang *et al.* studied the g -good-neighbor diagnosability of CT_n under the PMC model and MM* model for $g = 1, 2$. In 2015, Zhang *et al.* [12] proposed a new measure for fault diagnosis of the system, namely, g -extra diagnosability, which restrains that every fault-free component has at least $(g + 1)$ fault-free nodes. In [12], they studied the g -extra diagnosability of the n -dimensional hypercube under the PMC model and MM* model. The n -dimensional bubble-sort star graph BS_n has many good properties. In 2016, Wang *et al.* [13] studied the 2-extra diagnosability of BS_n under the PMC model and MM* model. In this paper, we obtain some properties of the g -good-neighbor (g -extra) diagnosability of the system and give the g -good-neighbor (g -extra) diagnosability of some graphs under the PMC model and MM* model.

2. Preliminaries

In this section, some definitions and notations needed for our discussion, some results, the PMC model and the MM* model are introduced.

2.1. Diagnosability

Under the PMC model, to diagnose a system G , two adjacent nodes in G are capable to perform tests on each other. For two adjacent nodes u and v in $V(G)$, the test performed by u on v is represented by the ordered pair (u, v) . The outcome of a test (u, v) is 1 (resp. 0) if u evaluate v as faulty (resp. fault-free). In the PMC model, we usually assume that the testing result is reliable (resp. unreliable) if the node u is fault-free (resp. faulty). A test assignment T for a system G is a collection of tests for every adjacent pair of vertices. It can be modeled as a directed testing graph $T = (V(G), L)$, where $(u, v) \in L$ implies that u and v are adjacent in G . The collection of all test results for a test assignment T is called a syndrome. Formally, a syndrome is a function $\sigma : L \mapsto \{0, 1\}$.

The set of all faulty processors in the system is called a faulty set. This can be any subset of $V(G)$. For a given syndrome σ , a subset of vertices $F \subseteq V(G)$ is said to be consistent with σ if syndrome σ can be produced from the situation that, for any $(u, v) \in L$ such that $u \in V \setminus F$, $\sigma(u, v) = 1$ if and only if $v \in F$. This means that F is a possible set of faulty processors. Since a test outcome produced by a faulty processor is unreliable, a given set F of faulty vertices may produce a lot of different syndromes. On the other hand, different fault sets may produce the same syndrome. Let $\sigma(F)$ denote the set of all syndromes which F is consistent with.

Under the PMC model, two distinct sets F_1 and F_2 in $V(G)$ are said to be indistinguishable if $\sigma(F_1) \cap \sigma(F_2) \neq \emptyset$, otherwise, F_1 and F_2 are said to be distinguishable. Besides, we say (F_1, F_2) is an indistinguishable pair if $\sigma(F_1) \cap \sigma(F_2) \neq \emptyset$; else, (F_1, F_2) is a distinguishable pair.

Using the MM model [14], the diagnosis is carried out by sending the same testing task to a pair of processors and comparing their responses. Under the MM model, we always assume the output of a comparison performed by a faulty processor is unreliable. The comparison scheme of a system $G = (V, E)$ is modeled as a multigraph, denoted by $M = (V(G), L)$, where L is the labeled-edge set. A labeled edge $(u, v)_w \in L$ represents a comparison

in which two vertices u and v are compared by a vertex w , which implies $uw, vw \in E(G)$, where w is called a comparator for processors u and v . The collection of all comparison results in $M = (V(G), L)$ is called the syndrome, denoted by σ^* , of the diagnosis. If the comparison $(u, v)_w$ disagrees, then $\sigma^*((u, v)_w) = 1$, otherwise, $\sigma^*((u, v)_w) = 0$. Hence, a syndrome is a function from L to $\{0, 1\}$. The MM^* model is a special case of the MM model. In the MM^* model, all comparisons of G are in the comparison scheme of G , i.e., if $uw, vw \in E(G)$, then $(u, v)_w \in L$. Since the comparator processor w itself might be faulty, a comparison result $\sigma^*((u, v)_w) = 0$ implies that if the comparator processor w is fault-free, then the compared processors u and v are fault-free; on the other hand, a comparison result $\sigma^*((u, v)_w) = 1$ implies that at least one of three processors involved in the comparison must be faulty. The possible comparison results for the different conditions of the three processors involved in a comparison are shown in **Table 1**.

