The Application of FTA and Fuzzy Theory in Fault Diagnosis of Boeing 777 System

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Abstract: A new method is proposed in this paper to implement the fault diagnosis of Boeing 777 system. Considering the structure and function of modern aircraft, the characteristics of systems are often stochastic. But the uncertainty problem cannot be solved with the traditional FTA in the machinery diagnosis. In this paper, the FTA and fuzzy theory are combined and applied on the fault diagnosis of Boeing 777 air-conditioning system. The experimental results validate the effectiveness of the proposed method.

Keywords: fault diagnosis; fuzzy theory; FTA; Boeing 777

1 Introduction
Fault Tree Analysis (FTA) is a logical and diagrammatic method for evaluating the safety and reliability of complex systems[1]. It provides a deductive functional development of a specific final undesired event through logic statements of the conditions which could cause the event, and it is useful for system reliability analysis and risk quantification since which illustrates the failure logic of a system. Fault Tree Analysis involves an accident model that interprets the relationships among the malfunctioning of components and the observed symptoms using gates. A fault tree is composed of a number of symbols, describing different types of events, which are operated on by logic gates. The logical gates which are most frequently used to develop fault trees include the basic 'and' and 'or' logical expressions. Since it can provide a qualitative analysis and a quantitative evaluation of faults, in recent years, Fault Tree Analysis has been recognized as a very powerful technique for fault diagnosis[2].

For the Boeing 777 aircraft air-conditioning system, there is a series uncertainty between the system fault events and fault reasons, this uncertainty problem is mainly manifested in Fuzziness and randomness. This requires the introduction of fuzzy theory to deal with these uncertain causes of the fault events. This paper will combine fuzzy theory and fault tree analysis to set up a better method to reflect the nature of the analysis which is a more flexible and adaptable method to fault analysis of aircraft air-conditioning system.

2 Fault Tree and Structure function
A Fault trees include the basic “and” and “or” logical gates to connect the logical relationship, which can use the structure function as a mathematical tool to establish fault tree mathematical expression for a qualitative analysis and a quantitative evaluation of faults.

We assume that the components and systems can only have two states, normal or failure, and the fault components are independent.

Set $X_i$ is the state variables of bottom event, values are 0 or 1.Set $\phi(\chi)$ is the state variables of top event, values are 0 or 1. The $X_i$ and the $\phi(\chi)$ can defined as follows:

$$X_i = \begin{cases} 1, & \text{(components fault)} \\ 0, & \text{(components normal)} \end{cases} \quad (1)$$

$$\phi(\chi) = \begin{cases} 1, & \text{(system fault)} \\ 0, & \text{(system normal)} \end{cases} \quad (2)$$

The top event of fault tree is the fault states, that is. With this state, the bottom events are fault states, that is $X_f=1$. So the state of top event $\phi(\chi)$ depends on the state of the bottom event $X_i$ entirely, the top event state is necessarily a function of the state of the bottom event, that is, $\phi(\chi) = \phi(X_1, \ldots, X_n)$. Where $\phi(\chi)$ is called the structure function of fault tree, which is a logical function of system states. “And” gate structure function:

$$\phi(\chi) = \bigwedge_{i} X_i = \min\{x_1, x_2, \ldots, x_n\} \quad (3)$$

Where $i = 1, 2, \ldots, n$, $n$ is the number of bottom event. When $X_i = 1$ or $X_i = 0$ structure function as follows:

$$\phi(\chi) = \prod_{i} X_i \quad (4)$$

"Or" gate structure function:
\[ \phi(X) = \bigcup_{i=1}^{n} x_i = \max\{x_1, x_2, \cdots, x_n\} \]  

(5)

Where \( i = 1, 2, \cdots, n \), \( n \) is the number of bottom event. When \( X_i = 1 \) or \( X_i = 0 \) structure function as follows:

\[ \phi(x) = \sum_{i=1}^{n} X_i = 1 - \prod_{i=1}^{n} (1 - X_i) \]  

(6)

3 Based on Fuzzy Theory of Fault Tree Analysis

3.1 Qualitative Fault Tree Analysis

Qualitative fault tree analysis is intended to find out all the possible failure modes which leading to the top event occur, that is, to find all the fault tree minimal cut sets, from which to determine the weakest parts in the system and take measures to remedy the problem[3].

