Supply Chain Permissible Delay in Payments Depending on the Order Quantity in Vendor —— buyer relationship inventory model

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Abstract: This study is to develop an improved inventory model and its solution algorithm to help the enterprises to advance their cost decreasing in a single vendor-single buyer environment with permissible delay in payments which depending on the ordering quantity and imperfect production. The goal of this paper is to derive simultaneously the optimal order quantity and the number of lots which are delivered from the vendor to the buyer. So for more closely conforming to the actual inventories and responding to the factors that contribute to inventory costs, our proposed new model can be the references to the business applications.

Keywords: Inventory model, Permissible delay in payments, Order quantity, Imperfect production

1. Introduction
In the previous EPQ/EOQ model was researched for a long time on both practical and academic sides. It arises that it is still extensively accepted by many industries today. Chiu et al. [1] have shown a study of the issue of production lot size problem with random defective rate, backlogging and rework considerations.

The problem for a buyer is how lot to order quantity. The economic order quantity model supposes that the buyer must pay for the purchased items when these items are received in traditionally.

This research developed an integrated vendor-buyer inventory model with a fixed demand and adjustable production rate under the condition of trade credit, which is dependent on order quantity. To optimize the joint expected total cost per unit time, two basic issues are determined in this study: how large the replenishment order should be, and how often the vendor should ship to the buyer during a production run. An interactive procedure is developed to find the optimal solution and numerical examples are presented to illustrate the proposed model.

2. Literature Review
Goyal [3] first developed an integrated inventory model for the single supplier and single customer problem. He concluded that the optimal order time interval and production cycle time can be obtained by supposing that the supplier’s production cycle time is an integer multiple of the customer’s order time interval. Banerjee [2] produced joint economic lot-size model from Goyal [4] by assuming that a vendor has a limited production rate and produces to order for a buyer on a lot-for-lot basis. Next, Goyal [3] extended Banerjee’s [2] model by less exacting the lot-for-lot policy and supposed that the vendor’s economic production quantity should be an integer multiple of the buyer’s purchase quantity that provided a lower joint total applicable cost. A review of related literature was proved by Goyal and Gupta [3]. Based on the same sized shipments to the buyer, Lu [6] presented a heuristic approach for the one-vendor multi-buyer integrated inventory case. He relaxed the assumption of Goyal [3] about completing a batch before starting transportations and investigated a model that allows transportations to occur during production period.

3. Problem Statement and Solution Methodology
In this section, we will develop the integrated inventory model under permissible delay in payments to take the order quantity into account. The payment for the items must be made immediately when the order quantity is less than the fixed quantity at which the delay in payments is permitted (Q < W). In order words, the fixed
trade credit period is permitted. Besides, this paper attempts to consider some alternations to move capital to match the policy of enterprise. We assume that the buyer will borrow 100% purchasing cost from the bank to pay off the account and the buyer does not return money to the bank during period of the inventory cycle when the buyer needs cash to pay off the account. The following nine assumptions and notation of formulation will be used in this paper.

In this section, the figure 1 describes the behavior of the inventory levels for the vendor and buyer, which is along the assumptions and the notations by Chung, Goyal and Huang which extend the integrated inventory model proposed by Ouyang et al. The joint total cost for the vendor and buyer consist of (a) the vendor’s total cost, and (b) the buyer’s total cost.

3.1 The Vender’s Total Annual Cost

The vendor produces with the rate of and carries a setup cost in each production run. The cycle time per production run for the vendor is . Therefore, the setup cost per year for the vendor is . At the end of the production period , the vendor’s accumulated inventory level increases to , and the vendor discontinue production from Ouyang shows as figure 1.

![Figure 1. The proposed inventory model for the vendor and buyer](image)

The accumulated reduction of inventory for the vendor, delivering shipments, each equals to , in a production run is given by

\[ L \times \left[ Q + 2Q + \cdots + (n-1)Q \right] \]  

(2)

The vendor’s inventory per production run can be calculated by removing the vendor’s accumulated consumption of inventory from the vendor’s total accumulation of inventory. The vendor’s average inventory per year can be determined as follows:

\[ \frac{\left( nQ(n-1) - \frac{1}{2} nQ(n-2) \right) L}{nL} \]

(3)

Considering the vendor’s holding cost, the holding cost per unit per year is , and the opportunity holding cost per unit per year is . Therefore, the vendor’s holding cost per year is

\[ \frac{C_u}{D} \left[ \frac{Q(n-1)}{2} + \frac{1}{\lambda} \frac{nQ(n-2)}{2} \right] \]

(4)