For a given syndrome σ^* , a subset of vertices $F \subseteq V(G)$ is said to be consistent with σ^* if syndrome σ can be produced from the situation that, for any $(u, v)_w \in L$ such that $w \in V \setminus F$, $\sigma(u, v)_w = 1$ if and only if $u \in F$ or $v \in F$. Similar to the PMC model, we can define two distinct sets F_1 and F_2 in $V(G)$ are indistinguishable (resp. distinguishable) under the MM^* model.

A system $G = (V, E)$ is g -good-neighbor t -diagnosable if F_1 and F_2 are distinguishable, for each distinct pair of g -good-neighbor faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$. The g -good-neighbor diagnosability $t_g(G)$ of G is the maximum value of t such that G is g -good-neighbor t -diagnosable.

Proposition 1. ([5]) For any given system G , $t_g(G) \leq t_{g'}(G)$ if $g \leq g'$.

In a system $G = (V, E)$, a faulty set $F \subseteq V$ is called a conditional faulty set if it does not contain all of neighbors of any vertex in G . A system G is conditional t -diagnosable if every two distinct conditional faulty subsets $F_1, F_2 \subseteq V$ with $|F_1|, |F_2| \leq t$, are distinguishable. The conditional diagnosability $t_c(G)$ of G is the maximum number of t such that G is conditional t -diagnosable. By [15], $t_c(G) \geq t(G)$.

Theorem 2. [10] For a system $G = (V, E)$, $t(G) = t_0(G) \leq t_1(G) \leq t_c(G)$.

In [10], Wang *et al.* proved that the 1-good-neighbor diagnosability of the Bubble-sort graph B_n under the PMC model is $2n - 3$ for $n \geq 4$. In [16], Zhou *et al.* proved the conditional diagnosability of B_n is $4n - 11$ for $n \geq 4$ under the PMC model. Therefore, $t_1(G) < t_c(G)$ when $n \geq 5$ and $t_1(G) = t_c(G)$ when $n = 4$.

In a system $G = (V, E)$, a faulty set $F \subseteq V$ is called a g -extra faulty set if every component of $G - F$ has more than g nodes. G is g -extra t -diagnosable if and only if for each pair of distinct faulty g -extra vertex subsets $F_1, F_2 \subseteq V(G)$ such that $|F_i| \leq t$, F_1 and F_2 are distinguishable. The g -extra diagnosability of G , denoted by $\tilde{t}_g(G)$, is the maximum value of t such that G is g -extra t -diagnosable.

Proposition 3. [13] For any given system G , $\tilde{t}_g(G) \leq \tilde{t}_{g'}(G)$ if $g \leq g'$.

Theorem 4. [13] For a system $G = (V, E)$, $t(G) = \tilde{t}_0(G) \leq \tilde{t}_g(G) \leq t_g(G)$.

Theorem 5. [13] For a system $G = (V, E)$, $\tilde{t}_1(G) = t_1(G)$.

2.2. Connectivity

A multiprocessor system is modeled as an undirected simple graph $G = (V, E)$, whose vertices (nodes) represent processors and edges (links) represent communication links. Given a nonempty vertex subset V' of V , the induced subgraph by V' in G , denoted by $G[V']$, is a graph, whose vertex set is V' and the edge set is the set of all the edges of G with both endpoints in V' . The degree $d_G(v)$ of a vertex v is the number of edges incident with v . The minimum degree of a vertex in G is denoted by $\delta(G)$. For any vertex v , we define the neighborhood $N_G(v)$ of v in G to be the set of vertices adjacent to v . u is called a neighbor vertex or a neighbor of v for $u \in N_G(v)$. Let $S \subseteq V$. We use $N_G(S)$ to denote the set $\bigcup_{v \in S} N_G(v) \setminus S$. For neighborhoods and degrees, we will usually omit the subscript for the graph when no confusion arises. Certain types

Table 1. The possible comparison results for different conditions of three processors in a comparison.

Comparator processor	Of the two compared processor	
	None is faulty	At least one is faulty
Fault-free	0	1
Faulty	0 or 1	0 or 1

of graphs play prominent roles in graph theory. A graph G is said to be k -regular if for any vertex v , $d_G(v) = k$. A complete graph is a simple graph in which any two vertices are adjacent. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left when G is complete. Let F_1 and F_2 be two distinct subsets of V , and let the symmetric difference $F_1 \Delta F_2 = (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$. For graph-theoretical terminology and notation not defined here we follow [17].