We assume that \( X_1, X_2, \cdots, X_n \) is the bottom event, \( C \in \{X_1, \cdots, X_n\} \) is the set of the bottom event. When all the bottom events, the top event is taken place, we get that \( C \) is a fault tree cut sets. If \( C \) is a cut-set, but remove any bottom event is not a cut-set, such a cut set is called minimal cut sets.

There are mainly two methods to solve the minimum cut set: upstream and downstream. Following this line of law as to strike a fault tree minimal cut set method, the specific idea is: only on the upper and lower adjacent two sets of view, the only increase the order of cut sets (the number of the bottom event in a cut set), does not increase the cut set number; only increase the number of cut-set order. The process to solve the minimum cut set is like this: sequential logic gate output events will be replaced as input events. If it is a “and” gate, we put the input events in the same line. If it is a “or” gate, we put the own input events in the same line up. When all of the events are replaced, we get all the cut sets. Then apply the rules of set operations to be simplify, remove the part of those non-minimum cut set, we will get the full fault tree minimal cut sets.

In this paper, all the bottom events of fault tree are confirmed. Qualitative fault tree analysis of the air-conditioning systems for the Boeing 777 is still use the ordinary fault tree analysis.

3.2 Based on Fuzzy Theory Quantitative Analysis of Fault Tree

In this paper, all of the bottom events are independent. So fault tree form is still used to describe the structure function:

\[ \Phi(X) = \bigcup_{i=1}^{k} k_i(X) = \bigcup_{j=1}^{m} \bigcap_{i=1}^{k} x_i \]  

(7)

Where \( i = 1, 2, \cdots, k \), \( k \) is the number of minimal cut set.

Now we assume that \( \bar{g} \) is fuzzy probability of the top event occur, which can be calculated by \( \bar{g} = (\bar{q}_1, \bar{q}_2, \cdots, \bar{q}_i) \) and \( \Phi(X) \).

Where \( \bar{q}_i \) is the subordinate function of the component fault. Therefore, calculating the probability of occurrence of top event can be divided into the following steps:

Step 1. Calculate each individual probability of occurrence of the minimum cut set.

\[ P[k(X)] = \bigcap_{i=1}^{k} \bar{q}_i \]  

(8)

Where \( n \) is the number of the bottom event in minimum cut set I

Step 2. Calculate probability of occurrence of a single minimum cut set.

\[ F_i = \sum_{j=1}^{k} P[k(X)] \]  

(9)

Step 3. Calculate probability of occurrence of two minimal cut set:

\[ F_2 = \sum_{i=1}^{k} \{p[k(X)] \cap p[k(X)]\} \]  

(10)

Step 4. With this method, we can calculate \( F_3, F_4, F_5, \cdots \)

Then the fuzzy probability of top event occurs as follows:

\[ \bar{g} = F_1 - F_2 + F_3 - F_4 + \cdots + (-1)^{n-1} F_n \]  

(11)

4 The Method of Fuzzy Fault Tree Fault Diagnosis

4.1 The General Concept of Fuzzy Fault Diagnosis

The traditional theory considers that the failure cause and fault symptom is one to one corresponding. If a fault symptom occurred, it must be a certain causes, which is obviously not in line with the actual situation. Fuzzy fault diagnosis considers that a symptom are often caused by a variety of failures; while a fault cause has different influence on a variety of fault symptom. The method of fuzzy reasoning is this: According to the different levels of causality (fuzzy matrix) between failure cause and fault symptom. Based on considering all fault symptoms, we diagnose the possible reasons for equipment failure and the possible extent[5].

Fuzzy fault diagnosis is use to find the cause of various failures by the subordinate degree of some symptoms to failure characterize the variety possibilities of failure cause.

4.2 Construction of Fuzzy Matrix

According to the basic principles of fuzzy transform and
the relationship between fault symptoms and fault causes. We defined $U$ as the possible of fault symptom set:

$$U = \{U_1, U_2, \cdots, U_m\}$$  \hspace{1cm} (12)

Where $m$ is the total number of fault symptom types. Defined $V$ is the possible of fault cause set:

$$V = \{V_1, V_2, \cdots, V_n\}$$  \hspace{1cm} (13)

Where $n$ is the total number of fault cause types. According to the importance of each element. The system’s fault symptoms fuzzy vector can be constructed:

$$u = (u_1, u_2, \cdots, u_m)$$  \hspace{1cm} (14)

If the symptom sample is caused by fault cause $V_i$, and $v_i$ is the subordinate degree of $V_i$, then the fault cause’s fuzzy vector

$$v = (v_1, v_2, \cdots, v_n)$$  \hspace{1cm} (15)