To motivate purchasing, the vendor offers a permissible delay period of time to the buyer. Hence, the opportunity cost per year for offering the permissible delay period is . The vendor produces one unit product for and sells to the buyer for . The vendor’s total annual cost can be presented as

\[ TC_v(n, Q) = \text{setup cost} + \text{holding cost} + \text{opportunity cost} \]

\[ = \frac{S}{nQ} + C_uQ(0_0 + I) \left[ \frac{n-1}{2} + \frac{1}{\lambda} \left( \frac{1}{(2-n)} \right) \right] + C_dP_iD \]

(5)

Hence, the , so the Eqs. (4) and (5) will be modified as follows:

\[ TC_v(n, L) = \frac{S}{nL} + C_uQ(0_0 + I) \left[ \frac{n-1}{2} + \frac{1}{\lambda} \left( \frac{1}{(2-n)} \right) \right] + C_dP_iD \]

(6)

3.2 The Buyer’s Total Annual Cost

The buy’s inventory model is developed and the payment for the items must be made immediately when the
order quantity is less than the fixed quantity at which the
delay in payments is permitted \((Q < W)\). In order words, the fixed trade credit period \(m\) is permitted. To the buyer, the total cost consists of the following elements. Two situations may we appear.

(A) \(W/D \leq m\) and (B) \(W/D > m\).

Suppose that

The buyer’s annual ordering cost is \(S_{ord}/Q = S_{ord}/L\). And the buyer’s annual stock holding cost (excluding interest charges) is \(DLC_{bh}h_{c}/2\). There are three cases to occur in interest payable per year.

Case I : \(0 < L < W/D\), shown in Figure 2.

In this case, the buyer must pay the amount of purchasing cost as soon as the items were received since \(Q < W\). According to assumption (9), the buyer will borrow 100% purchasing cost, \(C_{DL}\), from the bank to payoff the account with rate \(I_{23}\) and return money to the bank at the end of the inventory cycle. So, the loan period is \(L\).

Buyer’s interest payable cycle

\[C_{23}I_{23}DL + C_{23}I_{23}DL\] (year).

Figure 2. The total accumulation of interest payable when \(0 < L < W/D\).

Case II : \(W/D < L \leq m\).

We show that the buyer’s annual total cost function, \(TC_{B}(Q, L)\), is given by

\[TC_{B}(Q, L) = TC_{B1}(Q, L)\] if \(0 < L < W/D\)

\[TC_{B}(Q, L) = TC_{B2}(Q, L)\] if \(W/D \leq L \leq m\)

\[TC_{B}(Q, L) = TC_{B3}(Q, L)\] if \(m < L\)

(7)

where

\[TC_{B1}(QL) = S_{ord}/L + DLC_{bh}h_{c}/2 + F/C_{DL} - DLPI_{23}(m - L)/2\]

\[TC_{B2}(QL) = S_{ord}/L + DLC_{bh}h_{c}/2 + F/C_{DL} - DlP_{23}(m - L)/2\]

\[TC_{B3}(QL) = S_{ord}/L + DLC_{bh}h_{c}/2 + F/C_{DL} - DlP_{23}(m - L)/2\]

(9)

(10)

(A) Suppose that \(W/D > m\).

If \(W/D > m\), Eqs. (8) will be modified as follows:

\[TC_{B4}(QL) = TC_{B2}(QL)\] if \(0 < L < W/D\)

\[TC_{B4}(QL) = TC_{B3}(QL)\] if \(W/D < L\)

\[TC_{B4}(QL) = TC_{B3}(QL)\] if \(m < L\)

(11)

3.3 The Joint Total Annual Cost

Owing to the above investigations on the related cost between the vendor and the buyer, the joint total annual cost function, \(JTC(n, L)\), can be written as

(A) Suppose that \(W/D \leq m\).

\[JTC(n, L) = \begin{cases} JTC_{1}(n, L) & \text{if } 0 < L < W/D \text{ or } W/D < L < m \\ JTC_{2}(n, L) & \text{if } m < L \end{cases}\]

(12)

(B) Suppose that \(W/D > m\).

\[JTC(n, L) = \begin{cases} JTC_{4}(n, L) & \text{if } 0 < L < W/D \text{ or } W/D < L < m \\ JTC_{2}(n, L) & \text{if } m < L \end{cases}\]

(13)

Where

\[JTC_{4}(n, L) = JTC_{4}(n, L)\]

and

\[JTC_{2}(n, L) = JTC_{2}(n, L)\]

Case (A) : \(W/D \leq m\).