Let $G = (V, E)$. A fault set $F \subseteq V$ is called a g -good-neighbor faulty set if $|N(v) \cap (V \setminus F)| \geq g$ for every vertex v in $V \setminus F$. A g -good-neighbor cut of G is a g -good-neighbor faulty set F such that $G - F$ is disconnected. The minimum cardinality of g -good-neighbor cuts is said to be the g -good-neighbor connectivity of G , denoted by $\kappa^{(g)}(G)$. A fault set $F \subseteq V$ is called a g -extra faulty set if every component of $G - F$ has at least $(g + 1)$ vertices. A g -extra cut of G is a g -extra faulty set F such that $G - F$ is disconnected. The minimum cardinality of g -extra cuts is said to be the g -extra connectivity of G , denoted by $\tilde{\kappa}^{(g)}(G)$.

Proposition 6. *Let G be a connected graph. Then $\tilde{\kappa}^{(g)}(G) \leq \kappa^{(g)}(G)$.*

Proof. Let F be a minimum g -good-neighbor cut of G . Then $\delta(G - F) \geq g$. Therefore, every component of $G - F$ has at least $(g + 1)$ vertices and hence F is a g -extra cut of G . By the definition of the g -extra connectivity, $\tilde{\kappa}^{(g)}(G) \leq |F| = \kappa^{(g)}(G)$. \square

Proposition 7. *Let G be a connected graph. Then $\kappa^{(1)}(G) = \tilde{\kappa}^{(1)}(G)$.*

Proof. By Proposition 6, $\tilde{\kappa}^{(1)}(G) \leq \kappa^{(1)}(G)$. Let F be a minimum 1-extra cut of G . Then every component of $G - F$ has at least two vertices. Note that every component of $G - F$ is connected. Therefore, $\delta(G - F) \geq 1$ and hence F is a 1-good-neighbor cut of G . By the definition of the g -good-neighbor connectivity, $\kappa^{(1)}(G) \leq |F| = \tilde{\kappa}^{(1)}(G)$. Therefore, $\kappa^{(1)}(G) = \tilde{\kappa}^{(1)}(G)$. \square

3. Basic Results

Theorem 8. *Let a distinct pair of g -good-neighbor (g -extra) faulty subsets of V be S_1 and S_2 in a system $G = (V, E)$. If $S_1 \cup S_2 = V$, then (S_1, S_2) is indistinguishable under the PMC model and MM^* model.*

Proof. Consider two case as follows.

Case 1. MM^* model.

Consider a directed testing graph $M = (V(G), L)$ for the system $G = (V, E)$.

- 1). If $i, j, k \in S_1 \setminus S_2$ and $ik, jk \in E(G[S_1 \setminus S_2])$, then $(i, j)_k \in L$.
- 1'). If $i, j, k \in S_2 \setminus S_1$ and $ik, jk \in E(G[S_2 \setminus S_1])$, then $(i, j)_k \in L$.
- 1''). If $i, j, k \in S_2 \cap S_1$ and $ik, jk \in E(G[S_2 \cap S_1])$, then $(i, j)_k \in L$.
- 2). If $i, k \in S_1 \setminus S_2$, $j \in S_2$ and $j \notin S_1 \setminus S_2$, $ik \in E(G[S_1 \setminus S_2])$ and $jk \in E(G)$, then $(i, j)_k \in L$.
- 2'). If $i, k \in S_2 \setminus S_1$, $j \in S_1$ and $j \notin S_2 \setminus S_1$, $ik \in E(G[S_2 \setminus S_1])$ and $jk \in E(G)$, then $(i, j)_k \in L$.
- 2''). If $i, k \in S_2 \cap S_1$, $j \in F_1 \Delta F_2$, $ik \in E(G[S_2 \cap S_1])$ and $jk \in E(G)$, then $(i, j)_k \in L$.
- 3). If $k \in S_1 \setminus S_2$, $i, j \in S_2$ and $i, j \notin S_1 \setminus S_2$, $ik, jk \in E(G)$, then $(i, j)_k \in L$.
- 3'). If $k \in S_2 \setminus S_1$, $i, j \in S_1$ and $i, j \notin S_2 \setminus S_1$, $ik, jk \in E(G)$, then $(i, j)_k \in L$.
- 4). If $k \in S_2 \cap S_1$, $i, j \in F_1 \Delta F_2$, $ik, jk \in E(G)$, then $(i, j)_k \in L$.