We use the function $v = u * R$ to show relationship between $U$ and $V$, that is fuzzy decision function. $R$ is the fuzzy matrix:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$  \hspace{1cm} (16)

$R$ is a fuzzy relation from the fault domain $U$ to the symptoms domain $V$. $u_k = (u_1, v_1) = r_0$ is used to describe the subordinate degree, R is called Fuzzy Matrix then, $r_0 = 0$ the failure of discourse $U$ to the of the. The degree of membership. R is called fuzzy diagnosis matrix. Then The occurrence $u_k$ have no relationship with $v_j$, the larger $r_j$ is, the relationship more closely, then $r_0 = 1$, the fault must cause the symptoms occur.

### 4.3 Important Degree Analysis

Important Degree is calculated and formulated as:

$$I_u = \frac{E[\Phi(X)]}{g}$$  \hspace{1cm} (17)

Important Degree Analysis is based on statistical data and obtained by logical inference Analysis. $I_u = 0$, where fault symptom $k$ occur, fault cause $j$ does not occur

$I_u = 1$, where fault symptom $k$ occur, fault cause $j$ must occur.

The value of $I_u$ is the important degree between fault symptoms and fault causes. In this paper we use $I_u$ as the subordinate degree for reducing the subjective determination, and combining the important degree of fault cause and fuzzy decision for multiple fault analysis in the system.

### 4.4 The Fault Symptom Fuzzy Vector

Fault symptom fuzzy vector $u=(u_1, u_2, \cdots, u_n)$, Reflecting the various fault symptoms in the fuzzy decision, it directly affects the fuzzy decision results. In general, the value of fault symptom fuzzy vector is the fault of the various components of the severity of symptoms, usually invites experts to rate by their experience with statistical methods, the degree of failure as shown in Table 1.

<table>
<thead>
<tr>
<th>Fault Level</th>
<th>Rating Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very serious</td>
<td>1.0</td>
</tr>
<tr>
<td>Serious</td>
<td>0.8</td>
</tr>
<tr>
<td>General</td>
<td>0.6</td>
</tr>
<tr>
<td>Minor</td>
<td>0.3</td>
</tr>
<tr>
<td>Little</td>
<td>0.1</td>
</tr>
<tr>
<td>No</td>
<td>0</td>
</tr>
</tbody>
</table>

### 5 Examples of Fault Diagnosis

In this paper, the example uses the Boeing 777 air-conditioning system. The fault symptom is the compressor outlet temperature is too high. Inspection of the equipment by the analysis of fault diagnosis, there are three fault symptoms [5]:

$U = \{u_1, u_2, u_3\} = \text{(Main heat exchangers cooling inefficient, punching the cold air flow inside the pipe is too small, compressor overloaded)}$.

According to the experience of the aircraft line maintenance, Fault symptom fuzzy vector is established by worker.

$U = (u_1, u_2, u_3) = (0.6, 0.3, 0.1)$

From the previous maintenance experience in the work to select the possible cause of the malfunction fault symptoms set as follows:

$V = \{V_1, V_2, \cdots, V_n\} = \text{(Main heat exchangers dust, CTC control failure, non-return valve failure)}$.

We use the three fault symptoms as the top event to establish fault tree respectively, and important degree analysis. Using the formula (17) to get the three important degree set:
\[
I_1 = (I_{11}, I_{12}, I_{13}) = (0.51, 0.13, 0.10)
\]
\[
I_2 = (I_{21}, I_{22}, I_{23}) = (0.23, 0.48, 0.15)
\]
\[
I_3 = (I_{31}, I_{32}, I_{33}) = (0.00, 0.45, 0.00)
\]

According to \( r_{ij} = I_{kj} \), we obtain the fuzzy matrix \( R \):
\[
R = \begin{bmatrix}
0.51 & 0.13 & 0.10 \\
0.23 & 0.48 & 0.15 \\
0.00 & 0.45 & 0.00
\end{bmatrix}
\]

After getting \( R \), the fuzzy decision function as:
\[
v = u \ast R = [0.375, 0.267, 0.105]
\]

The largest element of the vector is the most probable fault. So in this example, the main heat exchangers dust much the most likely cause of the malfunction, the result is in line with the actual situation.

6 Conclusion
In this paper, the FTA and fuzzy theory are combined to solve some uncertainty problems and make up the traditional fault tree analysis theory inadequate. At last by the air-conditioning system on the Boeing 777 fault diagnosis example to verify the practicality and reliability.

References