The Impact of Multi-path Angular Spread of MIMO System Capacity

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Abstract: Multi-path angular spread in the MIMO system is analyzed in this paper. The relation between correlation coefficient with the arrival scattering angle and the average arrival direction angle is obtained, and on this basis the multi-path correlation coefficient on the impact of system capacity is verified by the simulation which provides a theoretical basis for further research on expanding channel capacity.

Keywords: MIMO; multi-path angular spread; channel; capacity

1 Introduction

Multiple Input Multiple Output (MIMO) systems can increase system capacity and improve the signal transmission rate and spectrum efficiency without increasing the transmitter power and bandwidth. The literature[1-2] analyzes the MIMO system channel capacity in the environment of single-user narrow-band Raleigh fading, and proves that if the decline is independent and identically distributed among different antennas, even if the transmitter channel parameters are unknown, the system channel capacity grows with the number of the antenna linearly. Literature [3-5] has derived the approximate formula of the ergodic channel capacity in some cases (such as the small signal to noise ratio or large SNR) respectively. Recently, the closed solution of the capacity expression is given in literature [6-7], but all these results are complex and the calculation is larger. Most of these documents analyze channel capacity with the independent and identically distributed between the antennas, but in the actual spread environment, certain relevance among declines exists because of effect of the antenna spacing and scattering objects around the antenna array and other factors, which will cause decline of channel capacity.

In order to analyze the impact of antenna spacing and scattering angle channel physical parameters on the channel capacity of multi-antenna systems, ULA(Uniform line Array, ULA) is used in the receiver in the paper. The formula relation between correlation coefficient with the scattering angle and the average arrival angle relations is given, then a system of frequency-selective MIMO channel model is build, and thus the expression of multi-path channel capacity in MIMO system is derived. Also the effect of the scattering angle and the antenna spacing and other physical parameters on channel capacity is analyzed, after that the final simulation findings and conclusions are given.

2 Multi-path Angular Spread

Plane wave received by array antenna usually is constituted of direct component and multi-path scattering component. Supposing plane wave’s incidence elevation $\gamma = 90^\circ$, the received signal’s power density distribution along the angle of arrival $\phi$ in the two-dimensional plane is $p(\phi)$, multi-path scattering component’s total power is $p_s$, and its power density distribution is $p_s(\phi)$ . Simultaneously supposing direct component without shadow whose power is $p_d(\phi)$ incidence along multi-path scattering component average direction $\phi_0$, its power density distribution is $p_s(\phi) = \sigma(\phi - \phi_0)$, while the total normalized received power density distribution is[8]:

$$p(\phi) = \frac{P_d(\phi) + K\sigma(\phi - \phi_0)}{1 + K}$$

(1)

where $K = p_d/p_s$ is the Rice factor, $\sigma(\bullet)$ is Dirac coefficient.

The angle expansion of received signal along the direction of wave arrival angle is caused by multi-path scattering which is closed to signal power density distribution $p(\phi)$ . When elevation of the incidence plane wave $\gamma = 90^\circ$, the expansion of multi-path angle is defined as

$$\Delta \phi = \sqrt{1 - \frac{|P_d|^2}{P_s^2}}$$

(2)
where \( F_n = \int_0^{2\pi} p(\phi) \exp(jn\phi)d\phi \) is the \( n \)th complex Fourier coefficient of \( p(\phi) \).

Isotropy multi-path scattering is satisfied with \( \Lambda_\phi = 1 \), anisotropy multi-path scattering is satisfied with \( 0 < \Lambda_\phi < 1 \).

In the past, in order to calculate conveniently, usually see \( p_\phi(\phi) \) as uniform distribution where \( \phi \in [0,2\pi] \), but if we assume the multi-path scattering is isotropy, the analysis of second-order moment have a greater error. Actually, \( \text{von Mises} \) distribution with wide adaptability is very suitable to describe distribution of random variables whose modulus is \( 2\pi \). Supposing the power density distribution of multi-path scattering component which along the wave arrival angle \( \phi \) obey the \( \text{von Mises} \) distribution, that is

\[
p_\phi(\phi) = \frac{e^{\alpha \cos(\phi-\phi_0)}}{2\pi I_0(\alpha)}, \phi \in [0,2\pi], \alpha \geq 0	ag{3}\]

where \( I_0(\alpha) \) is the first kind of zero-order amendment Bessel function, \( \phi_0 \) is average direction of scattering, \( \alpha \geq 0 \) is the parameter which denote the distribution concentration degree of scattering angle.