Consider a syndrome σ for each $(i, j)_k \in L$.

- 1). If $i, j, k \in S_1 \setminus S_2$ and $ik, jk \in E(G[S_1 \setminus S_2])$, and $(i, j)_k \in L$ then, $\sigma((i, j)_k) = 0$.
- 1'). If $i, j, k \in S_2 \setminus S_1$ and $ik, jk \in E(G[S_2 \setminus S_1])$, and $(i, j)_k \in L$ then, $\sigma((i, j)_k) = 0$.
- 1''). If $i, j, k \in S_2 \cap S_1$ and $ik, jk \in E(G[S_2 \cap S_1])$, and $(i, j)_k \in L$ then, $\sigma((i, j)_k) = 0$.
- 2). If $i, k \in S_1 \setminus S_2$, $j \in S_2$ and $j \notin S_1 \setminus S_2$, $ik \in E(G[S_1 \setminus S_2])$ and $jk \in E(G)$, and $(i, j)_k \in L$, then $\sigma((i, j)_k) = 1$.
- 2'). If $i, k \in S_2 \setminus S_1$, $j \in S_1$ and $j \notin S_2 \setminus S_1$, $ik \in E(G[S_2 \setminus S_1])$ and $jk \in E(G)$, and $(i, j)_k \in L$, then $\sigma((i, j)_k) = 1$.
- 2''). If $i, k \in S_2 \cap S_1$, $j \in F_1 \Delta F_2$, $ik \in E(G[S_2 \cap S_1])$ and $jk \in E(G)$, and $(i, j)_k \in L$, then $\sigma((i, j)_k) = 1$.
- 3). If $k \in S_1 \setminus S_2$, $i, j \in S_2$ and $i, j \notin S_1 \setminus S_2$, $ik, jk \in E(G)$, and $(i, j)_k \in L$, then $\sigma((i, j)_k) = 1$.
- 3'). If $k \in S_2 \setminus S_1$, $i, j \in S_1$ and $i, j \notin S_2 \setminus S_1$, $ik, jk \in E(G)$, and $(i, j)_k \in L$, then $\sigma((i, j)_k) = 1$.
- 4). If $k \in S_2 \cap S_1$, $i, j \in F_1 \Delta F_2$, $ik, jk \in E(G)$, and $(i, j)_k \in L$, then $\sigma((i, j)_k) = 1$.