4. Solution Procedure

4.1 Determination of the optimal number of shipments \(n\) for any given \(L\)
In this section, we examine the effect of \( n \) on the joint total annual cost function \( JTC(n, L) \) for given \( L \). We take the first-order and second-order partial derivatives of \( JTC(n, L) \), for \( i = 1, 2, 3 \) with respect to \( n \), it follows that
\[
\frac{\partial JTC(n, L)}{\partial n} = \frac{D}{2} \left( x - \frac{S}{L} - \frac{S}{n + L} \right) \quad \text{for} \quad i = 1, 2, 3 \quad (14)
\]
\[
\frac{\partial^2 JTC(n, L)}{\partial n^2} = \frac{2S}{n + L} > 0 \quad \text{for} \quad i = 1, 2, 3 \quad (15)
\]

Therefore, for fixed \( p \) and \( L \), \( JTC(n, L) \) is strictly convex on \( n > 0 \). Thus, in each production run, determining the optimal number of shipments \( n^* \), is simplified to get a local optimal solution.

4.2 Determination of the optimal number of shipments \( L \) for any given \( n \)

At last part, note that the above measure on \( L \) is to get the unique closed-form solution for the optimal \( L \). And by comparing, we have the following theorem.

**Theorem:** For any given \( n \), we can obtain the following results.

I. If
\[
2S + n(S_p + F) < n\exp\left(-\frac{p(D)}{2}\right)^{-\frac{1}{2}[x(1 - \frac{1}{2}(2 - n) + C_p + 2C_f + p)]} < 2S + n(S_p + F) < n\exp\left(-\frac{p(D)}{2}\right)^{-\frac{1}{2}[x(1 - \frac{1}{2}(2 - n) + C_p + 2C_f + p)]}
\]

then \( L^* = L_1 \)

II. If
\[
2S + n(S_p + F) < n\exp\left(-\frac{p(D)}{2}\right)^{-\frac{1}{2}[x(1 - \frac{1}{2}(2 - n) + C_p + 2C_f + p)]} < 2S + n(S_p + F) < n\exp\left(-\frac{p(D)}{2}\right)^{-\frac{1}{2}[x(1 - \frac{1}{2}(2 - n) + C_p + 2C_f + p)]}
\]

then \( L^* = L_2 \)

III. If
\[
2S + n(S_p + F) > n\exp\left(-\frac{p(D)}{2}\right)^{-\frac{1}{2}[x(1 - \frac{1}{2}(2 - n) + C_p + 2C_f + p)]} > 2S + n(S_p + F) > n\exp\left(-\frac{p(D)}{2}\right)^{-\frac{1}{2}[x(1 - \frac{1}{2}(2 - n) + C_p + 2C_f + p)]}
\]

then \( L^* = L_3 \)

IV. If
\[
2S + n(S_p + F) = n\exp\left(-\frac{p(D)}{2}\right)^{-\frac{1}{2}[x(1 - \frac{1}{2}(2 - n) + C_p + 2C_f + p)]} = 2S + n(S_p + F) = n\exp\left(-\frac{p(D)}{2}\right)^{-\frac{1}{2}[x(1 - \frac{1}{2}(2 - n) + C_p + 2C_f + p)]}
\]

then \( L^* = W/D \)

V. If
\[
2S + n(S_p + F) = n\exp\left(-\frac{p(D)}{2}\right)^{-\frac{1}{2}[x(1 - \frac{1}{2}(2 - n) + C_p + 2C_f + p)]} = 2S + n(S_p + F) = n\exp\left(-\frac{p(D)}{2}\right)^{-\frac{1}{2}[x(1 - \frac{1}{2}(2 - n) + C_p + 2C_f + p)]}
\]

then \( L^* = m \)

Theorem display that an integrated inventory model with some fixed permissible delay period \( D \), fixed quantity at which the delay in payments \( W \) and the adjusted demand \( D \), the buyer’s replenishment time interval \( L \) increases when the vendor’s setup cost \( S_p \), the buyer’s ordering cost \( SB \) or the transportation cost \( F \) increase.

5. Conclusion

This chapter investigates the production/inventory situation into an integrated inventory model under permissible delay in payments depending on order quantity. We formulate an integrated vendor–buyer inventory model in this chapter with the assumptions that the market demand is fixed to the retail price and the credit terms are linked to the order quantity. Finally, numerical examples are presented to explain the solution procedure, and sensitivity analysis of the optimal solution is also indicated. These results express that applying the permissible delay in payments depending on order quantity strategy between the vendor and the buyer can promote the cost reduction. Based on our result analysis, it was found that a longer trade credit term can decrease costs for the complete supply chain. They also illustrate that if the JIT cooperation between vendor and buyer could be implemented well, the cost of the complete inventory model will decrease.

**Reference**