When the concentration degree parameter \( \alpha \rightarrow 0 \) is corresponded to \( \Lambda \phi \rightarrow 1 \), \( p_\phi(\phi) \) tends to uniformly distributed in \([0,2\pi]\); when \( \alpha \) is bigger, \( p_\phi(\phi) \) tends to Gaussian distribution; when \( \alpha \rightarrow \infty \) is corresponded to \( \Lambda \phi \rightarrow 1 \), \( p_\phi(\phi) \) tends to \( \sigma(\phi-\phi_0) \). When the received signal has both multi-path scattering and direct component, there is

\[
F_0 = \int_0^{2\pi} p(\phi)d\phi = 1	ag{4}
\]

\[
F_1 = \int_0^{2\pi} p(\phi)e^{\alpha \phi}d\phi = \frac{1}{1+K} \left[ K + \frac{I_0(\alpha)}{I_0(\alpha)} \right] e^{\alpha \phi_0}	ag{5}
\]

where \( I_0(\alpha) \) is the first kind of first order amendment Bessel function. Now angle of multi-path scattering signal with the stable direct scattering component is extended to

\[
\Lambda_\phi(\alpha,K) = \frac{1}{1+K} \left[ 1 - \frac{I_1(\alpha)}{I_0(\alpha)} \right] + 2K \left[ 1 - \frac{I_1(\alpha)}{I_0(\alpha)} \right]	ag{6}
\]

where \( \alpha \) reflects the concentration degree of power density distribution of the multi-path scattering component when the center of \( \phi_0 \); \( K \) reflects the ratio between the direct power and the total power of scattering volume.

### 3 The Relation Between Correlation Coefficient with Scattering Angle and the Average Direction of Arrival Angle

Assuming the scattering angle of signal reaching antenna arrays is \( \Delta \), and is uniformly distributed in \([\phi-\Delta, \phi+\Delta]\), \( \phi \) is the signal’s average arrival direction angle. For the space of two adjacent antenna of uniform linear array \( d \), while the space correlation coefficient of two adjacent antenna in ULA antenna array is

\[
R(d) = \frac{1}{2\Delta} \int_{\phi-\Delta}^{\phi+\Delta} \exp\left( \frac{2\pi d}{\lambda} \sin(\gamma) \right) d\gamma
\]

that is

\[
R(d) = \frac{1}{2\Delta} \int_{\phi-\Delta}^{\phi+\Delta} \exp\left[ \frac{2\pi d}{\lambda} (\sin(t) \cos(\phi) + \cos(t) \sin(\phi)) \right] dt
\]

When the scattering angle is smaller, there is \( \sin(t) = t \), \( \cos(t) = 1 \). So this formula (8) can be simplified as

\[
R(d) = \exp\left[ \frac{2\pi d}{\lambda} \sin(\phi) \right] \sin\left[ \frac{2\pi d}{\lambda} \cos(\phi) \right]	ag{9}
\]

Based on the formula above, we can build spatial correlation matrix \( S \) when receiver is uniform linear array.

\[
S = \begin{bmatrix}
1 & R(d) & \cdots & R(M-1)d \\
R(d) & 1 & \cdots & R(M-2)d \\
\vdots & \vdots & \ddots & \vdots \\
R(M-1)d & R(M-2)d & \cdots & 1
\end{bmatrix}	ag{10}
\]

According to the analysis, spatial correlation matrix \( R \) and \( S \) are both non-negative set, and their eigenvalues are real numbers greater than zero.

Figure1 shows the relationship between receive correlation coefficient with antenna spacing and scattering angle. The received signal’s average arrival direction angle \( \phi = 60^\circ \), the number of receiving antennas \( M=4 \). It can be seen that, the bigger of the scattering angle, the smaller of required antenna spacing when received spatial correlation coefficient is minimum, and with the increasing of scattering angle, the range of space-related is gradually reduced. By increasing the distance between the receiving antenna, received spatial correlation coefficient will be
gradually decreasing until convergent to zero. When the receiving antenna spacing increases to a certain extent, received correlation coefficient Amplitude is small which has little effect on system capacity and performance.