Suppose that S_2 is a g -good-neighbor faulty subset of V . 1) Let $i, j, k \in S_1 \setminus S_2$ and $ik, jk \in E(G[S_1 \setminus S_2])$. Since i, j and k are fault-free, and $(i, j)_k \in L$, we have $\sigma((i, j)_k) = 0$. 1'). Let $i, j, k \in S_2 \setminus S_1$ and $ik, jk \in E(G[S_2 \setminus S_1])$. Since i, j and k are fault, and $(i, j)_k \in L$, we set $\sigma((i, j)_k) = 0$. 1''). Let $i, j, k \in S_2 \cap S_1$ and $ik, jk \in E(G[S_2 \cap S_1])$. Since i, j and k are fault, and $(i, j)_k \in L$, we set $\sigma((i, j)_k) = 0$. 2) Let $i, k \in S_1 \setminus S_2$, $j \in S_2$ and $j \notin S_1 \setminus S_2$, $ik \in E(G[S_1 \setminus S_2])$, $jk \in E(G)$, and $(i, j)_k \in L$. Since i and k are fault-free and j is fault, and $(i, j)_k \in L$, we have $\sigma((i, j)_k) = 1$. 2'). Let $i, k \in S_2 \setminus S_1$, $j \in S_1$ and $j \notin S_2 \setminus S_1$, $ik \in E(G[S_2 \setminus S_1])$, $jk \in E(G)$, and $(i, j)_k \in L$. Since i and k are fault, j is fault-free, and $(i, j)_k \in L$, we set $\sigma((i, j)_k) = 1$. 2''). Let $i, k \in S_2 \cap S_1$, $j \in F_1 \Delta F_2$, $ik \in E(G[S_2 \cap S_1])$, $jk \in E(G)$, and $(i, j)_k \in L$. Suppose that $j \in S_2 \setminus S_1$. Since i, k and j are fault and $(i, j)_k \in L$, we set $\sigma((i, j)_k) = 1$. Suppose that $j \in S_1 \setminus S_2$. Since i and k are fault, j is fault-free, and $(i, j)_k \in L$, we set $\sigma((i, j)_k) = 1$. 3) Let $k \in S_1 \setminus S_2$, $i, j \in S_2$ and $i, j \notin S_1 \setminus S_2$, $ik, jk \in E(G)$, and $(i, j)_k \in L$. Since k is fault-free, i and j are fault, and $(i, j)_k \in L$, we have $\sigma((i, j)_k) = 1$. 3'). Let $k \in S_2 \setminus S_1$, $i, j \in S_1$ and $i, j \notin S_2 \setminus S_1$, $ik, jk \in E(G)$, and $(i, j)_k \in L$. Since k is fault, i and j are fault-free, and $(i, j)_k \in L$, we set $\sigma((i, j)_k) = 1$. 4) Let $k \in S_2 \cap S_1$, $i, j \in F_1 \Delta F_2$, $ik, jk \in E(G)$, and $(i, j)_k \in L$. Since k is fault and $(i, j)_k \in L$, we set $\sigma((i, j)_k) = 1$. Therefore, $\sigma \in \sigma(S_2)$. Suppose that S_1 is a g -good-neighbor faulty subset of V . Similar to that of above paragraph, we have that $\sigma \in \sigma(S_1)$. Thus, the above syndrome belongs to $\sigma(F_1) \cap \sigma(F_2)$. Therefore, S_1 and S_2 are indistinguishable.

Case 2. PMC model.

Consider a directed testing graph $T = (V(G), L)$ for the system $G = (V, E)$.

1) If $i, j \in S_1 \setminus S_2$ and $ij \in E(G[S_1 \setminus S_2])$, then $(i, j) \in L$.

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2) If $i \in S_1 \setminus S_2$, $j \in S_2 \setminus S_1$ and $ij \in E(G)$, then $(i, j) \in L$.

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3) If $i \in S_1 \setminus S_2$, $j \in S_2 \cap S_1$ and $ij \in E(G)$, then $(i, j) \in L$.

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4) If $i \in S_2 \setminus S_1$, $j \in S_2 \cap S_1$ and $ij \in E(G)$, then $(i, j) \in L$.

4') If $i \in S_2 \cap S_1$, $j \in S_2 \setminus S_1$, and $ij \in E(G)$, then $(i, j) \in L$.

5) If $i, j \in S_2 \cap S_1$ and $ij \in E(G)$, then $(i, j) \in L$.

Consider a syndrome σ for each $(i, j) \in L$.

1) If $i, j \in S_1 \setminus S_2$ and $ij \in E(G[S_1 \setminus S_2])$, then $(i, j) \in L$ and $\sigma((i, j)) = 0$.

1') If $i, j \in S_2 \setminus S_1$ and $ij \in E(G[S_2 \setminus S_1])$, then $(i, j) \in L$ and $\sigma((i, j)) = 0$.

2) If $i \in S_1 \setminus S_2$, $j \in S_2 \setminus S_1$ and $ij \in E(G)$, then $(i, j) \in L$ and $\sigma((i, j)) = 1$.

2') If $i \in S_2 \setminus S_1$, $j \in S_1 \setminus S_2$ and $ij \in E(G)$, then $(i, j) \in L$ and $\sigma((i, j)) = 1$.

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4') If $i \in S_2 \cap S_1$, $j \in S_2 \setminus S_1$, and $ij \in E(G)$, then $(i, j) \in L$ and $\sigma((i, j)) = 1$.

5) If $i, j \in S_2 \cap S_1$ and $ij \in E(G)$, then $(i, j) \in L$ and $\sigma((i, j)) = 1$.

Suppose that S_2 is a g -good-neighbor faulty subset of V . 1) Let $i, j \in S_1 \setminus S_2 = V \setminus S_2$ and $ij \in E(G[S_1 \setminus S_2])$. Since i and j are fault-free, and $(i, j) \in L$, we have $\sigma((i, j)) = 0$. 1'). Let $i, j \in S_2 \setminus S_1$ and $ij \in E(G[S_2 \setminus S_1])$. Since i and j are fault, and $(i, j) \in L$, we set $\sigma((i, j)) = 0$. 2) Let $i \in S_1 \setminus S_2$, $j \in S_2 \setminus S_1$ and $ij \in E(G)$. Since i is fault-free and j is fault, and $(i, j) \in L$, we have $\sigma((i, j)) = 1$. 2'). Let $i \in S_2 \setminus S_1$, $j \in S_1 \setminus S_2$ and $ij \in E(G)$. Since i is fault and j is fault-free, and $(i, j) \in L$, we set $\sigma((i, j)) = 1$. 3) Let $i \in S_1 \setminus S_2$, $j \in S_2 \cap S_1$ and $ij \in E(G)$. Since i is fault-free and j is fault, and $(i, j) \in L$, we have $\sigma((i, j)) = 1$. 3'). Let $i \in S_2 \cap S_1$, $j \in S_1 \setminus S_2$, and $ij \in E(G)$. Since i is fault and j is fault-free, and $(i, j) \in L$, we set $\sigma((i, j)) = 1$. 4) Let $i \in S_2 \setminus S_1$, $j \in S_2 \cap S_1$ and $ij \in E(G)$. Since i and j are fault, and $(i, j) \in L$, we set $\sigma((i, j)) = 1$. 4') Let $i \in S_2 \cap S_1$, $j \in S_2 \setminus S_1$, and $ij \in E(G)$. Since i and j are fault, and $(i, j) \in L$, we set $\sigma((i, j)) = 1$. 5) Let $i, j \in S_2 \cap S_1$ and $ij \in E(G)$. Since i and j are fault, and $(i, j) \in L$, we set $\sigma((i, j)) = 1$. Therefore, $\sigma \in \sigma(S_2)$. Suppose that S_1 is a g -good-neighbor faulty subsets of V . Similar to that of above paragraph, we have that $\sigma \in \sigma(S_1)$. Thus, the above syndrome belongs to $\sigma(F_1) \cap \sigma(F_2)$. Therefore, S_1 and S_2 are

indistinguishable. □

Corollary 1. *Let a distinct pair of g -good-neighbor (g -extra) faulty subsets of V be F_1 and F_2 in a system $G = (V, E)$. If $F_1 \cup F_2 = V$, and $|F_1| \leq m$, $|F_2| \leq m$, then the g -good-neighbor (g -extra) diagnosability of G is less than m , $t_g(G) \leq m-1$ ($\tilde{t}_g(G) \leq m-1$), under the PMC model and MM^* model.*

Proof. By Theorem 8, G is not g -good-neighbor (g -extra) m -diagnosable under PMC model and MM^* model. Hence, by the definition of g -good-neighbor (g -extra) diagnosability, we conclude that the g -good-neighbor (g -extra) diagnosability of G is less than m , i.e., $t_g(G) \leq m-1$ ($\tilde{t}_g(G) \leq m-1$). □

Corollary 2. *Let a system $G = (V, E)$ with v vertices be g -good-neighbor (g -extra) t -diagnosable. If there is a distinct pair of g -good-neighbor (g -extra) faulty subsets F_1 and F_2 of V such that $|F_1| \leq t$ and $|F_2| \leq t$, and $v = |F_1 \cup F_2|$, then $v \geq 2t+1$.*

Proof. Suppose, on the contrary, that $v \leq 2t$. Let a distinct pair of g -good-neighbor (g -extra) faulty subsets of V be F_1 and F_2 such that $|F_1| \leq t$ and $|F_2| \leq t$, and $v = |F_1 \cup F_2|$. By Corollary 1, G is not g -good-neighbor (g -extra) t -diagnosable, a contradiction. □

4. The g -Good-Neighbor (g -Extra) Diagnosability of Some Graphs under the PMC Model and the MM^* Model

In this section, we will give the g -good-neighbor (g -extra) diagnosability of some graphs under the PMC model and the MM^* model.