4 Analysis of Multi-path Channel Capacity

4.1 Channel Correlation Model

The channel model of frequency selectively MIMO system is:

\[ y(n) = \sum_{i=0}^{L-1} H_i(n)x(n-i) + n(n) \]  

(11)

Where channel matrix of each delay path is combinedly written as H. The frequency selectively fading MIMO channel described by (11), has the relevance of different delay between the channel states which caused by pulse shaping filter of transceivers at both ends and response characteristics of the actual physical channel, that is to say, even if the space physical channel meets the broad dependent scattering model, it can not ignore the relevance of delay path in generalized channel caused by the factors such as pulse shaping filter of transceivers at both ends. We can call the relevance of time delay between the path components as the multi-path relevance.

Promoting the Kroneeker related model in the case of flat decline into frequency selectively channel. Assuming that the path delay between the correlation matrix is indicated by L×L-dimensional Hermite matrix \( \phi_{op} \), there is:

\[ R_H = \phi_{op} \otimes R_c \otimes R_r \]  

(12)

This is Kronecker correlation model of the frequency selectively decline channel.

4.2 Channel Correlation Model

Assuming that the antennas are independent and uniform distribution, that the channel capacity

\[ C = E_H \left\{ \log_2 \det(I_{N_c} + \frac{\rho}{N_t} H_H H) \right\} \]  

(13)

where \( E_H \) is the expectations of matrix \( H \), \( I_{N_c} \) is unit matrix, \( \rho \) is average ratio of signal to noise, \( N_t \) is the number of transmission antennas, \( H \) is channel spreading matrix.

To consider the number of carrier and the relevance of decline, formula (13) can be written

\[ C = \frac{1}{N_c} \sum_{k=1}^{N_c} E_H \left\{ \log_2 \det(I_{N_c} + \frac{\rho}{N_t} \gamma_s R_c H_R R_R H_H) \right\} \]  

(14)
Where $N_c$ is the number of carrier, $R_t$ and $R_r$ are the correlation coefficient of transmitter and receiver.

$$R_t = \frac{1}{N_r} E\{HH^H\} = \frac{1}{N_r} \sum_{r=1}^{N_r} E\{h_r h_r^H\}$$  \hspace{1cm} (15)$$

$$R_r = \frac{1}{N_r} E\{(H^H H)^r\} = \frac{1}{N_r} \sum_{r=1}^{N_r} E\{h_r h_r^H\}$$  \hspace{1cm} (16)$$

$h_r$ and $h_t$ indicate the coefficient of fading channel between the $c$th and $r$th transmitting antenna and each receiving antenna.

$$\gamma_k = \frac{1}{L}
\sum_{l=0}^{L-1} \phi_{ap} [I_l I_l^*] e^{j(l-l_0)\phi}$$

$$= 1 + 2 \sum_{l=0}^{L-1} \sum_{l'=0}^{L-1} R_l \phi_{ap} [I_l I_l^*] e^{j(l-l_0)\phi}$$  \hspace{1cm} (17)$$

Multi-path are independent to each other, that is $\phi_{ap}$ is a diagonal matrices, thus for arbitrary $k$, there is $\gamma_k = 1$. Then only considering the situation of the existence of receiver spatial correlation, that is $R_l = 1$, the system capacity can be reduced to

$$C = E_H \{ \log_2 \det ( I_{N_r} + \frac{\rho}{N_r} R H H^H) \}$$  \hspace{1cm} (18)$$

Figure 3 shows the curve of relationship between channel related capacity with spatial correlation coefficient and multi-path correlation coefficient. $\alpha$ is defined as spatial correlation coefficient. When SNR=10dB , $\alpha=0.9, 0.5, 0.1$, it is easy to see from the figure that multi-path correlation will reduce the system.

5 Conclusion

The capacity of MIMO systems is influenced by the relevance of array antenna largely. The greater the spatial correlation coefficient, the more the system capacity will be reduced. When the spatial correlation coefficient is zero, the system obtains the maximum capacity. In the actual spreading environment, the correlation can not be ignored, so how to reduce or eliminate such relevance is the main direction of further studies.

References