Theorem 9. ([8] [13]) *A system $G = (V, E)$ is g -good-neighbor (g -extra) t -diagnosable under the PMC model if and only if there is an edge $uv \in E$ with $u \in V \setminus (F_1 \cup F_2)$ and $v \in F_1 \Delta F_2$ for each distinct pair of g -good-neighbor (g -extra) faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$.*

Theorem 10. ([1] [8] [13]) *A system $G = (V, E)$ is g -good-neighbor (g -extra) t -diagnosable under the MM^* model if and only if for each distinct pair of g -good-neighbor (g -extra) faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$ satisfies one of the following conditions.*

- 1) There are two vertices $u, w \in V \setminus (F_1 \cup F_2)$ and there is a vertex $v \in F_1 \Delta F_2$ such that $uw \in E$ and $vw \in E$.
- 2) There are two vertices $u, v \in F_2 \setminus F_1$ and there is a vertex $w \in V \setminus (F_1 \cup F_2)$ such that $uw \in E$ and $vw \in E$.
- 3) There are two vertices $u, v \in F_2 \setminus F_1$ and there is a vertex $w \in V \setminus (F_1 \cup F_2)$ such that $uw \in E$ and $vw \in E$.

Theorem 11. *Let $0 \leq g \leq \lfloor \frac{n}{2} \rfloor - 1$. If $n \geq 3$, the g -good-neighbor (g -extra) diagnosability of the complete graph K_n under the PMC model, $t_g(K_n) = \lfloor \frac{n}{2} \rfloor - 1$ ($\tilde{t}_g(K_n) = \lfloor \frac{n}{2} \rfloor - 1$). If $n \geq 4$, the g -good-neighbor (g -extra) diagnosability of K_n under the MM^* model, $t_g(K_n) = \lfloor \frac{n}{2} \rfloor - 1$ ($\tilde{t}_g(K_n) = \lfloor \frac{n}{2} \rfloor - 1$).*

Proof. Let $n \geq 3$, $0 \leq g \leq \lfloor \frac{n}{2} \rfloor - 1$. Firstly, prove that $t_g(K_n) \leq \lfloor \frac{n}{2} \rfloor - 1$ ($\tilde{t}_g(K_n) \leq \lfloor \frac{n}{2} \rfloor - 1$). If

$X, Y \subseteq V(K_n)$ such that $|X| = |Y| = \lfloor \frac{n}{2} \rfloor$ and $X \cup Y = V(K_n)$. By Corollary 1, $t_g(K_n) \leq \lfloor \frac{n}{2} \rfloor - 1$

($\tilde{t}_g(K_n) \leq \lfloor \frac{n}{2} \rfloor - 1$) for $|X| \leq \lfloor \frac{n}{2} \rfloor - 1$ and $|Y| \leq \lfloor \frac{n}{2} \rfloor - 1$. Note that $K_n - X$ is connected and

$\delta(K_n[V(K_n - X)]) = n - |X| - 1 = \lfloor \frac{n}{2} \rfloor - 1 \geq g$. Therefore, $|V(K_n - X)| \geq g + 1$. Next, prove that

$t_g(K_n) \geq \lfloor \frac{n}{2} \rfloor - 1$ ($\tilde{t}_g(K_n) \geq \lfloor \frac{n}{2} \rfloor - 1$). By the definition of g -good-neighbor (g -extra) diagnosability, it is

sufficient to show that K_n is g -good-neighbor (g -extra) $(\lfloor \frac{n}{2} \rfloor - 1)$ -diagnosable. Consider two cases as follows.

Case 1. PMC model.

Let $n \geq 3$. By Theorem 9, to prove K_n is g -good-neighbor (g -extra) $\left(\left\lceil \frac{n}{2} \right\rceil - 1\right)$ -diagnosable, it is equivalent to prove that there is an edge $uv \in E(K_n)$ with $u \in V(K_n) \setminus (F_1 \cup F_2)$ and $v \in F_1 \Delta F_2$ for each distinct pair of g -good-neighbor (g -extra) faulty subsets F_1 and F_2 of $V(K_n)$ with $|F_1| \leq \left\lceil \frac{n}{2} \right\rceil - 1$ and $|F_2| \leq \left\lceil \frac{n}{2} \right\rceil - 1$.

Since $|F_1| \leq \left\lceil \frac{n}{2} \right\rceil - 1$ and $|F_2| \leq \left\lceil \frac{n}{2} \right\rceil - 1$, $V(K_n) \setminus (F_1 \cup F_2) \neq \emptyset$ holds. Thus, there is an $u \in V(K_n) \setminus (F_1 \cup F_2)$. Since g -good-neighbor (g -extra) faulty subsets F_1 and F_2 of $V(K_n)$ are distinct, without loss of generality, suppose that $F_2 \setminus F_1 \neq \emptyset$. Then there is a $v \in F_2 \setminus F_1 \subseteq F_1 \Delta F_2$. Since K_n is complete, uv is an edge between $V(K_n) \setminus (F_1 \cup F_2)$ and $F_1 \Delta F_2$. By Theorem 9, K_n is g -good-neighbor (g -extra) $\left(\left\lceil \frac{n}{2} \right\rceil - 1\right)$ -diagnosable.

Case 2. MM^* model.

Let $n \geq 4$. By the definition of g -good-neighbor (g -extra) diagnosability, it is sufficient to show that K_n is g -good-neighbor $\left(\left\lceil \frac{n}{2} \right\rceil - 1\right)$ -diagnosable. By Theorem 10, to prove K_n is g -good-neighbor (g -extra) $\left(\left\lceil \frac{n}{2} \right\rceil - 1\right)$ -diagnosable, it is equivalent to prove that for each distinct pair of g -good-neighbor (g -extra) faulty subsets F_1 and F_2 of $V(K_n)$ with $|F_1| \leq \left\lceil \frac{n}{2} \right\rceil - 1$ and $|F_2| \leq \left\lceil \frac{n}{2} \right\rceil - 1$ satisfies one of the following conditions. 1).

There are two vertices $u, w \in V(K_n) \setminus (F_1 \cup F_2)$ and there is a vertex $v \in F_1 \Delta F_2$ such that $uw \in E(K_n)$ and $vw \in E(K_n)$. 2). There are two vertices $u, v \in F_1 \setminus F_2$ and there is a vertex $w \in V(K_n) \setminus (F_1 \cup F_2)$ such that $uw \in E(K_n)$ and $vw \in E(K_n)$. 3). There are two vertices $u, v \in F_2 \setminus F_1$ and there is a vertex $w \in V(K_n) \setminus (F_1 \cup F_2)$ such that $uw \in E(K_n)$ and $vw \in E(K_n)$.

Since $|F_1| \leq \left\lceil \frac{n}{2} \right\rceil - 1$ and $|F_2| \leq \left\lceil \frac{n}{2} \right\rceil - 1$, $V(K_n) \setminus (F_1 \cup F_2) \neq \emptyset$. Thus, there is $u, w \in V(K_n) \setminus (F_1 \cup F_2)$ when n is even. Since g -good-neighbor (g -extra) faulty subsets F_1 and F_2 of $V(K_n)$ are distinct, without loss of generality, suppose that $F_2 \setminus F_1 \neq \emptyset$. Thus, there is a $v \in F_2 \setminus F_1 \subseteq F_1 \Delta F_2$. Since K_n is complete, we have that $uw \in E(K_n)$ and vw is an edge between $V(K_n) \setminus (F_1 \cup F_2)$ and $F_1 \Delta F_2$. Let n be odd as follows. Note that there is an $u \in V(K_n) \setminus (F_1 \cup F_2)$. If $u, w \in V(K_n) \setminus (F_1 \cup F_2)$, by the above proof, $uw \in E(K_n)$ and $vw \in E(K_n)$, where $v \in F_2 \setminus F_1$. Suppose that exactly one $w \in V(K_n) \setminus (F_1 \cup F_2)$. Then

$|F_1 \setminus F_2| = n - 1 - |F_2| \geq n - 1 - \left(\left\lceil \frac{n}{2} \right\rceil - 1\right) = n - \frac{n+1}{2} = \frac{n-1}{2}$. Since $n \geq 5$, we have $|F_1 \setminus F_2| \geq 2$. Therefore, let

$u, v \in F_1 \setminus F_2 \subseteq F_1 \Delta F_2$. Since K_n is complete, we have that $uw \in E(K_n)$ and $vw \in E(K_n)$. By Theorem 10, K_n is g -good-neighbor (g -extra) $\left(\left\lceil \frac{n}{2} \right\rceil - 1\right)$ -diagnosable. \square

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